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Extensions of Symmetric Rings

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Abstract: We first consider properties and basic extensions of symmetric rings. We next argue about the symmetry of some kinds of polynomial rings, and show that if R is a reduced ring then $R[x]/(x^n)$ is a symmetric ring, where (x^n) is the ideal generated by x^n and n is a positive integer. Consequently, we prove that for a right Ore ring R with Q its classical right quotient ring, R is symmetric if and only if Q is symmetric.

Key words: symmetric ring; trivial extension; polynomial ring; classical right quotient ring.
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1. Introduction

Throughout this paper, all rings are associative with identity, and n is a positive integer. A ring R is called symmetric if $abc = 0$ implies $bac = 0$ for $a, b, c \in R$. Anderson-Camillo[1] took the term ZC_3 for this notion. A ring R is called reversible if $ab = 0$ implies $ba = 0$ for $a, b \in R$. A ring R is called semicommutative if $ab = 0$ implies $aRb = 0$ for $a, b \in R$. Reduced rings (i.e., rings with no nonzero nilpotent elements) are symmetric by [1, Theorem I.3], commutative rings are clearly symmetric, symmetric rings are clearly reversible, and reversible rings are semicommutative by [2, Proposition 1.3]. But these implications are irreversible by [1, 3], and symmetric rings without identity need not be reversible by [4]. In this paper, we continue to study symmetric rings and related rings. First we consider properties and basic extensions of symmetric rings. We next argue about the symmetry of some kinds of polynomial rings, and show that if R is a reduced ring then $R[x]/(x^n)$ is a symmetric ring, where (x^n) is the ideal generated by x^n. Consequently, we prove that for a right Ore ring R with Q its classical right quotient ring, R is symmetric if and only if Q is symmetric.

2. Symmetric rings and related rings

We have the following lemma by [5, Proposition 1], while Anderson-Camillo proved the result in [1, Theorem I.1].

Lemma 2.1 *For a ring R the following statements are equivalent:*

(1) R is symmetric;

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(2) For $r_1, r_2, \ldots, r_n (n \geq 3) \in R$, $r_1 r_2 \cdots r_n = 0$ implies $r_{\sigma(1)} r_{\sigma(2)} \cdots r_{\sigma(n)} = 0$ for any permutation σ of the set $\{1, 2, \ldots, n\}$.

Given a ring R and a bimodule ${}_RM_R$, the trivial extension of R by M is the ring $T(R, M) = R \bigoplus M$ with the usual addition and the following multiplication

$$(r_1, m_1)(r_2, m_2) = (r_1 r_2, r_1 m_2 + m_1 r_2).$$

This is isomorphic to the ring of all matrics $\begin{pmatrix} r & m \\ 0 & r \end{pmatrix}$, where $r \in R$, $m \in M$ and the usual matrix operations are used.

Proposition 2.2 *Let R be a reduced ring. Then $T(R, R)$ is a symmetric ring.*

Proof Let

$$\begin{pmatrix} a & b \\ 0 & a \end{pmatrix}, \begin{pmatrix} c & d \\ 0 & c \end{pmatrix}, \begin{pmatrix} e & f \\ 0 & e \end{pmatrix} \in T(R, R)$$

with

$$\begin{pmatrix} a & b \\ 0 & a \end{pmatrix} \begin{pmatrix} c & d \\ 0 & c \end{pmatrix} \begin{pmatrix} e & f \\ 0 & e \end{pmatrix} = 0.$$

Then $ace = 0$ and $acf + ade + bce = 0$. Since R is reduced, we have $cae = cea = eac = 0$, and so $0 = ceacf + ceade + cebce = cebce$, which implies $(bce)^2 = 0$, hence $bce = 0 = cbe$. Similarly, we have $ade = acf = 0$ and so $dae = caf = 0$. Hence

$$\begin{pmatrix} c & d \\ 0 & c \end{pmatrix} \begin{pmatrix} a & b \\ 0 & a \end{pmatrix} \begin{pmatrix} e & f \\ 0 & e \end{pmatrix} = 0.$$

Based on Proposition 2.2, we may conjecture that if a ring R is reduced, then

(1) $T = \begin{pmatrix} R & R \\ 0 & R \end{pmatrix}$ is symmetric.

(2) $S = \left\{ \begin{pmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{pmatrix} \middle| a, b, c, d \in R \right\}$ is symmetric.

However the following example erases the possibility.

Example 2.3 Let R be a reduced ring. Then

(1) $T = \begin{pmatrix} R & R \\ 0 & R \end{pmatrix}$ is not symmetric. Because

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = 0,$$

but

$$\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \neq 0.$$

(2) $S = \left\{ \begin{pmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{pmatrix} \middle| a, b, c, d \in R \right\}$ is not symmetric. We have that S is semicommutative but not reversible by [3, Proposition 1.2], so S is not symmetric.

Lemma 2.4 *Symmetric rings are semicommutative.*

Example 2.3 shows that the converse of Lemma 2.4 is not true in general.

Lemma 2.5 *The class of symmetric rings is closed under subrings and direct products.*

The following example shows that if R is symmetric, then $T(R,R)$ is not symmetric.

Example 2.6 Let $\mathbb{H}$ be the Hamilton quaternions over the real number field and R be the trivial extension of $\mathbb{H}$ by $\mathbb{H}$. Then R is symmetric by Proposition 2.2. Let S be the trivial extension of R by R. However $S = T(R,R)$ is not semicommutative by [3, Example 1.7], hence S is not symmetric by Lemma 2.4.

One may suspect that R is a symmetric ring if for any symmetric nonzero proper ideal I of R, R/I symmetric, where I is considered as a ring without identity. However the following example erases the possibility.

Example 2.7 Let S be a division ring and consider the ring $R = \begin{pmatrix} S & S \\ 0 & S \end{pmatrix}$. Then R is not symmetric by Example 2.3. First notice that R has only the following nonzero proper ideals

$$I_1 = \begin{pmatrix} 0 & S \\ 0 & S \end{pmatrix},\ I_2 = \begin{pmatrix} S & S \\ 0 & 0 \end{pmatrix},\ I_3 = \begin{pmatrix} 0 & S \\ 0 & 0 \end{pmatrix}.$$

The following results are based on Lemma 2.5 and the result that division ring is clearly reduced ring.

(1) $I_1 = \begin{pmatrix} 0 & S \\ 0 & S \end{pmatrix}$ is not symmetric. Because

$$\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = 0,$$

but

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \neq 0.$$

(2) $I_2 = \begin{pmatrix} S & S \\ 0 & 0 \end{pmatrix}$. Let

$$\alpha = \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix},\ \beta = \begin{pmatrix} c & d \\ 0 & 0 \end{pmatrix},\ \gamma = \begin{pmatrix} e & f \\ 0 & 0 \end{pmatrix} \in I_2$$

with $\alpha\beta\gamma = 0$. Then $ace = acf = 0$ and so $cae = caf = 0$, hence $\beta\alpha\gamma = 0$. Therefore, I_2 is symmetric. R/I_2 is symmetric because $R/I_2 \cong S$.

(3) $I_3 = \begin{pmatrix} 0 & S \\ 0 & 0 \end{pmatrix}$. We easily prove that I_3 is symmetric and R/I_3 is also symmetric by $R/I_3 \cong S \bigoplus S$.

But we have an affirmative answer if we take a stronger condition as follows.

Proposition 2.8 *Suppose that R/I is a symmetric ring for some ideal I of a ring R, If I is reduced then R is symmetric.*

Proof Let $a, b, c \in R$ with $abc = 0$ and $\bar{a} = a + I$. Then $\bar{a}\bar{b}\bar{c} = 0$, which implies $\bar{b}\bar{a}\bar{c} = 0$ by condition, and so $bac \in I$. By R/I symmetric, we have that R/I is semicommutative. So R

is semicommutative by [6, Theorem 6], hence $abc=0$ implies $acbc=0$ implies $acbac=0$. So $bac=0$ and R is symmetric.

Dually, one may conjecture that if a ring R is symmetric then R/I is also symmetric for any ideal I in R. However, according to [3, Example 2.1], let $\mathbb{Z}_2$ be the field of integers modulo 2 and $A=\mathbb{Z}_2\langle a_0,a_1,a_2,b_0,b_1,b_2,c\rangle$ be the free algebra of polynomials with zero constant terms in noncommutative indeterminates $a_0,a_1,a_2,b_0,b_1,b_2,c$ over $\mathbb{Z}_2$. I is the ideal of the ring $\mathbb{Z}_2+A$, and generated by the following elements

$$a_0b_0, a_0b_1+a_1b_0, a_0b_2+a_1b_1+a_2b_0, a_1b_2+a_2b_1, a_2b_2, a_0rb_0, a_2rb_2,$$

$$b_0a_0, b_0a_1+b_1a_0, b_0a_2+b_1a_1+b_2a_0, b_1a_2+b_2a_1, b_2a_2, b_0ra_0, b_2ra_2,$$

$$(a_0+a_1+a_2)r(b_0+b_1+b_2), (b_0+b_1+b_2)r(a_0+a_1+a_2), \quad \text{and} \quad r_1r_2r_3r_4,$$

where $r,r_1,r_2,r_3,r_4\in A$. We have that $(\mathbb{Z}_2+A)[x]$ is a domain and so clearly symmetric, but the factor ring $(\mathbb{Z}_2+A)[x]/I[x]\cong((\mathbb{Z}_2+A)/I)[x]$ is not semicommutative and so not symmetric. Specially, I is an ideal of R that is an annihilator in R, the argument is true, and Proposition 1.14 (1) in [3] is the corollary of the following proposition.

Proposition 2.9 *Let R be a symmetric ring and I be an ideal of R that is an annihilator in R. Then R/I is a symmetric ring.*

Proof Set $I=r_R(J)$ with $J\subseteq R$ and $\bar{r}=r+I$. Let $\bar{a}\bar{b}\bar{c}=0$. Then $abc\in I$ and $Jabc=0$, so $Jbac=0$ since R is symmetric. We have $\bar{b}\bar{a}\bar{c}=0$ and hence R/I is symmetric.

Proposition 2.10 *For a ring R the following statements are equivalent:*

(1) R is symmetric;

(2) eR and $(1-e)R$ are symmetric for some central idempotent e of R;

(3) $\Delta^{-1}R$ is symmetric, where Δ is a multiplicatively closed subset of R consisting of central regular elements.

Proof It suffices to show (2)$\Longrightarrow$(1) and (1)$\Longrightarrow$(3) by Lemma 2.5.

(2)$\Longrightarrow$(1). Suppose that eR, $(1-e)R$ are symmetric, then R is symmetric. Let $a,b,c\in R$ with $abc=0$. Then $eabc=0$ and $(1-e)abc=0$, so we have $ebac=0$ and $(1-e)bac=0$ by supposition. Hence $bac=ebac+(1-e)bac=0$ and so R is symmetric.

(1)$\Longrightarrow$(3). Let $\alpha\beta\gamma=0$ with $\alpha=u^{-1}a,\beta=v^{-1}b,\gamma=w^{-1}c$, where $u,v,w\in\Delta$ and $a,b,c\in R$. Since Δ is contained in the center of R, we have $0=\alpha\beta\gamma=u^{-1}av^{-1}bw^{-1}c=u^{-1}v^{-1}w^{-1}abc$ and so $abc=0$. But R is symmetric by condition, so $bac=0$ and we have $\beta\alpha\gamma=v^{-1}bu^{-1}aw^{-1}c=v^{-1}u^{-1}w^{-1}bac=0$. Hence $\Delta^{-1}R$ is symmetric.

3. Polynomial rings and classical quotient rings

In this section we are concerned with the symmetry of two important kinds of extensions of symmetric rings. A ring R is called Armendariz if whenever polynomials $f(x)=$

$\sum_{i=0}^{m} a_i x^i, g(x) = \sum_{j=0}^{n} b_j x^j \in R[x]$ satisfy $f(x)g(x) = 0$, then $a_i b_j = 0$ for each i, j (see [8] for details). Reduced rings are Armendariz by [9, Lemma 1] and reduced rings are symmetric by [1, Theorem I.3]. Hence there may be relations between Armendariz rings and symmetric rings. The ring of Laurent polynomials in x with coefficients in a ring R consists of all formal sums $\sum_{i=k}^{n} m_i x^i$ with obvious addition and multiplication, where $m_i \in R$ and k, n are (possibly negative) integers. Denote it by $R[x; x^{-1}]$.

Lemma 3.1 *For a ring R, $R[x]$ is symmetric if and only if $R[x; x^{-1}]$ is symmetric.*

Proof By Lemma 2.5, it suffices to show the necessity. Let $\Delta = \{1, x, x^2, \ldots\}$. Then clearly Δ is a multiplicatively closed subset of $R[x]$. From $R[x; x^{-1}] = \Delta^{-1}R[x]$ it follows that $R[x; x^{-1}]$ is symmetric by Proposition 2.10.

Proposition 3.2 *Let R be an Armendariz ring. Then the following statements are equivalent:*

(1) R is symmetric;

(2) $R[x]$ is symmetric;

(3) $R[x; x^{-1}]$ is symmetric.

Proof By Lemmas 2.5 and 3.1, it suffices to show (1)$\Longrightarrow$(2). Let $f(x) = \sum_{i=0}^{m} a_i x^i, g(x) = \sum_{j=0}^{n} b_j x^j, h(x) = \sum_{k=0}^{l} c_k x^k \in R[x]$ such that $fgh = 0$. Then since R is Armendariz, $a_i b_j c_k = 0$ by [7, Proposition 1] for all i, j, k. But R is symmetric so $b_j a_i c_k = 0$ for all i, j, k. Thereby, $gfh = 0$ and so $R[x]$ is symmetric.

The following result, similar to [7, Theorem 5], extends the class of symmetric ring, and Theorem 2.5 in [3] is its corollary .

Theorem 3.3 *If R is reduced ring, then $R[x]/(x^n)$ is a symmetric ring, where (x^n) is the ideal generated by x^n.*

Proof Let $u = x + (x^n)$. Then $R[x]/(x^n) \cong R[u]$. Set $A = a_0 + a_1 u + \cdots + a_{n-1}u^{n-1}, B = b_0 + b_1 u + \cdots + b_{n-1}u^{n-1}$, $C = c_0 + c_1 u + \cdots + c_{n-1}u^{n-1} \in R[u]$ such that $ABC = 0$. Notice that $a_i b_j c_k u^{i+j+k} = 0$ for all $i + j + k \geq n$. So it suffices to check the cases of $i + j + k < n$. From $ABC = 0$, we have $0 = ABC = \sum_{l=0}^{n-1} \left(\sum_{i+j+k=l} a_i b_j c_k u^l\right)$ and so

$$\sum_{i+j+k=l} a_i b_j c_k = 0, \quad l = 0, 1, \ldots, n-1. \tag{1}$$

Let $l = 0$. Then $i = j = k = 0$. We have $a_0 b_0 c_0 = 0$ by (1). Inductively, we may assume that $a_i b_j c_k = 0$ since $l = i + j + k < n - 1$, and prove that $a_i b_j c_k = 0$ since $l = i + j + k = n - 1$. Set $Q = \{0, 1, \ldots, l\}$, with the usual order. Consider $Q^3 = Q \times Q \times Q$ with the lexicographic order. Set $W = \{(i, j, k) \in Q^3 | i + j + k = l\}$. Clearly, $(0, 0, l) \in W$ is the minimal element of W. From (1),

$$\sum_{i+j+k=l} a_i b_j c_k = 0.$$

If $i \geq 1$, then $0 + j + k < l = n - 1$. Thus, by the assumption, $a_0 b_j c_k = 0$ implies $b_j c_k a_0 = 0$.

Hence

$$0=(\sum_{\substack{i+j+k=l\\ i\geq 1}} a_ib_jc_k)a_0+(\sum_{0+j+k=l} a_0b_jc_k)a_0=(\sum_{0+j+k=l} a_0b_jc_k)a_0. \tag{2}$$

If $j\geq 1$, then $0+0+k<l$. Thus, by the assumption, $a_0b_0c_k=0$ implies $c_ka_0b_0=0$. From (2),

$$0=(\sum_{j+k=l} a_0b_jc_k)a_0b_0=(a_0b_0c_l)a_0b_0+(\sum_{\substack{j+k=l\\ j\geq 1}} a_0b_jc_k)a_0b_0=(a_0b_0c_l)a_0b_0.$$

It follows that $a_0b_0c_l=0$. Now supposing that $(r,s,t)\in W$ such that $a_ib_jc_k=0$ when $(i,j,k)\in W$ with $(i,j,k)<(r,s,t)$, we will show $a_rb_sc_t=0$. If we multiply (1) on the right side by a_r, then

$$0=(\sum_{i+j+k=l} a_ib_jc_k)a_r=(\sum_{r+j+k=l} a_rb_jc_k)a_r+(\sum_{\substack{i+j+k=l\\ i<r}} a_ib_jc_k)a_r+(\sum_{\substack{i+j+k=l\\ i>r}} a_ib_jc_k)a_r.$$

If $i<r$, then $(i,j,k)<(r,s,t)$. Thus, $a_ib_jc_k=0$ by the assumption. If $i>r$, then $r+j+k<l$. Thus, $a_rb_jc_k=0$ and so $b_jc_ka_r=0$. Hence

$$(\sum_{r+j+k=l} a_rb_jc_k)a_r=0. \tag{3}$$

If we multiply (3) on the right side by b_s, then

$$0=(\sum_{r+j+k=l} a_rb_jc_k)a_rb_s=(a_rb_sc_t)a_rb_s+(\sum_{\substack{r+j+k=l\\ j<s}} a_rb_jc_k)a_rb_s+(\sum_{\substack{r+j+k=l\\ j>s}} a_rb_jc_k)a_rb_s.$$

If $j<s$, then $(r,j,k)<(r,s,t)$. If $j>s$, then $r+s+k<l$. Thus $a_rb_sc_k=0$ and so $c_ka_rb_s=0$. Hence $a_rb_sc_ta_rb_s=0$ and so $a_rb_sc_t=0$. So we have shown that $a_ib_jc_k=0$ when $(i,j,k)\in W$. By induction, $a_ib_jc_k=0$ for any $i+j+k=0,1,2,\ldots,n-1$ and so $b_ja_ic_k=0$. Therefore, $BAC=0$. i.e., $R[x]/(x^n)$ is symmetric.

Proposition 2.2 can be a corollary of Theorem 3.3 since $T(R,R)\cong R[x]/(x^2)$. A ring R is called right Ore ring if given $a,b\in R$ with b regular there exist $a_1,b_1\in R$ with b_1 regular such that $ab_1=ba_1$. It is well-known that R is a right Ore ring if and only if the classical right quotient ring of R exists.

Theorem 3.4 *Let R be a right Ore ring and Q be the classical right quotient ring of R. Then R is symmetric if and only if Q is symmetric.*

Proof By Lemma 2.5, it suffices to show the necessity. Let $\alpha=ab^{-1},\beta=cd^{-1},\gamma=ef^{-1}$ with $\alpha\beta\gamma=0$. By the assumption, for b,c there exist $b_1,c_1\in R$ with b_1 regular such that $bc_1=cb_1$ and $b^{-1}c=c_1b_1^{-1}$; for d,e there exist $d_1,e_1\in R$ with d_1 regular such that $de_1=ed_1$ and $d^{-1}e=e_1d_1^{-1}$; for b_1,e_1 there exist $b_2,e_2\in R$ with b_2 regular such that $b_1e_2=e_1b_2$ and $b_1^{-1}e_1=e_2b_2^{-1}$. So $0=\alpha\beta\gamma=ab^{-1}cd^{-1}ef^{-1}=ac_1b_1^{-1}e_1d_1^{-1}f^{-1}=ac_1e_2b_2^{-1}d_1^{-1}f^{-1}=(ac_1e_2)(fd_1b_2)^{-1}$, which implies $ac_1e_2=0$. But, by the assumption, for d,a there exist $d_2,a_2\in R$ with d_2 regular such that

$da_2 = ad_2$ and $d^{-1}a = a_2d_2^{-1}$; for b, e there exist $b_3, e_3 \in R$ with b_3 regular such that $be_3 = eb_3$ and $b^{-1}e = e_3b_3^{-1}$; for e_3, d_2 there exist $e_4, d_4 \in R$ with d_4 regular such that $d_2e_4 = e_3d_4$ and $d_2^{-1}e_3 = e_4d_4^{-1}$. So $\beta\alpha\gamma = cd^{-1}ab^{-1}ef^{-1} = ca_2d_2^{-1}e_3b_3^{-1}f^{-1} = ca_2e_4d_4^{-1}b_3^{-1}f^{-1} = (ca_2e_4)(fb_3d_4)^{-1}$. Next we prove $ca_2e_4 = 0$. The following computations are based on Lemma 2.1 and R symmetric. Multiply $ac_1e_2 = 0$ on the left hand side by b, then $0 = bac_1e_2 = abc_1e_2 = acb_1e_2 = b_1ace_2$ implies $ace_2 = 0$. Similarly multiply $ace_2 = 0$ on the left hand side by d_2, then $a_2ce_2 = 0$; multiply $a_2ce_2 = 0$ on the right hand side by b_1, then $a_2ce_1 = 0$; multiply $a_2ce_1 = 0$ on the right hand side by d, then $a_2ce = 0$; multiply $a_2ce = 0$ on the right hand side by b_3, then $a_2ce_3 = 0$; multiply $a_2ce_3 = 0$ on the right hand side by d_4, then $a_2ce_4 = 0$, and so $ca_2e_4 = 0$ implies $\beta\alpha\gamma = 0$. Hence Q is symmetric.

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对称环的扩张

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摘要: 本文首先考虑了对称环的性质和基本的扩张. 其次讨论了几种多项式环的对称性, 且证明了：如果 R 是约化环, 则 $R[x]/(x^n)$ 是对称环, 其中 (x^n) 是由 x^n 生成的理想, n 是一个正整数. 最后证明了：对一个右 Ore 环 R, R 是对称环当且仅当 R 的古典右商环 Q 是对称环.

关键词: 对称环; 平凡扩张; 多项式环; 古典右商环.