

变系数 2-D 线性离散系统在一般模型下的状态响应及其观控性

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摘 要

本文研究了变系数 2-D 线性离散系统的一般模型,得出了系统的响应公式,给出了系统是可控和可观的充要条件,并解决了相应的最小能量控制问题。

关键词: 2-D系统,复杂系统,线性系统,可控性。

一、引 言

自 1975 年 Roesser 提出 2-D 常系数线性系统的离散模型以来, Fornasini、Marchesini 和 Attasi 又分别提出了第 I 类和第 II 类的 FM 模型、Attasi 模型^[3]等, 1985 年 Kurek^[2] 又提出了 2-D 常系数线性系统的一般形式,并对它们进行了深入的研究,取得了较为完满的结果。1986 年 Kaczorek^[1] 开始讨论变系数情形下的 Roesser 模型,并取得了许多富有开拓性的成果。本文参照 Kurek 的定义,给出了变系数 2-D 线性系统的一般离散模型的定义,并进行了较为深入的分析 and 讨论,从而得出了一些新的结果。

二、一般模型的状态响应公式

考虑如下的 2-D 离散模型

$$\begin{aligned} \mathbf{x}(i+1, j+1) = & A_0(i, j)\mathbf{x}(i, j) + A_1(i+1, j)\mathbf{x}(i+1, j) \\ & + A_2(i, j+1)\mathbf{x}(i, j+1) + B_0(i, j)\mathbf{u}(i, j) \\ & + B_1(i+1, j)\mathbf{u}(i+1, j) + B_2(i, j+1)\mathbf{u}(i, j+1), \end{aligned} \quad (1.1)$$

$$\mathbf{y}(i, j) = C(i, j)\mathbf{x}(i, j), \quad (i, j = 0, 1, 2, \dots). \quad (1.2)$$

这里 $\mathbf{x}(i, j) \in R^n$ 是状态向量; $\mathbf{u}(i, j) \in R^m$ 是输入向量; $\mathbf{y}(i, j) \in R^r$ 是一输出向量; $A_k(i, j), B_k(i, j) (k = 0, 1, 2), C(i, j)$ 各为适当维数的实矩阵。设给定边界条件为

$$\mathbf{x}(i, 0), \mathbf{x}(0, j), (i, j = 0, 1, 2, \dots). \quad (2)$$

定义式(1)的状态传递矩阵如下:

$$\Phi_{k,l}^{i,j} = A_0(k-1, l-1)\Phi_{k-1,l-1}^{i-1,j-1} + A_1(k, l-1)\Phi_{k,l-1}^{i-1,j-1} + A_2(k-1, l)\Phi_{k-1,l}^{i-1,j-1},$$

$$(k, l) > 0, (i, j) > 0, \quad (3.1)$$

$$\Phi_{k,l}^{i,j} = 0, \min\{i, j, k, l\} < 0, \quad (3.2)$$

$$\Phi_{k,l}^{0,0} = I, \quad (3.3)$$

$$A_0(k, l) = A_1(k, l) = A_2(k, l) = 0, \min\{k, l\} < 0. \quad (3.4)$$

定理 1. 系统 (1) 在边界条件 (2) 之下的状态响应为

$$\begin{aligned} \mathbf{x}(i, j) = & \Phi_{i,j}^{i-1,j-1} [A_0(0, 0), B_0(0, 0)] \begin{bmatrix} \mathbf{x}(0, 0) \\ \mathbf{u}(0, 0) \end{bmatrix} \\ & + \sum_{k=1}^i \{ \Phi_{i,j}^{i-k,j-1} [A_1(k, 0), B_1(k, 0)] + \Phi_{i,j}^{i-k-1,j-1} [A_0(k, 0), B_0(k, 0)] \} \begin{bmatrix} \mathbf{x}(k, 0) \\ \mathbf{u}(k, 0) \end{bmatrix} \\ & + \sum_{l=1}^j \{ \Phi_{i,j}^{i-1,j-l} [A_2(0, l), B_2(0, l)] + \Phi_{i,j}^{i-1,j-l-1} [A_0(0, l), B_0(0, l)] \} \begin{bmatrix} \mathbf{x}(0, l) \\ \mathbf{u}(0, l) \end{bmatrix} \\ & + \sum_{k=1}^i \sum_{l=1}^j [\Phi_{i,j}^{i-k-1,j-l-1} B_0(k, l) + \Phi_{i,j}^{i-k-1,j-l} B_2(k, l) \\ & + \Phi_{i,j}^{i-k,j-l-1} B_1(k, l)] \mathbf{u}(k, l). \end{aligned} \quad (4)$$

证明. a) 当 $(i, j) = (1, 1)$ 时, 利用式 (3) 直接从式 (1) 出发即知结论成立.

假设定理 1 在 $(1, j_0 - 1), (i_0 - 1, 1)$ 成立, 即有

$$\begin{aligned} \mathbf{x}(1, j_0 - 1) = & \Phi_{1,j_0-1}^{0,j_0-2} [A_0(0, 0), B_0(0, 0)] \begin{bmatrix} \mathbf{x}(0, 0) \\ \mathbf{u}(0, 0) \end{bmatrix} \\ & + \Phi_{1,j_0-1}^{0,j_0-2} [A_1(1, 0), B_1(1, 0)] \begin{bmatrix} \mathbf{x}(1, 0) \\ \mathbf{u}(1, 0) \end{bmatrix} + \sum_{l=1}^{j_0-1} \Phi_{01,j_0-1}^{0,j_0-2-l} B_1(1, l) \mathbf{u}(1, l) \\ & + \sum_{l=1}^{j_0-1} \{ \Phi_{1,j_0-1}^{0,j_0-1-l} [A_2(0, l), B_2(0, l)] + \Phi_{1,j_0-1}^{0,j_0-2-l} \\ & \times [A_0(0, l), B_0(0, l)] \} \begin{bmatrix} \mathbf{x}(0, l) \\ \mathbf{u}(0, l) \end{bmatrix}, \end{aligned} \quad (5)$$

$$\begin{aligned} \mathbf{x}(i_0 - 1, 1) = & \Phi_{i_0-1,1}^{i_0-2,0} [A_0(0, 0), B_0(0, 0)] \begin{bmatrix} \mathbf{x}(0, 0) \\ \mathbf{u}(0, 0) \end{bmatrix} \\ & + \sum_{k=1}^{i_0-1} \{ \Phi_{i_0-1,1}^{i_0-1-k,0} [A_1(k, 0), B_1(k, 0)] + \Phi_{i_0-1,1}^{i_0-2-k,0} [A_0(k, 0), B_0(k, 0)] \} \begin{bmatrix} \mathbf{x}(k, 0) \\ \mathbf{u}(k, 0) \end{bmatrix} \\ & + \Phi_{i_0-1,1}^{i_0-2,0} [A_2(0, 1), B_2(0, 1)] \begin{bmatrix} \mathbf{x}(0, 1) \\ \mathbf{u}(0, 1) \end{bmatrix} + \sum_{k=1}^{i_0-1} \Phi_{i_0-1,1}^{i_0-2-k,0} B_2(k, 1) \mathbf{u}(k, 1). \end{aligned} \quad (6)$$

以下证明定理在 $(i_0, 1), (1, j_0)$ 成立.

注意到如在式 (3) 中令 $j = 0$, 即有

$$\Phi_{k,l}^{i,0} = A_2(k-1, l) \Phi_{k-1,l}^{i-1,0}. \quad (7)$$

由此及 (1), (7) 两式即知

$$\begin{aligned} \mathbf{x}(i_0, 1) = & [A_0(i_0 - 1, 0), B_0(i_0 - 1, 0)] \begin{bmatrix} \mathbf{x}(i_0 - 1, 0) \\ \mathbf{u}(i_0 - 1, 0) \end{bmatrix} \\ & + [A_1(i_0, 0), B_1(i_0, 0)] \begin{bmatrix} \mathbf{x}(i_0, 0) \\ \mathbf{u}(i_0, 0) \end{bmatrix} + B_2(i_0 - 1, 1) \mathbf{u}(i_0 - 1, 1) \end{aligned}$$

$$\begin{aligned}
& + \Phi_{i_0,1}^{i_0-1,0} [A_0(0,0), B_0(0,0)] \begin{bmatrix} \mathbf{x}(0,0) \\ \mathbf{u}(0,0) \end{bmatrix} \\
& + \sum_{k=1}^{i_0-1} \{ \Phi_{i_0,1}^{i_0-k,0} [A_1(k,0), B_1(k,0)] + \Phi_{i_0,1}^{i_0-k-1,0} [A_0(k,0), B_0(k,0)] \} \begin{bmatrix} \mathbf{x}(k,0) \\ \mathbf{u}(k,0) \end{bmatrix} \\
& + \Phi_{i_0,1}^{i_0-1,0} [A_2(0,1), B_2(0,1)] \begin{bmatrix} \mathbf{x}(0,1) \\ \mathbf{u}(0,1) \end{bmatrix} + \sum_{k=1}^{i_0-2} \Phi_{i_0,1}^{i_0-1-k,0} B_2(k,1) \mathbf{u}(k,1) \\
& = \Phi_{i_0,1}^{i_0-1,0} [A_0(0,0), B_0(0,0)] \begin{bmatrix} \mathbf{x}(0,0) \\ \mathbf{u}(0,0) \end{bmatrix} + \Phi_{i_0,1}^{i_0-1,0} [A_2(0,1), B_2(0,1)] \begin{bmatrix} \mathbf{x}(0,1) \\ \mathbf{u}(0,1) \end{bmatrix} \\
& + \sum_{k=1}^{i_0} \{ \Phi_{i_0,1}^{i_0-k,0} [A_1(k,0), B_1(k,0)] + \Phi_{i_0,1}^{i_0-k-1,0} [A_0(k,0), B_0(k,0)] \} \begin{bmatrix} \mathbf{x}(k,0) \\ \mathbf{u}(k,0) \end{bmatrix} \\
& + \sum_{k=1}^{i_0-1} \Phi_{i_0,1}^{i_0-1-k,0} B_2(k,1) \mathbf{u}(k,1). \tag{8}
\end{aligned}$$

类似可证定理在 $(1, j_0)$ 也成立.

b) 假定定理在 $(i, j) < (i_0, j_0)$ 成立, 下面证在 (i_0, j_0) 也成立. 此时可以得到 $\mathbf{x}(i_0-1, j_0)$, $\mathbf{x}(i_0, j_0-1)$, $\mathbf{x}(i_0-1, j_0-1)$ 在上述假设之下的三个等式, 将这三个等式代入式 (1.1) 并整理即得

$$\begin{aligned}
\mathbf{x}(i_0, j_0) & = \Phi_{i_0, j_0}^{i_0-1, j_0-1} [A_0(0,0), B_0(0,0)] \begin{bmatrix} \mathbf{x}(0,0) \\ \mathbf{u}(0,0) \end{bmatrix} \\
& + \sum_{k=1}^{i_0} \{ \Phi_{i_0, j_0}^{i_0-k, j_0-1} [A_1(k,0), B_1(k,0)] \\
& + \Phi_{i_0, j_0}^{i_0-k-1, j_0-1} [A_0(k,0), B_0(k,0)] \} \begin{bmatrix} \mathbf{x}(k,0) \\ \mathbf{u}(k,0) \end{bmatrix} \\
& + \sum_{l=1}^{j_0} \{ \Phi_{i_0, j_0}^{i_0-1, j_0-l} [A_2(0,l), B_2(0,l)] \\
& + \Phi_{i_0, j_0}^{i_0-1, j_0-l-1} [A_0(0,l), B_0(0,l)] \} \begin{bmatrix} \mathbf{x}(0,l) \\ \mathbf{u}(0,l) \end{bmatrix} \\
& + \sum_{k=1}^{i_0} \sum_{l=1}^{j_0} [\Phi_{i_0, j_0}^{i_0-k-1, j_0-l-1} B_0(k,l) + \Phi_{i_0, j_0}^{i_0-k-1, j_0-l} B_2(k,l) \\
& + \Phi_{i_0, j_0}^{i_0-k, j_0-l-1} B_1(k,l)] \mathbf{u}(k,l). \tag{9}
\end{aligned}$$

综上所述, 由此及数学归纳法即知 (4) 式成立, 证毕.

三、能控性与能观性

定义 1. 由系统 (1) 在 2-D 矩形域 $[(0,0), (p,q)]$ 中能控, 系指对任何边界条件 $\mathbf{x}(i,0), \mathbf{x}(0,j)$, $i \in [0,p], j \in [0,q]$ 及任何向量 $\bar{\mathbf{x}} \in R^n$ 存在输入序列 $\mathbf{u}(i,j), (0,0) \leq (i,j) < (p,q)$, 使得 $\mathbf{x}(p,q) = \bar{\mathbf{x}}$.

定理 2. 系统 (1) 在 $[(0,0), (p,q)]$ 中能控的充要条件是

$$\text{rank}[\bar{M}_{p,q}^{0,0}, \bar{M}_{p,q}^{0,1}, \bar{M}_{p,q}^{1,0}, \dots, \bar{M}_{p,q}^{k,l}, \dots, \bar{M}_{p,q}^{p,q}] = n, \quad (10)$$

$$\text{其中 } \bar{M}_{p,q}^{k,l} = \Phi_{p,q}^{p-l-1, q-l-1} B_0(k, l) + \Phi_{p,q}^{p-k-1, q-l} B_2(k, l) + \Phi_{p,q}^{p-k, q-l-1} B_2(k, l), \\ k \in [0, p], l \in [0, q], \quad (11)$$

$$\text{在这里规定 } \Phi_{p,q}^{p,l} = \Phi_{p,q}^{k,q} = 0. \quad (12)$$

证明. 由定理 1 得

$$\begin{aligned} \mathbf{x}(p, q) &= \Phi_{p,q}^{p-1, q-1} A_0(0, 0) \mathbf{x}(0, 0) - \sum_{k=1}^p \Phi_{p,q}^{p-k, q-1} A_1(k, 0) \mathbf{x}(k, 0) \\ &\quad - \sum_{k=1}^p \Phi_{p,q}^{p-k-1, q-1} A_0(k, 0) \mathbf{x}(k, 0) - \sum_{l=1}^q \Phi_{p,q}^{p-1, q-l} A_2(0, l) \mathbf{x}(0, l) \\ &\quad - \sum_{l=1}^q \Phi_{p,q}^{p-1, q-l-1} A_0(0, l) \mathbf{x}(0, l) = \Phi_{p,q}^{p-1, q-1} B_0(0, 0) \mathbf{u}(0, 0) \\ &\quad + \sum_{k=1}^p \Phi_{p,q}^{p-k, q-1} B_1(k, 0) \mathbf{u}(k, 0) + \sum_{k=1}^p \Phi_{p,q}^{p-k-1, q-1} B_0(k, 0) \mathbf{u}(k, 0) \\ &\quad + \sum_{l=1}^q \Phi_{p,q}^{p-1, q-l} B_2(0, l) \mathbf{u}(0, l) + \sum_{l=1}^q \Phi_{p,q}^{p-1, q-l-1} B_0(0, l) \mathbf{u}(0, l) \\ &\quad + \sum_{k=1}^p \sum_{l=1}^q [\Phi_{p,q}^{p-k-1, q-l-1} B_0(k, l) + \Phi_{p,q}^{p-k-1, q-l} B_2(k, l) \\ &\quad + \Phi_{p,q}^{p-k, q-l-1} B_1(k, l)] \mathbf{u}(k, l) \\ &= \sum_{k=0}^p \sum_{l=0}^q \bar{M}_{p,q}^{k,l} \mathbf{u}(k, l) = [\bar{M}_{p,q}^{0,0}, \bar{M}_{p,q}^{0,1}, \bar{M}_{p,q}^{1,0}, \dots, \bar{M}_{p,q}^{k,l}, \dots, \\ &\quad \bar{M}_{p,q}^{p,q}] [\mathbf{u}(0, 0), \mathbf{u}(0, 1), \mathbf{u}(1, 0), \dots, \mathbf{u}(k, l), \dots, \mathbf{u}(p, q)]^T. \quad (13) \end{aligned}$$

显然由此及定义 1 即知系统 (1) 在 $[(0, 0), (p, q)]$ 中是能控的, 当且仅当式 (11) 成立.

定义 2. 称系统 (1) 在矩形域 $[(0, 0), (p, q)]$ 中是能观的, 系指不存在局部初始状态 $\mathbf{x}(0, 0) \neq 0$, 使得对零输入 $\mathbf{u}(i, j) = 0, (0, 0) \leq (i, j) \leq (p, q)$ 及零边界条件 $\mathbf{x}(i, 0), i \in [1, p], \mathbf{x}(0, j) = 0, j \in [1, q]$ 有输出 $\mathbf{y}(i, j) = 0, (0, 0) \leq (i, j) \leq (p, q)$.

定理 3. 系统 (1) 在 $[(0, 0), (p, q)]$ 中是能观的充要条件是

$$\text{rank} \begin{bmatrix} C(0, 0) \\ C(1, 1) A_0(0, 0) \\ C(1, 2) \Phi_{1,2}^{0,1} A_0(0, 0) \\ C(2, 1) \Phi_{2,1}^{1,0} A_0(0, 0) \\ \dots \\ C(p, q) \Phi_{p,q}^{p-1, q-1} A_0(0, 0) \end{bmatrix} = n, \quad (14)$$

其中 $\Phi_{k,i}^{j,i}$ 由式 (3) 定义.

证明. 将 $\mathbf{x}(i, 0) = 0, i \in [1, p], \mathbf{x}(0, j) = 0, j \in [1, q]$ 和 $\mathbf{u}(i, j) = 0, (0, 0) \leq (i, j) \leq (p, q)$ 代入式 (4) 即得 $\mathbf{x}(i, j) = \Phi_{i,j}^{i-1, j-1} A_0(0, 0) \mathbf{x}(0, 0), (i, j) \in [(0, 0), (p, q)]$. 代入式 (1.2) 即得

$$\mathbf{y}(i, j) = C(i, j)\Phi_{i,j}^{i-1, j-1}A_0(0, 0)\mathbf{x}(0, 0). \quad (15)$$

令 $(0, 0) \leq (i, j) \leq (p, q)$ 时, $\mathbf{y}(i, j) = 0$, 则由式 (15) 即得

$$\begin{bmatrix} C(0, 0) \\ C(1, 1)A_0(0, 0) \\ C(1, 2)\Phi_{1,2}^{0,1}A_0(0, 0) \\ C(2, 1)\Phi_{2,1}^{1,0}A_0(0, 0) \\ \dots \\ C(p, q)\Phi_{p,q}^{p-1, q-1}A_0(0, 0) \end{bmatrix} \mathbf{x}(0, 0) = 0, \quad (16)$$

而该方程对 $\mathbf{x}(0, 0)$ 有且仅有零解的充要条件是式 (14) 成立.

四、最小能量控制问题

考虑系统 (1) 及性能指标

$$I(\mathbf{u}) = \sum_{k=0}^p \sum_{l=0}^q \mathbf{u}^T(k, l)Q\mathbf{u}(k, l), \quad (17)$$

其中 Q 是适当维数的对称正定加权矩阵. 系统 (1) 的最小能量控制问题可以描述如下: 定式 (1) 的系数阵 $A_\alpha(k, l)$, $B_\alpha(k, l)$, $\alpha = 0, 1, 2$ 和加权阵 Q 及边界条件 $\mathbf{x}(k, 0)$, $k \in [1, p]$; $\mathbf{x}(0, l)$, $l \in [1, q]$, 求输入序列 $\mathbf{u}(k, l)$, $k \in [0, p]$, $l \in [0, q]$, 使系统从初始局部状态 $\mathbf{x}_0 = \mathbf{x}(0, 0)$ 转移到预定的终端局部状态 $\mathbf{x}_f = \mathbf{x}(p, q)$, 并使式 (17) 达到最小. 定义

$$D(p, q) = [\bar{M}_{p,q}^{0,0}, \bar{M}_{p,q}^{0,1}, \bar{M}_{p,q}^{1,0}, \dots, \bar{M}_{p,q}^{k,l}, \dots, \bar{M}_{p,q}^{p,q}], \quad (18)$$

$$\bar{W}_Q(p, q) = \sum_{(0,0) \leq (k,l) \leq (p,q)} M_{p,q}^{k,l} Q^{-1} (\bar{M}_{p,q}^{k,l})^T, \quad (19)$$

其中 $\bar{M}_{p,q}^{k,l}$ 由式 (11) 定义, 则有下列结论.

定理 4. 系统 (1) 在 $[(0, 0), (p, q)]$ 内能控, 当且仅当矩阵 $\bar{W}_Q(p, q)$ 正定.

证明. 由式 (18) 即知

$$\bar{W}_Q(p, q) = D(p, q) \underbrace{\begin{bmatrix} Q^{-1} & & & \\ & Q^{-1} & & \\ & & \ddots & \\ & & & Q^{-1} \end{bmatrix}}_{pq \text{ 个 } Q^{-1}} D^T(p, q). \quad (20)$$

对任何向量 \mathbf{r} 有

$$\mathbf{r}^T \bar{W}_Q(p, q) \mathbf{r} = [\mathbf{r}^T D(p, q)] \begin{bmatrix} Q^{-1} & & & \\ & Q^{-1} & & \\ & & \ddots & \\ & & & Q^{-1} \end{bmatrix} [D^T(p, q) \mathbf{r}] \geq 0, \quad (21)$$

$$\text{且 } \mathbf{r}^T \bar{W}_Q(p, q) \mathbf{r} = 0. \quad (22)$$

当且仅当

$$\mathbf{r}^T D(p, q) = 0. \quad (23)$$

由式(23)显然知: 当且仅当 $\text{rank}D(p, q) = n$ 时, 有

$$\mathbf{r}^T \bar{W}_Q(p, q) \mathbf{r} = 0 \iff \mathbf{r} = 0. \quad (24)$$

由此即知矩阵 $\bar{W}_Q(p, q)$ 正定当且仅当 $\text{rank}D(p, q) = n$, 即系统(1)在 $[(0, 0), (p, q)]$ 内能控, 证毕.

定义如下输入序列:

$$\hat{\mathbf{u}} = \hat{\mathbf{u}}(k, l) = Q^{-1}(\bar{M}_{p,q}^{k,l})^T \bar{W}_Q^{-1}(p, q) \bar{S}(p, q), \quad (25)$$

其中

$$\begin{aligned} \bar{S}(p, q) = & \mathbf{x}_f - \Phi_{p,q}^{p-1, q-1} A_0(0, 0) \mathbf{x}(0, 0) - \sum_{k=1}^p \Phi_{p,q}^{p-k, q-1} A_1(k, 0) \mathbf{x}(k, 0) \\ & - \sum_{k=1}^p \Phi_{p,q}^{p-k-1, q-1} A_0(k, 0) \mathbf{x}(k, 0) - \sum_{l=1}^q \Phi_{p,q}^{p-1, q-l} A_2(0, l) \mathbf{x}(0, l) \\ & - \sum_{l=1}^q \Phi_{p,q}^{p-1, q-l-1} A_0(0, l) \mathbf{x}(0, l). \end{aligned} \quad (26)$$

定理 5. 设系统(1)在矩形域 $[(0, 0), (p, q)]$ 内能控. 输入序列(25)使系统从 \mathbf{x}_0 转移到 \mathbf{x}_f , 且 $I(\hat{\mathbf{u}}) = \min$, 进一步地, 式(17)的最小值由下式给为

$$I(\hat{\mathbf{u}}) = \bar{S}^T(p, q) \bar{W}_Q^{-1}(p, q) \bar{S}(p, q).$$

证明. 1) 首先证明输入序列(25)使 $\mathbf{x}(p, q) = \mathbf{x}_f$, 这由如下解的计算即得, 将(11), (19), (25), (26)四式代入式(4)即有

$$\begin{aligned} \mathbf{x}(p, q) = & \Phi_{p,q}^{p-1, q-1} A_0(0, 0) \mathbf{x}(0, 0) + \sum_{k=1}^p \Phi_{p,q}^{p-k, q-1} A_1(k, 0) \mathbf{x}(k, 0) \\ & + \sum_{k=1}^p \Phi_{p,q}^{p-k-1, q-1} A_0(k, 0) \mathbf{x}(k, 0) + \sum_{l=1}^q \Phi_{p,q}^{p-1, q-l} A_2(0, l) \mathbf{x}(0, l) \\ & + \sum_{l=1}^q \Phi_{p,q}^{p-1, q-l-1} A_0(0, l) \mathbf{x}(0, l) + \sum_{k=0}^p \sum_{l=0}^q \bar{M}_{p,q}^{k,l} \hat{\mathbf{u}}(k, l) \\ = & \mathbf{x}_f - \bar{S}(p, q) + \sum_{k=0}^p \sum_{l=0}^q \bar{M}_{p,q}^{k,l} Q^{-1}(\bar{M}_{p,q}^{k,l})^T W_Q^{-1}(p, q) \bar{S}(p, q) \\ = & \mathbf{x}_f. \end{aligned} \quad (27)$$

2) 设 $\bar{\mathbf{u}} = \bar{\mathbf{u}}(k, l)$, 亦使系统从 \mathbf{x}_0 转移到 \mathbf{x}_f , 则从式(13)可得

$$\sum_{k=0}^p \sum_{l=0}^q \bar{M}_{p,q}^{k,l} \bar{\mathbf{u}}(k, l) = \sum_{k=0}^p \sum_{l=0}^q M_{p,q}^{k,l} \hat{\mathbf{u}}(k, l), \quad (28)$$

亦即

$$\sum_{k=0}^p \sum_{l=0}^q \bar{M}_{p,q}^{k,l} [\bar{\mathbf{u}}(k, l) - \hat{\mathbf{u}}(k, l)] = 0. \quad (29)$$

由此及式(25)可得

$$\sum_{k=0}^p \sum_{l=0}^q [\bar{\mathbf{u}}(k, l) - \hat{\mathbf{u}}(k, l)]^T Q \hat{\mathbf{u}}(k, l)$$

$$= \sum_{k=0}^p \sum_{l=0}^q [\bar{\mathbf{u}}(k, l) - \hat{\mathbf{u}}(k, l)] [\bar{M}_{p,q}^{k,l}]^T \bar{W}_0^{-1}(p, q) \bar{S}(p, q) = 0. \quad (30)$$

反复应用式 (30) 即知:

$$\begin{aligned} \sum_{k=0}^p \sum_{l=0}^q \bar{\mathbf{u}}^T(k, l) Q \bar{\mathbf{u}}(k, l) &= \sum_{k=0}^p \sum_{l=0}^q \hat{\mathbf{u}}^T(k, l) Q \hat{\mathbf{u}}(k, l) \\ &+ \sum_{k=0}^p \sum_{l=0}^q [\bar{\mathbf{u}}(k, l) - \hat{\mathbf{u}}(k, l)]^T Q [\bar{\mathbf{u}}(k, l) - \hat{\mathbf{u}}(k, l)] \end{aligned} \quad (31)$$

由于 (31) 式右端第二项总是非负, 故 $I(\bar{\mathbf{u}}) \geq I(\hat{\mathbf{u}})$.

把式 (25) 代入式 (17) 并利用 Q^{-1} , $\bar{W}_0^{-1}(p, q)$ 对称即得

$$\begin{aligned} I(\hat{\mathbf{u}}) &= \sum_{k=0}^{p-1} \sum_{l=0}^{q-1} \hat{\mathbf{u}}^T(k, l) Q \hat{\mathbf{u}}(k, l) \\ &= \sum_{k=0}^{p-1} \sum_{l=0}^{q-1} \bar{S}(p, q) W_0^{-1}(p, q) (\bar{M}_{p,q}^{k,l}) Q^{-1} (\bar{M}_{p,q}^{k,l})^T \bar{W}_0^{-1}(p, q) \bar{S}(p, q) \\ &= [\bar{S}^T(p, q) W_0^{-1}(p, q)] \left[\sum_{k=0}^{p-1} \sum_{l=0}^{q-1} (\bar{M}_{p,q}^{k,l}) Q^{-1} (\bar{M}_{p,q}^{k,l})^T \right] \bar{W}_0^{-1}(p, q) \bar{S}(p, q) \\ &= \bar{S}^T(p, q) W_0^{-1}(p, q) \bar{S}(p, q). \end{aligned} \quad \text{证毕}$$

五、结 论

本文讨论并解决了变系数 2-D 系统在一般模型下的状态响应及能控、能观性和最小能量控制问题, 推广了现有的有关结论. 显然当系统退化为相应的常系数情形时, 就得到了关于定系数 2-D 线性离散系统在一般模型下相应的有关结果^[4], 计算结果表明, 若将上述模型取为 Roesser 模型, 则本文的最小能量控制问题的有关结果与现有的结果^[3]一致.

参 考 文 献

- [1] Wieslaw Marszalek, Two-dimensional Discrete Model with Varying Coefficients Ph. D Dissertation, *IEE Proceedings*, 133(1986), No. 4.
- [2] Kurek, The General State-space Model for a Two-dimensional Linear Digital System, *IEEE Trans. on Automat. Contr.*, 30(1985), No. 6.
- [3] 杨成梧, 二维线性多变量系统, 华东工学院, 1987.
- [4] Tadeusz Kacorek et al., Minimum energy Control for General Model of 2-D Linear Systems, *Int. J. Contr.*, 47(1988), 5, 1555—1562.

**THE RESPONSE FORMULA AND CONTROLLABILITY &
OBSERVABILITY OF THE GENERAL STATE-SPACE
MODEL FOR A TWO-DIMENSIONAL LINEAR
DISCRETE SYSTEM WITH VARYING
COEFFICIENTS**

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ABSTRACT

In this paper, we consider the general state-space model for the two-dimensional linear discrete systems with varying coefficients. The general formula for response of such systems is derived. Necessary and sufficient conditions for local controllability and observability are obtained. In addition, minimum energy problem is solved.

Key words: 2-D systems; complicated systems; linear systems; controllability.