Dynamic Interaction of Intense Rain with Water Waves

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ABSTRACT

A theory for determining the dynamic effect of intense rain on water waves is established, based on momentum exchange. The theory takes into account the rain intensity, angle of incidence and fall velocity, and the wave amplitude, frequency and water depth. It is found that the rain induces a uniform increase of pressure in the water column and a uniform mass transport in a thin boundary layer affected by the momentum exchange. The rain also induces a fluctuating pressure and shear stress on the free surface. For vertical or near vertical rainfall, these fluctuating free surface forces are responsible for a non-negligible wave amplitude decay, particularly in the high frequency range. In the case of high winds, the rain horizontal velocity component is large and the corresponding stress on the free surface is nearly in phase with free surface slope. Then instead of causing a decay, the rain adds its effect to the wind and enhances the growth of high frequency waves. It is concluded that this effect, previously neglected, should be considered for insertion as a sink-source mechanism in advanced air-sea interaction models.

1. Introduction

Any mariner has observed the changing appearance of the sea surface upon the arrival of a squall. Despite the sudden increase of wind, the choppy sea surface is slowly replaced by an apparently smoother surface, as the rain interferes with the waves. More difficult to observe, but nevertheless present, the generation of wind waves under hurricane or stormy conditions, is also affected by the intense rain which accompanies the depression for hours and even days. Therefore, it would appear that the rain–wave interaction mechanism should normally be added to the other sink–sources mechanisms in the radiation-transfer equation which governs the generation of waves by wind (for example, see Hasselman et al. 1976).

This effect is generally neglected in air–sea interaction studies, even though the damping of water waves by rain was mentioned by Reynolds as early as 1875. More recently the subject has been investigated by Manton (1973), Nystuen (1989), Tsimplis and Thorpe (1989). In all these studies, it is assumed that the main mechanism for wave damping by rain is the result of the turbulent dissipation induced by the rain drops penetrating the free surface of the waves. Philips (1987) also mentions its relative importance in the case of a heavy vertical downpour and alludes to the effect of vertical momentum exchanges.

The purpose of this paper is to establish a theory on the interaction between rain and water waves, based on momentum exchanges and to assess its relative importance regarding the wave prediction models. Only the dynamical interference of rain by momentum exchange with an existing wave field is taken into account. The collapse of water cavities generated by raindrops (Le Méhauté et al. 1987) and generation of gravity–capillary rings resulting from these has been investigated elsewhere (Le Méhauté 1988). The interference of these cavities and rings with a prevailing wave field is neglected. Also, the effect of the freshwater rain on surfactant films and the subsequent variations of wave damping in the mixed gravity–capillary and pure capillary range is also a subject which remains an open question.

Initially, the momentum exchange between rain and water waves is investigated and translated into a free surface boundary condition. Then a wave damping (or wave growth) coefficient is formulated in terms of the rain and wave parameters. Finally, the formulation is applied to storm and hurricane conditions, and its relative importance is assessed.

2. Momentum exchange between rain and water waves

The motion is defined with respect to a clockwise cartesian coordinate system (ox, oz). ox is horizontal positive in the wave traveling direction, oz is vertical positive upwards from the still wave level (Fig. 1).

Consider a drop of rain of mass (ρvol) falling at a clockwise angle α with the vertical oz at a velocity $\vec{V}_r$. The drop reaches the free surface of a liquid of density ρ_s which moves at a velocity $\vec{V}_s(u_s, w_s)$ where u_s and w_s are the horizontal and vertical components

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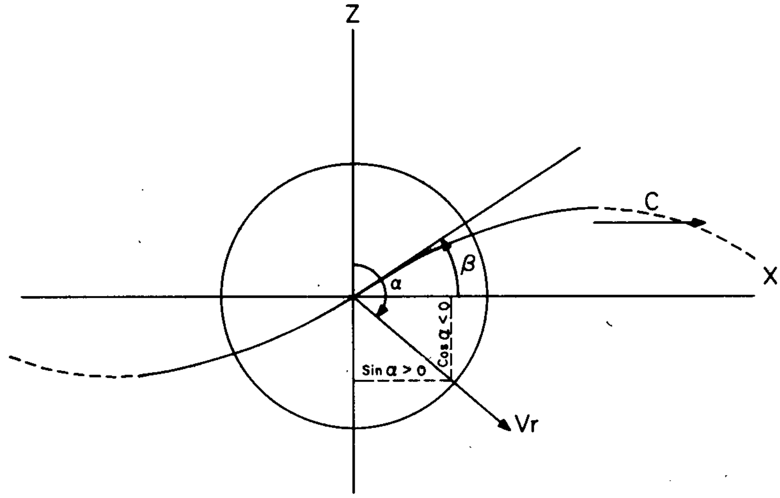

FIG. 1. Notation and sign convention.

of $\vec{V}_s$, respectively. As soon as the drop reaches the free surface, its mass is entrained by the current at velocity $\vec{V}_s$ in a thin boundary layer affected by the momentum exchange. Therefore, the drop is subjected to a change of momentum:

$$\Delta M = \rho \text{vol}[\vec{V}_s - \vec{V}_r]. \quad (1)$$

Consider now a flow of raindrops falling at a representative fall velocity V_r with a concentration C_r. The vertical discharge per unit area, or precipitation q is ($\cos\alpha < 0$, $\sin\alpha > 0$),

$$q = -C_r V_r \cos\alpha. \quad (2)$$

Since the surface of the moving fluid has a vertical component of velocity w_s, the discharge which reaches the free surface varies with w_s. For example, in the case of a water wave, it will be higher on the forward face of the wave, when the free surface moves upwards ($w_s > 0$). It will be smaller on the rear of the wave when the free surface moves downwards ($w_s < 0$). Therefore, the amount of rain that is entrained at velocity $\vec{V}_s$ per unit of time is

$$q' = C_r[-V_r \cos\alpha + w_s]. \quad (3)$$

Since the momentum transfer in gravity capillary rings is neglected, the corresponding change of momentum, or alternately the force exerted per unit area by the rain on the current is

$$F = \rho q'[\vec{V}_r - \vec{V}_s]. \quad (4)$$

Projected on an horizontal and vertical axis, the forces components are

$$F_x = \rho q'[V_r \sin\alpha - u_s], \quad (5)$$

$$F_z = \rho q'[V_r \cos\alpha - w_s]. \quad (6)$$

Consider now an infinitely long field of uniform plane (x, z) progressive water waves on a horizontal bottom. As a result of the rain, it is assumed that the amplitude of the wave varies exponentially with time. Such a wave field can then be defined by the linear Airy wave solution where the amplitude (a) is related to an initial amplitude (a_0) at time $t = 0$. The assumption of linearity implies that the wave field can be considered as irrotational and defined by a potential function. Such type of solution can be described by a complex potential function $\phi(x, z, t)$ such as

$$\phi = D \frac{\cosh k(d + z)}{\cosh kd} \text{Re}(-ie^{i\theta}), \quad (7)$$

where

$$D = a_0 g/\sigma_R, \quad (8)$$

g is the gravity acceleration and σ_R is the real part of the wave frequency. Water depth d is constant and the phase θ is defined by,

$$\theta = kx - \sigma t, \quad (9)$$

where k is the wavenumber and σ is the complex wave frequency. The complex frequency may be written in terms of its real and imaginary part as

$$\sigma = \sigma_R + i\sigma_I, \quad (10)$$

where σ_I the imaginary part corresponds to the wave decay ($\sigma_I < 0$) or wave growth ($\sigma_I > 0$) coefficient. Therefore, the wave amplitude $a(t)$ is

$$a(t) = a_0 \exp(\sigma_I t). \quad (11)$$

Differentiating Eq. (7) with respect to x and z at the free surface ($z \approx 0$), u_s and w_s are obtained. Then inserting these into Eqs. (5) and (6), assuming $V_s \ll$

V_r, and neglecting the nonlinear terms (w_s^2, $u_s w_s$), one finds

$$F_x = \rho q[V_r \sin\alpha - Dk(1 - i \tanh kd \tan\alpha)e^{i\theta}] + O(a_0^2k^2), \quad (12)$$

$$F_z = \rho q[V_r \cos\alpha + Dk2i \tanh kd\, e^{i\theta}] + O(a_0^2k^2), \quad (13)$$

respectively. Both of these forces can be considered as the sum of pressure force component p perpendicular to the free surface and tangential stress component τ parallel to the free surface. The slope of the free surface η (measured positively counterclockwise from ox) is

$$\tan\beta \approx \beta = \eta_x = ia_0ke^{i\theta}. \quad (14)$$

By changing sign, in order to obtain the reaction of the current on the rain, and assuming $\beta^2 \ll \beta(\cos\beta \approx 1, \sin\beta \approx \beta)$, one obtains

$$p = -F_z \cos\beta + F_x \sin\beta \approx -F_z + F_x\beta, \quad (15)$$

$$\tau = -F_z \sin\beta - F_x \cos\beta \approx -F_z\beta - F_x. \quad (16)$$

Inserting Eqs. (12) and (13) into (15) and (16), and neglecting the nonlinear terms, yields

$$p = \rho q[-V_r \cos\alpha + (ia_0kV_r \sin\alpha - 2iDk \tanh kd)e^{i\theta}] + O(a_0^2k^2), \quad (17)$$

$$\tau = \rho q[-V_r \sin\alpha + (Dk - iDk \tanh kd \tan\alpha - ia_0kV_r \cos\alpha)e^{i\theta}] + O(a_0^2k^2). \quad (18)$$

3. Free surface condition

Referring to Eqs. (17) and (18), it is seen that p and τ are given by the sum of a steady uniform force and a fluctuating force varying with θ. Reverting to the direct force of the rain on the current ($p^* = -p$), it is seen that the rain exerts a steady uniform downward pressure force $\overline{p^*}$:

$$\overline{p^*} = \rho q V_r \cos\alpha = -\rho C_r V_r^2 \cos^2\alpha, \quad (19)$$

which is maximum in the case of a vertical rain.

This force acts throughout the water column and is balanced by the force of the seafloor on the water. It does not have any dynamic effect on the wave, except for changing the Bernoulli constant.

The permanent horizontal shear stress τ induces a uniform mass transport in a near surface thin boundary layer δ affected by the transfer of momentum from the rain. The average value of δ is constant, so that the mass transfer velocity $\bar{U}$ in the ox direction is

$$\bar{U} = \int_0^\delta \bar{\tau} dz, \quad (20)$$

where

$$\bar{\tau} = -\rho q[-V_r \sin\alpha] = \rho V_r^2 \sin\alpha \cos\alpha. \quad (21)$$

$\bar{\tau}$ and $\bar{U}$ are nil in the case of a vertical rain.

This uniform layer δ is equivalent in its effects on the waves, to an additional uniform pressure force $\delta p = \rho g \delta$ on the upper surface. Therefore it does not affect the wave dynamic either.

Consider further the shearing force τ given by Eq. (18). This can still be written as:

$$\tau = \bar{\tau} + \tau' = \bar{\tau} + \tau_1 e^{i(\theta+\gamma)}, \quad (22)$$

where

$$\tau_1 = \rho q D k, \quad (23)$$

and

$$\gamma = \tan^{-1}\left[-\left(\tanh kd \tan\alpha + \frac{\sigma_R}{g} V_r \cos\alpha\right)\right]. \quad (24)$$

Following an approach developed by Longuet-Higgins (1968) on the effect of wind stress on water waves, let u' denote the fluctuating component of boundary layer velocity and m' the corresponding mass flux over a variable boundary layer thickness δ', such as

$$m' = \int_\delta^{\delta'} \rho u' dz. \quad (25)$$

Then by conservation of momentum parallel to the boundary, we have

$$\frac{\partial m'}{\partial t} = \tau_1 e^{i(\theta+\gamma)}. \quad (26)$$

Note δ' is conceived as always consisting of the same marked particles. If w' denotes the additional component of velocity normal to the boundary layer, i.e., normal to the free surface, then

$$\frac{\partial \delta'}{\partial t} = [w'] = \int_\delta^{\delta'} w'_z dz = -\int_\delta^{\delta'} u'_x dz, \quad (27)$$

by continuity. If C is the phase velocity, and since

$$\frac{\partial}{\partial x} = -\frac{1}{C}\frac{\partial}{\partial t}, \quad (28)$$

hence, inserting Eqs. (25) and (26) into Eq. (27):

$$\frac{\partial \delta'}{\partial t} = \frac{1}{C}\frac{\partial}{\partial t}\int_\delta^{\delta'} \rho u' dz = \frac{1}{\rho C}\frac{\delta m'}{\partial t} = \frac{\tau_1 e^{i(\theta+\gamma)}}{\rho C}. \quad (29)$$

Integrating with respect to time, it follows that the thickness of the fluctuating boundary layer δ' is

$$\delta' = \frac{\tau_1 e^{i(\theta+\gamma)}}{-i\sigma_R \rho C} + \text{const.} \quad (30)$$

The phase of τ' being $(\theta + \gamma)$, it follows that the total boundary layer thickness ($\delta + \delta'$) is in phase with ($i\tau'$), i.e., with $[\theta + \gamma + (\pi/2)]$. As in the case of the steady part of the boundary layer δ, the fluctuating part of boundary layer thickness δ' is equivalent, in its effect on waves, to an additional pressure $\delta p'$, acting on the upper surface of the wave. Therefore, from Eq. (30) neglecting the constant, the significance of which is irrelevant here, and since

$$C \approx \frac{g}{\sigma_R} \tanh kd, \tag{31}$$

we can write

$$\delta p' = \rho g \delta' = \frac{i\tau'}{\tanh kd}. \tag{32}$$

In other words, a fluctuating tangential stress applied to the free surface is dynamically equivalent to a normal pressure fluctuation given by equation (32) which has a phase difference of $\pi/2$ with the shearing stress. Since the τ value which is derived is the reaction force, the applied force is $\tau^* = -\tau$. Therefore, the total pressure force acting on the free surface is

$$p_T = p + (-\tau)e^{i(\pi/2)} \coth kd$$
$$= p + \tau e^{-i(\pi/2)} \coth kd. \tag{33}$$

The free surface boundary condition is obtained by the linearized Bernoulli equation, where the pressure p_T is now given by Eq. (33). Differentiating with respect to t, and inserting the linearized kinematic condition at the free surface ($\eta_t = \phi_z$), one obtains the free surface condition

$$\rho_s \phi_{tt} + \rho_s g \phi_z + p_{Tt} = 0, \tag{34}$$

where ρ_s is the density of the sea water.

4. Dispersion relationship and wave amplitude variation coefficient

Inserting Eqs. (7), (33) with Eqs. (17) and (18) into Eq. (34), and eliminating θ, one obtains after some arithmetic, the linear dispersion relationship:

$$\sigma^2 = gk \tanh kd + \sigma k \frac{\rho}{\rho_s} q[F_{\eta_x} + F_{u_s} + F_{w_s}], \tag{35}$$

where the F_i functions result from the relative effects of the free surface slope η_x, and the free surface velocity components, u_s and w_s as follows:

$$F_{\eta_x} = +\frac{i\sigma V_r}{g}(\sin\alpha + i\cos\alpha \coth kd), \tag{36}$$

$$F_{u_s} = -i \coth kd, \tag{37}$$

$$F_{w_s} = -2i \tanh kd - \tan\alpha. \tag{38}$$

Expressing σ in terms of σ_R and σ_I [Eq. (10)], the real and imaginary parts can be separated. Then a set of two equations with two unknowns, σ_R and σ_I, is obtained:

$$\sigma_R^2 - \sigma_I^2 = gk \tanh kd - \frac{\rho}{\rho_s} qk\left[\frac{V_r}{g}[(\sigma_I^2 - \sigma_R^2)\cos\alpha \coth kd + 2\sigma_R\sigma_I \sin\alpha] + k\sigma_R \tan\alpha - \sigma_I(2\tanh kd + \coth kd)\right], \tag{39}$$

$$2\sigma_R\sigma_I = \frac{\rho}{\rho_s} qk\left[\frac{V_r}{g}[(\sigma_R^2 - \sigma_I^2)\sin\alpha - 2\sigma_R\sigma_I \cos\sigma \coth kd] - \sigma_R(2\tanh kd + \coth kd) - \sigma_I k \tan\sigma\right]. \tag{40}$$

These two equations cannot be solved analytically. Numerical solutions have been obtained iteratively for σ_R and σ_I without neglecting any terms. They show that σ_R is very little influenced by the rain, and that for all practical purposes

$$\sigma_I \ll \sigma_R \approx (gk \tanh kd)^{1/2}. \tag{41}$$

Referring to Eq. (40), and neglecting $\sigma_I k \tan\sigma$ (which implies that the result is not applicable to nearly horizontal rain), one finds that

$$\sigma_I \approx \frac{\rho}{\rho_s}\frac{qk}{D_{en}}\left[\frac{V_r \sin\alpha}{2g}\sigma_R - \tanh kd - \frac{\coth kd}{2}\right], \tag{42}$$

where

$$D_{en} = 1 + \frac{\rho}{\rho_s}\frac{qk}{g}\coth kd V_r \cos\alpha. \tag{43}$$

In most cases $D_{en} \approx 1$. In shallow water this reduces to

$$\sigma_I \to -\frac{\rho}{\rho_s}\frac{q}{2d}, \tag{44}$$

regardless of the rain angle. In deep water it becomes

$$\sigma_I = \frac{\rho}{\rho_s} qk\left[\frac{V_r}{2}\left(\frac{k}{g}\right)^{1/2}\sin\alpha - \frac{3}{2}\right], \tag{45}$$

which has a minimum for a value of k, given by

$$k = k_m = g\left(\frac{2}{V_r \sin\alpha}\right)^{1/2}, \tag{46}$$

and in the case of vertical rain in deep water, Eq. (45)

reduces simply to

$$\sigma_I = -\frac{3}{2}\frac{\rho}{\rho_s}qk. \quad (47)$$

It is found that the horizontal change of momentum due to u_s is half of the vertical change of momentum due to w_s. In the case where the vertical change of momentum only is taken into account, [Eq. (38)], then $\sigma_I \approx -qk$, a result already mentioned by Phillips (1987) in reference to a communication by F. L. Bliven.

Referring to Eq. (11), and inserting the values of σ_I given by Eqs. (42), (44), (45) or (47), the variation of the wave amplitude at a given point as a function of time is obtained.

5. Physical interpretation and applications

A detailed investigation on the relative influence of rain–sea interaction under various meteorological conditions, and its incorporation in the radiation transfer equation which governs the evolution of wind waves are beyond the scope of the present paper. Nevertheless, the determination of the sign of σ_I and its relative importance is pertinent. This is achieved by inserting the magnitude of typically observed physical parameters.

Referring to Eq. (42), it is seen that σ_I could either be positive or negative, depending upon the sign of the sum of the terms within the bracket. In the former case, the wave height increases with time, in the latter it decreases [Eq. (11)]. For example in deep water, the wave grows with time when [Eq. (45)]

$$\frac{V_r}{3}\left(\frac{k}{g}\right)^{1/2}\sin\alpha > 1, \quad (48)$$

In the general case [Eq. (42)], the first term in the bracket is generally positive and contributes to wave growth. It would be negative in the case of a wind blowing in the opposite direction to wave travel ($\sin\alpha < 0$), an unlikely occurrence. This term is related to F_{η_x} resulting from the horizontal component of momentum transfer acting on the wave slope in the wave travel direction.

The two other terms in the bracket are always negative and contribute to wave decay. They are related to F_{u_s} and F_{w_s} [Eqs. (37) and (38)], and they translate the forces required to entrain the mass of the rain drops at the wave particle velocities. In order to assess their relative importance, one has to resort to quantitative values of various parameters.

The value of the precipitation varies considerably. There is a considerable amount of data inland, less at sea. Values as high as 600 mm h^{-1} have been quoted in Hong Kong (Cline 1926). The monsoon produces torrential rain everywhere it occurs. More recently Willis et al. (1989) measured 429 mm h^{-1} in Miami during a tropical depression. Precipitations larger than 100 mm h^{-1} are frequently recorded inland.

Precipitation at sea during tropical cyclone conditions are given by Cline (1927). Rainfall may cover an area of 200 to 400 n mi diameter, lasting for hours and even days depending upon their speed of propagation. The highest precipitation in fast moving hurricanes is in the right front quadrant and is of the order of inches per hour. In the case of a stalling hurricane, it is at the rear of the cyclone, where the wind has abated.

Consider initially the simplest case of a vertical rainfall in deep water [Eq. (47)]. Then σ_I is always negative and the wave height decays with time. Let us assume a relatively typical precipitation rate, q, of 50 mm h^{-1}. Then by application of Eq. (47), for waves of 1 m, 10 m, 100 m wave length, their amplitude decay to 62%, 95%, 99.5% of their initial values, respectively, after 1 hour duration. If the rain persists for 10 hours, then the 10 m (100 m) wave decays to 62% (95%) of their initial values. Therefore, the rain is most likely to affect the tail of the spectrum (towards the high frequencies) and in some cases of high precipitation of long duration, its peak energy frequency.

Similar results would be obtained, if instead of investigating the variation of wave height at a given location as function of time, one investigates its damping with respect to distance. [It suffices to define the k value in Eq. (9) as the sum of a real and an imaginary component k_I, or since the waves are monochromatic and of constant amplitude, to simply replace t by x/C]. Since the waves propagate in the same direction as the storm, a long duration of precipitation is the norm.

In the limit of the shallow water case, it is recalled that Eq. (44) is valid regardless of the rain angle and frequency. The expression for σ_I can then easily be retraced to the horizontal velocity component u_s as the main culprit for wave damping. Consider a water depth of 3 m for example. It is found that after one hour the wave heights are reduced to 99% of their initial values for all frequencies. The interest of this limit case is only theoretical.

Consider now the case of rain at an angle in deep water [Eq. (45)]. The vertical fall velocity of water drops seldom exceeds 9 m s^{-1}. However, when subjected to a strong wind, the wind velocity is added vectorially to the free fall velocity. Under hurricane conditions the wind can reach 100 m s^{-1}, then V_r is of the order of the wind velocity. The drops continuing on their own momentum in the wind boundary layer, $\sin\alpha$ is also close to unity. It has already been mentioned that the theory would not be valid for nearly horizontal rain. Nevertheless, it indicates a definite trend.

Indeed, in the latter case, σ_I is generally positive, the wave amplitude grows with time and the rain adds positively its effect to the wind. In the present example of $q = 50$ mm h^{-1}, it is found that σ_I is 12.04, 0.35 and 0.0078 for the 1 m, 10 m, 100 m wavelengths, respectively. Therefore, high frequency waves are found to grow very rapidly under the added influence of the rain.

That under certain circumstances rain can contribute to wave growth is an interesting fact, which deserves some comments. The first term in brackets in Eq. (42) or (45) results from the action of the horizontal rain component acting on a wavy surface. The phase γ is nearly $\pi/2$ [Eq. (24)] and the momentum transfer from the rain is nearly in phase with the free surface slope, i.e., acts positively on the rear of the wave and negatively on its front. Therefore, it adds its effects to the wind-induced pressure field, which is also in phase with the wave slope (Phillips 1966). The increase of surface stress due to wind under heavy rainfall has also been noticed by Van Dorn (1953) and Caldwell and Elliott (1971).

An order of magnitude of the relative effect of rain as compared to the wind is obtained as follows. In the extreme case of a nearly horizontal rain ($\sin\alpha \sim 1$, V_r large), the rain horizontal momentum transfer is

$$\tau_R^* = -\tau = \rho q V_r. \qquad (49)$$

The wind shear is

$$\tau_w = \rho_a U_*^2. \qquad (50)$$

where ρ_a is the air density and U_* the wind shear velocity. Taking, for example, (Wu 1969)

$$U_* = 5.10 \times 10^{-2} U_{10}, \qquad (51)$$

and since the wind velocity $U_{10} \approx V_r$, then in the previous example, ($q = 50$ mm h^{-1}) with a wind velocity $U_{10} = 50$ m s^{-1}, one finds

$$\frac{\tau_R}{\tau_w} \approx 8.4\%. \qquad (52)$$

The values of σ_I can be deduced in the deep-water case from Fig. 2 which presents the value of the σ_I/q as function of the wavenumber k for various value of $V_r \sin\alpha$. Figure 3 is a shallow water case ($d = 3$ m) obtained by application of the general equation (42) with (43). For the value of k larger than 0.6, the water depth has very little effect, and Fig. 2 is applicable.

6. Conclusion

A theory to determine the dynamic interaction between rain and water waves has been established. The theory is based on momentum exchange. It takes into account the rain intensity, the rain drop fall velocity and angle, and the water wave characteristics (wave amplitude, wave frequency and water depth). It is found that in the general case the rain induces

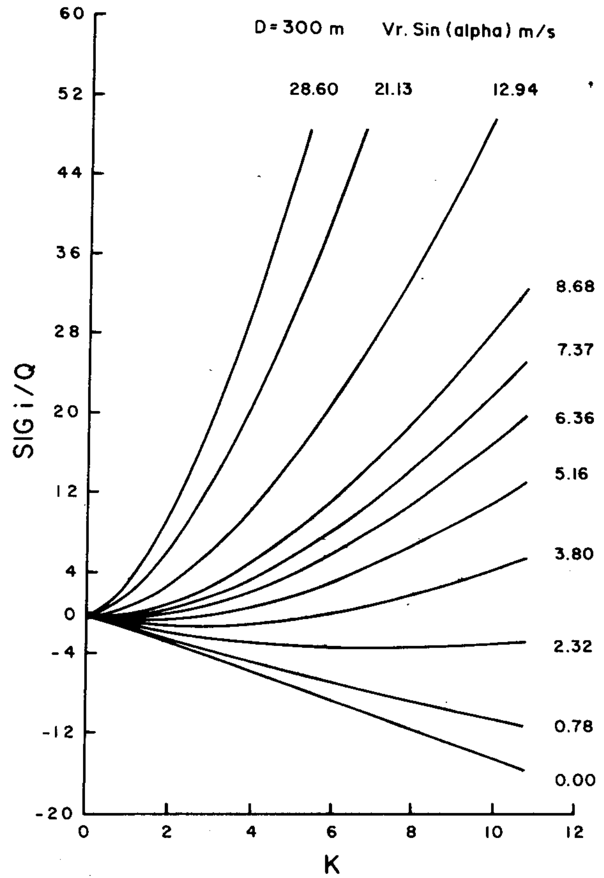

FIG. 2. Values of the wave amplitude variation coefficient σ_I divided by the precipitation q (m s^{-1}), as function of the wavenumber k (m^{-1}) and the product of the raindrop velocity $\vec{V}_r$ (in m s^{-1}) by $\sin\alpha$. α is the clockwise angle of the rainfall velocity vector with the vertical positive upwards ($\sin\alpha > 0$). Deep-water case.

1) a uniform increase of pressure throughout the water column

2) a steady mass transport in a thin uniform near surface layer affected by the momentum exchange

3) a fluctuating pressure on the free surface

4) a fluctuating shearing stress which induces a fluctuating boundary layer thickness and mass transport. This fluctuating stress multiplied by cothkd is dynamically equivalent to a pressure force acting with a phase difference of $\pi/2$.

The fluctuating pressure and stress combined allows us to determine a complex dispersion relationship, which yields the wave amplitude variation coefficient σ_I.

It is found that in the case of high winds, the rain contributes to the growth of the amplitude of high frequency waves in addition to the effect of wind ($\sigma_I > 0$).

In the case of vertical rain, intense rain causes significant wave damping, particularly in the high frequency range ($\sigma_I < 0$).

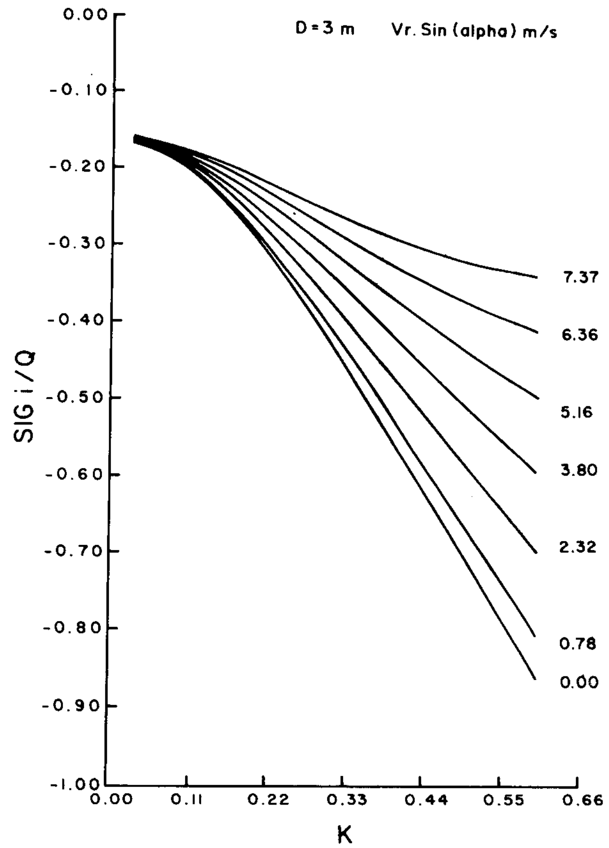

FIG. 3. Blow-up presentation of σ_I/q as function of the wavenumber k for various value of $\vec{V}_r \sin\alpha$ in a shallow water case (water depth $d = 3$ m).

It is concluded that this effect may not be negligible and, at times, should be introduced as a sink–source mechanism in advanced models of the radiation transfer equation, which translates the generation of waves by wind.

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APPENDIX

Notation

- C wave phase velocity
- C_r rain volume concentration
- D coefficient (Eq. 8)
- D_{en} denominator (Eq. 43)
- F force exerted by the rain on a wave (Eq. 4)
- F_x horizontal F component
- F_z vertical F component
- F_{η_x} relative force component due to wave slope (Eq. 36)
- F_{u_s} relative force component due to horizontal velocity (Eq. 37)
- F_{w_s} relative force component due to vertical velocity (Eq. 38)
- $\bar{U}$ average mass transport velocity in the near surface boundary layer
- U_* wind friction velocity
- U_{10} wind velocity
- vol volume of a drop
- $\vec{V}_s$ free surface velocity
- $\vec{V}_r$ raindrop velocity
- a wave amplitude
- a_0 wave amplitude at time $t = 0$
- d water depth
- g gravity acceleration
- k wavenumber
- k_m wavenumber corresponding to the minimum value of σ_I (Eq. 46)
- m' fluctuating mass transport near the free surface (Eq. 25)
- p pressure
- p^* average rain pressure on the free surface
- p_T total pressure [Eq. (33)]
- q precipitation
- q' rate of rain discharge reaching the free surface of a water wave
- t time
- u_s horizontal wave free surface velocity component
- w_s vertical wave free surface velocity component
- x horizontal axis (positive in the wave travel direction)
- z vertical axis (positive upwards from the still water level)
- ϕ potential function
- θ wave phase $(kx - \sigma t)$
- ΔM change of momentum of a drop
- α rainfall angle with the axis oz
- β free surface slope
- γ phase difference of the shearing stress τ with θ
- δ'_p increase of pressure due to the fluctuating near surface boundary layer (equation 32)
- σ complex frequency
- σ_R real value component of σ or wave frequency
- σ_I imaginary part of σ or wave amplitude variation coefficient
- ρ rain drop density
- ρ_a air density
- ρ_s sea water density
- τ shearing stress exerted by the rain on the water waves
- $\bar{\tau}$ steady component of the shearing stress on the free surface
- τ_f amplitude of the fluctuating shear stress at the free surface
- τ^*_R horizontal rain shear stress
- τ_w wind shear stress

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