第26卷第4期 上海师范大学学报(自然科学版) Vol. 26, No. 4
1997年12月 J. of Shanghai Teachers Univ. (Natural Sciences) Dec. 1997

On Uniquely r-bipancyclic Graphs

Shi Yongbing *Sun Jiashu*

Abstract Let $r \geqslant 4$ be an even integer. A bipartite graph G if order $2n$ is said to be uniquely r-bipancyclic if G contains exactly one cycle of every even length t, $r \leqslant t \leqslant 2n$, and G contains no cycle of length less than r. If G is a uniquely r-bipancyclic graph, then G is called an r-graph. In this paper, we prove that there exist exactly six outerplanar r-UB-graphs and exactly twelve r-UB-graphs of order $2n$ and size $2n+m$ for $m \leqslant 3$.

Key words cycle; bipartite graph; uniquely r-bipancyclic graph

1 Introduction and Notation

In 1973, Entringer R C raised the problen of determining which graph G is uniquely pancyclic[1], that is, which G contains exactly one cycle of each length t, $3 \leqslant t \leqslant |V(G)|$. In 1983, Yap H P and Teo S K generalized the notion of a uniquely pancyclic graph and defined a notion of a uniquely r-pancyclic graph[2]. A graph G of order v is said to be uniquely r-pancyclic if G contains exactly one cycle of length t, for each $r \leqslant t \leqslant v$, and G contains no cycle of length less than r. In [3] and [4], several important results of uniquely r-pancyclic graphs have been obtained. The main objective of this paper is to study analogous questions relating to bipartite graphs.

Let $r \geqslant 4$ be an even integer. A bipartite graph G of order $2n$ is said to be uniquely r-bipancyclic if G contains exactly one cycle of every even length t, $r \leqslant t \leqslant 2n$, and G contains no cycle of length less than r. If G is a uniquely r-bipancyclic graph, then G is called an r-UB-graph. We

Received: 1997-06-28

First author Shi Yongbing, male, professor, Department of Mathematics, Shanghai Teachers University, Shanghai, 200234

usually abbreviate '4-UB-graph' to 'UB-graph'.

The main results of this paper are stated in the abstract above.

We shall require the following definitions and notation.

Suppose that G is a graph drawn on a plane P. If C is a cycle of G, then C divides P into two regions. The bounded (unbounded) region is called the interior (exterior) of C and is denoted by int C (ext C).

Let G be an r-UB-graph and let C be the Hamilton cycle of G. Then G is obtained from C by adding some edges joining some pairs of vertices of C. We assume that the edges of G other than those edges of C are drawn in the interior of C, and we call these edges the bridges of G. Two bridges b and b' are said to be skew if they have no common end-vertex and they cross each other.

Let G be an r-UB-graph and let b be a bridge of G. Then G has precisely two cycles containing b and containing no other bridges. Of these two cycles, the one that is of smaller length is called the side cycle of b and is denoted by $C(b)$. If the length of $C(b)$ is k, then b is called a k-bridge and k is called the order of b. If there are no bridges in int $C(b)$ and there is no other bridge b' such that b and b' are skew, then b is called a strict bridge and $C(b)$ is called a strict side cycle. If C' is not a side cycle, then C' is called an inner cycle. If C' is a cycle containing only one bridge b, then the other cycle containing only b is denoted by $\overline{C'}$. A cycle of length k is called a k-cycle.

2 Outerplanar r-UB-graphs

In this section, the bridge having order 2^i+2 is denoted by b_i. We shall determine all outerplanar r-UB-graphs. We first prove the following:

Lemma 2.1 If G is an outerplanar r-UB-graph and G is not a cycle, then $r=4$.

Proof Let C' be the $(2n-2)$-cycle of G. Since G is outerplanar, C' contains only one bridge. Thus $\overline{C'}$ is a 4-cycle. Hence $r=4$. □

From this lemma, it follows that the only outerplanar r-UB-graphs, which are not cycles, are the UB-graphs. The following lemma enables us to find all outerplanar UB-graphs (See Theorem 2.3).

Lemma 2.2 If G is an outerplanar UB-graph having $m \geqslant 3$ bridges, then G contains exactly one (2^i+2)-strict bridge b_i for each $1 \leqslant i \leqslant m$.

Proof We prove this lemma by induction on i. Since G has exactly one $(2n-2)$-cycle and one $(2n-4)$-cycle and G is outerplanar, G has exactly one (2^1+2)-strict bridge b_1 and one $(2^2+$

2)-strict bridge b_2.

Assume that G has exactly one (2^i+2)-strict bridge b_i for each $i<k\leqslant m$. Since any positive even integer $s\leqslant 2^k-2$ has a unique expression

$$2^{i_1}+2^{i_2}+\cdots+2^{i_t}, \qquad 1\leqslant i_1<i_2<\cdots<i_t\leqslant k-1,$$

G has a unique $(2n-s)$-cycle containing exclusively the strict bridges $b_{i_1}, b_{i_2}, \cdots, b_{i_t}$. Hence G does not contain any other strict bridge whose order is smaller than 2^k+2.

Let C' be the $(2n\text{-}2^k)$-cycle of G. Then C' contains one bridge $b\notin B_1=\{b_1,b_2,\cdots,b_{k-1}\}$. If C' contains another bridge $b'\neq b$, then replacing the bridge b' by the path $C(b')\cap C$, we obtain a cycle C'' of length $2n-(2^k-p)$ for $p=|V(C(b')\cap C)|-2\geqslant 2$. Thus G has two cycles of length $2n-(2^k-p)$, which is false. Hence C' contains only one bridge b, and b is a (2^k+2)-bridge b_k.

We shall now prove by contradiction that b_k is strict.

Suppose that b_k is not strict. Then there is at least one bridge in ext C'. Since $\overline{C}'$ is a (2^k+2)-cycle and G contains no bridge $b'\notin B_1$ such that the order of b' is smaller than 2^k+2, the only bridges in ext C' are the bridges b_j, $j\leqslant k-1$.

Clearly b_{k-1} is contained in int C'. Otherwise, since $2^k+2-2^{k-1}=2^{k-1}+2$, it follows that G has an inner cycle of length $2^{k-1}+2$ containing exclusively the bridges b_k and b_{k-1}, which contradicts the fact that the side cycle $C(b_{k-1})$ is also of length $2^{k-1}+2$. We can thus assume that there is a smallest index $j\leqslant k-2$ such that $b_j,\cdots,b_{j+q-1}$ are contained in ext C' and b_{j+q} is contained in int C', where $j+q\leqslant k-1$. In the following we assume that $j\neq 1$. However, when $j=1$, the proof is similar.

Next, since any even integer s satisfying $2^k<s\leqslant 2^k+2^j-2$ can be written uniquely in the form

$$2^{i_1}+2^{i_2}+\cdots+2^{i_t}+2^k, \qquad 1\leqslant i_1<i_2<\cdots<i_t\leqslant j-1,$$

G has a $(2n-s)$-cycle containing exclusively the bridges $b_{i_1}, b_{i_2}, \cdots, b_{i_t}$ and b_k. Hence G has no bridge $b'\notin B_2=B_1\cup\{b_k\}$ whose order is smaller than 2^k+2^j+2.

Let C'' be the $(2n-2^k-2^j)$-cycle of G. Clearly C'' contains at least one bridge $b^*\notin B_2$. A similar discussion to that of the $(2n-2^k)$-cycle C' yields that b^* is a (2^k+2^j+2)-bridge. (See Fig. 1.) There are two cases, depending on whether b_j is contained in int C'' or not.

Case 1　b_j is contained in int C''. In this case, b_k is contained in int C''. It follows that G has two $(2n-2^k-2^{j+q})$-cycles, one of which is the cycle containing exclusevely the two bridges b_k and b_{j+q}, the other is the cycle containing exclusively the bridges $b_j, b_{j+1}, \cdots, b_{j+q-1}$ and b^*, which is false.

Case 2 b_j is contained in ext C''. In this case, G has two (2^k+2)-cycles, one of which is the side cycle containing exclusively the bridge b_k, and the other is the cycle containing exclusively the two bridges b_j and b^*, which is false.

Thus b_k is a strict bridge, and hence the proof of the lemma is completed.

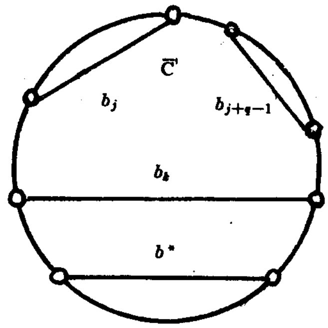

Figure 1

Figure 2 shows six outerplanar UB-graphs.

Theorem 2.3 Let G be an outerplanar r-UB-graph and G is not a cycle, then $G \in \{H_8, H_{14}^{(1)}, H_{14}^{(2)}, H_{14}^{(3)}, H_{14}^{(4)}\}$.

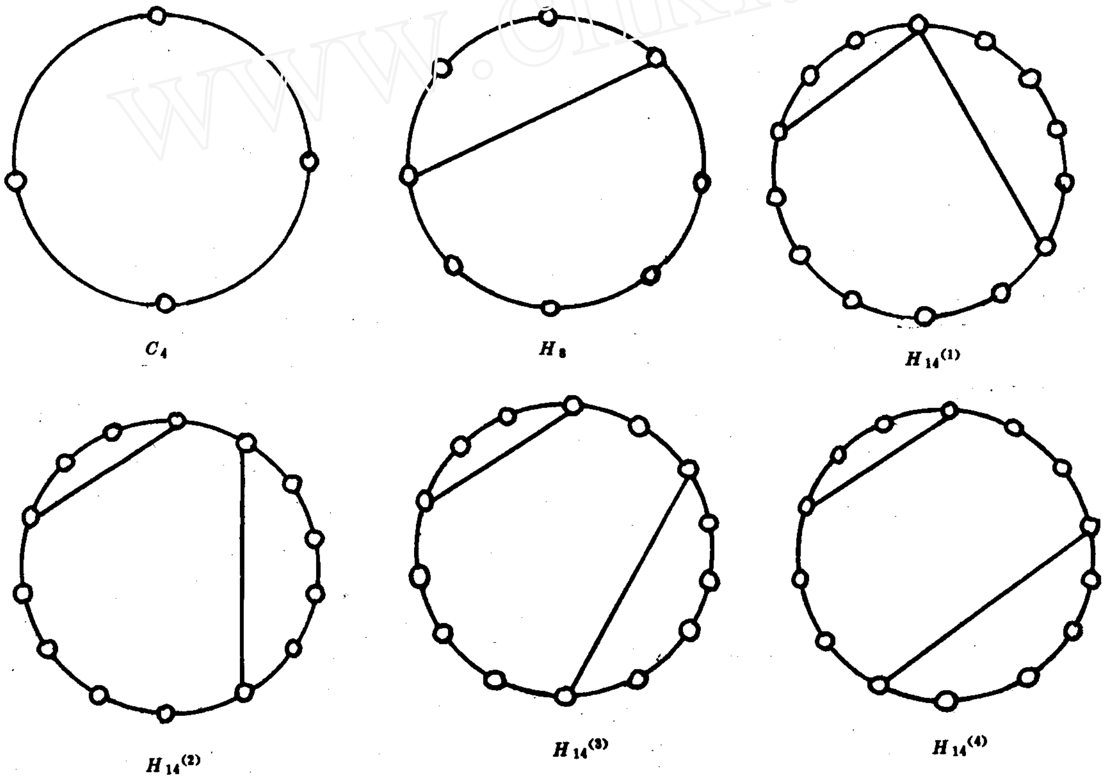

Figure 2

Proof By Lemma 2.1, $r=4$. Let G have m bridges.

If $m \geqslant 3$, then by Lemma 2.2, these m bridges are (2^i+2)-strict bridges b_i, $1 \leqslant i \leqslant m$.

Thus G has m strict side cycles $C(b_1), C(b_2), \cdots, C(b_m)$. Let $P_i = C(b_i) \cap C$ for $i = 1, 2, \cdots, m$ and let C' be any inner cycle of G. Then C' contains either b_i or P_i for $i = 1, 2, \cdots, m$. It follows that G has 2 inner cycles, and hence G has $2^m + m$ cycles. Since the inner cycle of minimum length is the cycle containing all the m bridges, the length of this cycle is

$$2(2^m + m) + 2 - \sum_{i=1}^{m} 2^i = 2m + 4 \geqslant 10.$$

Hence G does not contain an 8-cycle and G is not an outerplanar UB-graph.

If $m = 1$, then $G = H_8$. If $m = 2$, then $G \in \{H_{14}^{(1)}, H_{14}^{(2)}, H_{14}^{(3)}, H_{14}^{(4)}\}$. □

3　r-UB-graphs with $m \leqslant 3$ bridges

Let $b_1', b_2', \cdots, b_m'$ be the bridges of G and let $v_{\alpha_1}, v_{\alpha_2}, \cdots, v_{\alpha_t}$ (α_i is an integer, $\alpha_1 < \alpha_2 < \cdots < \alpha_t$) be the vertices of attachment of these bridges and these vertices appear in the clockwise order $v_{\alpha_1}, v_{\alpha_2}, \cdots, v_{\alpha_t}$ on C, where $b_i' = x_i y_i$, $x_i, y_i \in \{v_{\alpha_1}, v_{\alpha_2}, \cdots, v_{\alpha_t}\}$, $i = 1, 2, \cdots, m$. Then the graph which satisfies the above conditions is represented by $G(x_1y_1, x_2y_2, \cdots, x_my_m)$. It is stressed that the bridges between parentheses appear in the order $b_1', b_2', \cdots, b_m'$. We frequently regard the $G(x_1y_1, x_2y_2, \cdots, x_my_m)$ as a diagram of G which manifests the relation of relative positions holding between some of the bridges in G. For example, a diagram $G(v_1v_4, v_5v_{10}, v_7v_{11}, v_5v_9)$ of a given graph G is shown in Figure 3. It shows that G contains four bridges $b_1' = v_1v_4$, $b_2' = v_5v_{10}$, $b_3' = v_7v_{11}$, $b_4' = v_5v_9$, where b_3' and b_2' are skew, b_3' and b_4' are also skew, but b_1' does not skew to the other.

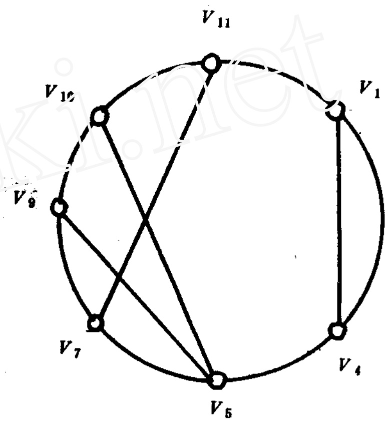

Figure　3

Let $C[v_i, v_j]$ denote the (v_i, v_j)-path which follows the clockwise orientation of C. Similarly, the symbol $C(v_i, v_j)$ denotes the path $C[v_i, v_j] - \{v_i, v_j\}$. And the symbol $\overline{C[v_i, v_j]}$ is used to denote the reverse path of $C[v_i, v_j]$.

An r-UB-graph G is said to be a skew graph if G is not an outerplanar graph. An r-UB-graph G is said to be an m-skew graph if G is a skew graph with m bridges.

Lemma 3.1　If G is a UB-graph and b is a 4-bridge in G, then b does not skew to the other bridges in G.

Proof　Let $b_1' = v_1v_3$ and $b_1' \cup C[v_3, v_4]$ be a 4-cycle. Suppose that there is a bridge b_2'

which skews to b_1' (See $G(v_1v_3, v_2v_4.)$, where $b_2' = v_2v_4$). Clearly, there is exactly one vertex on $C(v_3, v_4) \cup C(v_4, v_1)$. We may assume that $C(v_4, v_1)$ contains exactly one vertex. Then G contains two cycles $b_1' \cup C[v_1, v_2] \cup b_2' \cup v_4v_3$ and $b_2' \cup C[v_4, v_2]$ which have the same length, a contradiction. □

Lemma 3.2 None of the 2-skew graphs is an r-UB-graph.

Proof Let G be any 2-skew graph. Clearly G has exactly 7 cycles denoted by $C_1, C_2, \cdots, C_7$. It is easy to verify that
$$\sum_{i=1}^{7} |E(C_i)| = 4|E(G)|.$$

Suppose that G is an r-UB-graph. Then
$$|V(G)| = 2n = r + (7-1) \times 2 = r + 12,\ |E(G)| = 2n + 2 = r + 14,$$
and
$$\sum_{i=1}^{7} |E(C_i)| = r + (r+2) + \cdots + (r+12) = 7r + 42.$$
Thus $4(r+14) = 7r + 42$, and hence $3r = 14$, which is false. □

The proof of the following lemma is not difficult and therefore is left to the reader.

Lemma 3.3 Let $|E(G)| = \sum_{i=1}^{m} a_i l_i$ and $a_1 \geqslant a_2 \geqslant \cdots \geqslant a_m > 0$. If $l_{k_1} l_{k_2} \cdots l_{k_m}$ is a permutation of $l_1 l_2 \cdots l_m$ with $0 < l_{k_1} \leqslant l_{k_2} \cdots \leqslant l_{k_m}$, then $|E(G)| = \sum_{i=1}^{m} a_i l_i$.

Lemma 3.4 If G is a 3-skew UB-graph, then G contains one 4-bridge.

Proof We shall prove this lemma by contradiction. Suppose that G contains no 4-bridge. Then G must contain one 4-inner cycle, say C_4. We shall now consider the number of the bridges contained in C_4. There are two possible cases only. □

Case 1 C_4 contains exactly two bridges, say b_1' and b_2' . In this case, C is not a skew cycle, otherwise G has two Hamilton cycles, a contradiction.

Now both b_1' and b_2' must skew to the third bridge b_3' . Otherwise we may assume that b_1' skews to b_3' and b_2' does not skew to b_3' . Let $b_2' = v_2v_4$. We may also assume that there are no bridges in int $C[v_2, v_4] \cup b_2'$. Further, let $G^* = G - C(v_2, v_4)$. Then b_1' is a 4-bridge of G^* . By the proof of Lemma 3.1, it is easily seen that G^* contains two cycles which have the same length, a contradiction.

We shall now consider two subcases, depending on whether b_1' and b_2' are adjacent or not.

Case 1.1 b_1' and b_2' are adjacent (See $G(v_1v_4, v_2v_4, v_3v_5)$). In this case, G contains two cycles $b_3' \cup C[v_5, v_3]$ and $C[v_5, v_1] \cup v_1v_4v_2 \cup C[v_2, v_3] \cup v_3v_5$ which have the same length, a contradiction.

Case 1.2 b_1' and b_2' are not adjacent (See $G(v_1v_5, v_2v_4, v_3v_6)$). In this case, G contains two cycles $C[v_6, v_1] \cup v_1v_5 \cup \overline{C[v_3, v_5]} \cup v_3v_6$ and $C[v_6, v_2] \cup v_2v_4 \cup \overline{C[v_3, v_4]} \cup v_3v_6$ which have the same length, again a contradiction.

Case 2 C_4 contains exactly three bridges. Since G is a skew graph, there are two bridges in G which are skew (See $G(v_1v_3, v_2v_4, v_1v_2)$). Let $G^* = G - C(v_1, v_2)$ and $v^* = |V(G^*)|$. Then G^* contains two v^*-cycles, once more a contradiction.

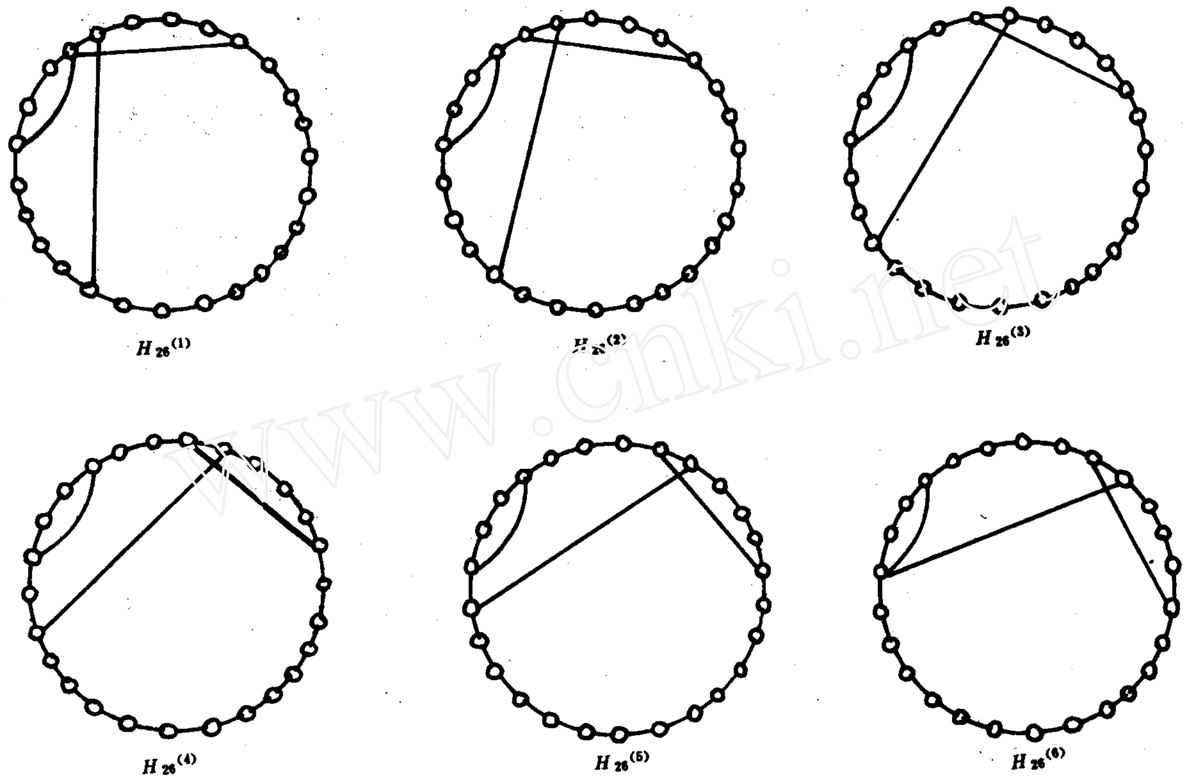

Figure 4

The graphs $H_{26}^{(1)}$, $H_{26}^{(2)}$, $H_{26}^{(3)}$, $H_{26}^{(4)}$, $H_{26}^{(5)}$ and $H_{26}^{(6)}$ are depicted in Figure 4.

Lemma 3.5 A 3-skew graph G is an r-UB-graph if and only if $G \in \{H_{26}^{(1)}, H_{26}^{(2)}, H_{26}^{(3)}, H_{26}^{(4)}, H_{26}^{(5)}, H_{26}^{(6)}\}$.

Proof The sufficiency is easily seen by immediately checking $H_{26}^{(i)} (i = 1, 2, \cdots, 6)$. We shall prove the necessity. Let b_1', b_2' and b_3' be the bridges of G and let b_1' skew to b_2'.

Consider the relation of the relative positions holding between b_3' and the other two bridges. We have three cases.

Case 1 b_3' does not skew to the other two bridges (See $G(v_1v_3, v_2v_4, v_5v_6)$). We allow $v_6 = v_1$ *or* $v_4 = v_5$. In this case G contains exactly 12 cycles. Let $C_1 = b_3' \cup C[v_5, v_6]$, $C_2 = b_1' \cup$

$C[v_1,v_3]$, $C_3 = b_2' \cup C[v_2,v_4]$, $C_4 = b_2' \cup \overline{C[v_1,v_2]} \cup b_1' \cup C[v_3,v_4]$ and let $C_5, C_6, \cdots, C_{12}$ be the other cycles. In the coming discussion, $|E(C_i)|$ is denoted by l_i. It is easy to verify that

$$4l_1 + 2(l_2 + l_3 + l_4) + \sum_{i=5}^{12} l_i = 8|E(G)|. \qquad (1)$$

Since G is a r-UB-graph, G contains exactly one k-cycle for each even k, $r \leqslant k \leqslant r+22$. By Lemma 3. 3,

$$|E(G)| \geqslant \frac{4r + 2(r+2+r+4+r+6) + \sum_{i=4}^{11}(r+2i)}{8} = \frac{9r+72}{4}.$$

Clearly $|E(G)| = r + 25$. Therefore $r + 25 \geqslant \frac{9r+72}{4}$, i e , $5r \leqslant 28$.

Since $r \geqslant 4$ is even, $r = 4$. Thus $2n = 26$.

By Lemmas 3. 1 and 3. 4, b_3' is a 4-bridge and $b_3' \cup C[v_5,v_6]$ is a 4-cycle. From (1), we have

$$3l_1 + (l_2 + l_3 + l_4) + \sum_{i=1}^{12} l_i = 8(2n+3).$$

Hence $$l_2 + l_3 + l_4 = 8 \times 29 - 3 \times 4 - \sum_{i=0}^{11}(4+2i) = 40.$$

Let $Q_i = C[v_i, v_{i+1}]$, for $i = 1,2,3,4,5$, and let $Q_6 = C[v_6,v_1]$, $q_i = |E(Q_i)|$ for $i = 1,2,3,4,5,6$.

It is easily seen that

$$40 = l_2 + l_3 + l_4 = 2(q_1 - 1 + q_2 - 1 + q_3 - 1) + 4, \text{ i e }, q_1 + q_2 + q_3 = 21.$$

Thus $(q_4 - 1) + (q_6 - 1) = 26 - (q_1 + q_2 + q_3 + r - 4) = 5$, i e , $q_4 + q_6 = 7$.

Since each cycle other than C_1, C_2, C_3, C_4 contains the paths Q_4 and Q_6, the 6-cycle of G must be one of the three cycles C_2, C_3 and C_4.

Suppose that C_4 is the 6-cycle of G, then $q_1 + q_3 = 6$. In this case, G has two 24-cycles $b_3' \cup C[v_6,v_5]$ and $b_2' \cup C[v_2,v_3] \cup b_1' \cup \overline{C[v_4,v_1]}$, a contradiction. Therefore the 6-cycle of G must be C_2 or C_3. We may assume that C_2 is the 6-cycle of G. Then $q_1 + q_2 = 7$. Since C_4 is even cycle and $q_3 = 21 - 7 = 14$, q_1 is even. Also $q_1 \geqslant 2$, $q_2 \geqslant 2$. It follows that $q_1 = 2$ or $q_1 = 4$.

Suppose that $q_1 = 4$. Then $q_2 = 3$. In this case G has two 12-cycles $b_2' \cup C[v_4,v_2]$ and $b_2' \cup C[v_2,v_3] \cup b_1' \cup \overline{C[v_4,v_1]}$, a contradiction. Therefore $q_1 = 2$. Consequently $G \in \{H_{26}^{(1)}, H_{26}^{(2)}, H_{26}^{(3)}, H_{26}^{(4)}, H_{26}^{(5)}, H_{26}^{(6)}\}$.

Case 2 b_3' skews to exactly one of the two bridges b_1' and b_2'. We may assume that b_3' skews to b_2' . Then $G = G(v_1v_3, v_2v_4, v_3v_5)$ or $G = G(v_1v_3, v_2v_5, v_4v_6)$.

Case 2. 1 $G = G(v_1v_3, v_2v_4, v_3v_5)$. Clearly G has exactly 13 cycles. Let $C_1 = b_2' \cup C[v_4, v_2]$ and let $C_2, C_3, \cdots, C_{13}$ be the other cycles. It is easy to verify that $\sum_{i=2}^{13} l_i = 6|E(G)|$.

Since G is an r-UB-graph, $2n = r + (13\text{-}1) \times 2 = 24 + r$. Using Lemma 3.3, we get

$$6(24 + r + 3) \geqslant \sum_{i=0}^{11}(r + 2i) = 12r + 132,$$

i e, $r \leqslant 5$. Thus $r = 4$. By Lemmas 3.4 and 3.1, G has a strict 4-bridge, which is false.

Case 2.2 $G = G(v_1v_3, v_2v_5, v_4v_6)$. Clearly G has exactly 14 cycles. Let $C_1 = b_1' \cup C[v_1, v_3]$, $C_2 = b_3' \cup C[v_4, v_6]$ and let $C_3, C_4, \cdots, C_{14}$ be the other cycles. It is easy to verify that

$$l_1 + l_2 + \sum_{i=1}^{14} l_i = 8|E(G)|.$$

Since G is an r-UB-graph, $2n = r + 26$. Using Lemma 3.3, we have

$$8(26 + r + 3) \geqslant 2r + 2(r + 2) + \sum_{i=2}^{13}(r + 2i) = 16r + 184,$$

i e, $r \leqslant 6$. Thus $r = 4$ or $r = 6$.

A discussion similar to that of the case 2.1 yields that $r \neq 4$. Therefore $r = 6$, and $2n = 26 + 6 = 32$. From (2), we have $8(2n + 3) = l_1 + l_2 + \sum_{i=3}^{16} 2i$. i e, $35 \times 8 = l_1 + l_2 + 266$, Hence $l_1 + l_2 = 14$.

Clearly $\{l_1, l_2\} = \{6, 8\}$ and $b_1' \cup C[v_3, v_4] \cup b_3' \cup C[v_6, v_1]$ is a 22-cycle.

Let C' be the 30-cycle of G. Clearly C' contains at least two bridges and C' is a skew cycle. If C' contains exactly two bridges, we may assume that C' contains bridges b_1' and b_2'. Then the other skew cycle containing exactly the two bridges b_1' and b_2' is a 6-cycle. Thus G has two 6-cycles, which is false. Therefore C' contains exactly three bridges. Thus the other skew cycle containing exactly the three bridges is an 8-cycle, and hence G has two 8-cycles, which is false.

Case 3 b_3' skews to both b_1' and b_2'. In this case $G = G(v_1v_4, v_2v_5, v_3v_6)$. Cleary G has exactly 15 cycles, say $C_1, C_2, \cdots, C_{15}$. It is easy to verify that, $\sum_{i=1}^{15} l_i = 8|E(G)|$.

Since G is an r-UB-graph, $2n = 28 + r$ and $8(28 + r + 3) = \sum_{i=0}^{14}(r + 2i)$, i e, $7r = 38$, which is false.

Theorem 3.6 Let G be an r-UB-graph with $m \leqslant 3$ bridges. Then $G \in \{C_4, H_8, H_{14}^{(1)}, H_{14}^{(2)}, H_{14}^{(3)}, H_{14}^{(4)}, H_{26}^{(1)}, H_{26}^{(2)}, H_{26}^{(3)}, H_{26}^{(4)}, H_{26}^{(5)}, H_{26}^{(6)}\}$.

Proof This theorem follows immediately from Theorem 2.3, Lemmas 3.2 and 3.5. □

We end this paper with the following conjecture.

Conjecture A graph G is an r-UB-graph if and only if $G \in \{C_4, H_8, H_{14}^{(1)}, H_{14}^{(2)}, H_{14}^{(3)}, H_{14}^{(4)}, H_{26}^{(1)}, H_{26}^{(2)}, H_{26}^{(3)}, H_{26}^{(4)}, H_{26}^{(5)}, H_{26}^{(6)}\}$.

References

1 Bondy J A, Murty U S R. Graph Theory with Applications. Macmillan Press, 1976

2 Yap H P, Teo S K. On uniquely r-pancyclic graphs. Lecture Notes in Math, 1984(1073):334～335

3 Shi Yongbing. Some theorems of uniquely pancyclic graphs. Discrete Math, 1986(59):167～180

4 Shi Yongbing, Yap H P, Teo S K. On uniquely r-pancyclic graphs. Annals of the New York Academy of Sciences, 1989(576):487～499

关于唯一 r-偶泛圈图

施永兵　　孙家恕
(上海师范大学)　　(淮南师范专科学校)

提　要　设 $r\geqslant 4$ 且 r 是偶整数. 阶为 $2n$ 的偶图 G 被称为唯一 r-偶泛圈图,如果对每个偶整数 t, $r\leqslant t\leqslant 2n$, G 恰含一个长为 t 的圈,且 G 不含长小于 r 的圈. 若 G 是唯一 r-偶泛圈图,则称 G 是 r-UB-图. 证明了恰好存在 6 个外可平面的 r-UB-图和对 $m\leqslant 3$ 恰好存在 12 个阶为 $2n$ 和边数为 $2n+m$ 的 r-UB-图.

关键词　圈;偶图;唯一 r-偶泛圈图
中图法分类号　O157.5