*Original Article*

# Nonlinear Large-Deflection Analysis of Orthodontic Wires

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**Abstract:** The purposes of this study were (1) to measure the nonlinear force-deflection behavior of selected orthodontic wires using a conventional tensile test, (2) to extend a mathematical model for simulating the force system produced by orthodontic wires based on the small-deflection linear theory to the large-deflection nonlinear theory, and (3) to examine the effects of the cross-section and mechanical properties of orthodontic wires on nonlinear characteristics. A method for extending a mathematical model for simulating the force system produced by orthodontic wires based on the small-deflection linear theory to the large-deflection nonlinear theory was established, and this can provide a clear view of the true nature of orthodontic wires. Furthermore, our results demonstrated that the nonlinear properties of orthodontic wires were affected more by the cross-sectional shape than by mechanical properties. (*Angle Orthod* 2004; 74:112–117.)

**Key Words:** Nonlinear large-deformation analysis; Mechanical property; Wire cross section; Force system

## INTRODUCTION

Orthodontic tooth movement is greatly influenced by the characteristics of the applied force, including its magnitude and direction, and the physiological condition of the periodontal tissue of individual patients.[1] Thus, the theoretical prediction of the forces and moments produced by orthodontic appliance is important to control treatment.[2–5] In many previous studies, the mathematical analysis of the force system produced by orthodontic appliances has been limited to the small-deflection linear theory.[6–10] However, the quantitative accuracy of such simulations is impaired because most appliances undergo large activations in clinical practice.[11] Koenig and Burstone[11] developed a new mathematical model for simulating the force system produced by orthodontic appliances, based on the large-deflection nonlinear theory. By comparing the results with those obtained with the small-deflection theory, the effectiveness of the large-deflection method was confirmed. Chen et al[12] created large-deformation nonlinear finite-element models of orthodontic springs. They also suggested the validity of the nonlinear large-deflection method. Unfortunately, the previous studies did not examine the effects of the cross section and mechanical properties of orthodontic wires on nonlinear characteristics. In addition, the theory underlying the calculations was not clarified.

The purposes of this study were (1) to measure the nonlinear force-deflection behavior of selected orthodontic wires using a conventional tensile test, (2) to extend a mathematical model for simulating the force system produced by orthodontic wires based on the small-deflection linear theory to the large-deflection nonlinear theory, and (3) to examine the effects of the cross section and mechanical properties of orthodontic wires on their nonlinear characteristics.

## MATERIALS AND METHODS

### Tensile test

Stainless steel wires (0.016-inch round and 0.016 $\times$ 0.022–inch rectangular; Stainless steel archwires, Ormco, Glendora, CA), cobalt-chromium wires (0.016-inch round), and heat-treated cobalt-chromium wires (0.016-inch round) (Elgiloy-Blue, RMO, Denver CO) were selected. Tensile strength was tested on a testing machine using an intercross head distance of 80 mm and a crosshead speed of 20 mm/minute. Five samples of each wire were tested. Stress-strain curves were determined from the load-deflection data. The 0.1 percent yield strength, elastic modulus, and breaking point were calculated.[13,14] The stress-strain curves were used to determine the deflection curves for each wire.

### Nonlinear large-deflection analysis

A four-point support statically indeterminate beam was considered as a simulation model of the force system pro-

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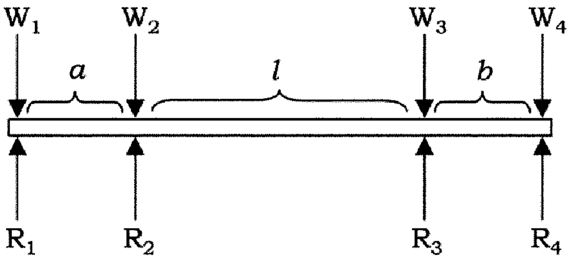


**FIGURE 1.** Four-point support statically indeterminate beam was considered as a simulation model for the force system produced by a straight wire in two brackets. R1−R4 and W1−W4 are the concentrated loads on the brackets, and *a, b,* and *l* are the span lengths of the orthodontic wires (*a* and *b* are the bracket widths, *l* is the interbracket distance).

duced by a straight wire in two brackets (Figure 1). For both force and moment, in equilibrium, the following equations are obtained:

$$R_1 + R_2 + R_3 + R_4 = W_1 + W_2 + W_3 + W_4 \quad (1)$$

$$(W_1 - R_1)(a + l/2) + (W_2 - R_2)l/2 = (W_3 - R_3)l/2 + (W_4 - R_4)(b + l/2) \quad (2)$$

where $R_1 - R_4$ and $W_1 - W_4$ are the concentrated loads from the brackets, and *a*, *b*, and *l* are the span lengths for each orthodontic wire (*a* and *b* are the bracket widths and *l* is the interbracket distance). The shear stress ($F_1 - F_3$) and moments ($M_1 - M_3$) applied to each cross section of the orthodontic wire were expressed as follows:

for $0 \leqq x < a$;

$$F_1 = W_1 - R_1, \qquad M_1 = (W_1 - R_1)x$$

$a \leqq x < a + 1$;

$$F_2 = W_1 - R_1 + W_2 - R_2,$$

$$M_2 = (W_1 - R_1)x + (W_2 - R_2)(x - a)$$

$a + l \leqq x < a + b + l$;

$$F_3 = W_1 - R_1 + W_2 - R_2 + W_3 - R_3,$$

$$M_3 = (W_1 - R_1)x + (W_2 - R_2)(x - a) + (W_3 - R_3)(x - a - l)$$

where x is an arbitrary span length of orthodontic wire. For $P_1 = W_1 - R_1$, $P_2 = W_2 - R_2$, $P_3 = W_3 - R_3$, and $P_4 = W_4 - R_4$, we can rearrange the above equations as follows:

$$P_1 + P_2 + P_3 + P_4 = 0, \quad (3)$$

$$P_1(a + l/2) + P_2(l/2) = P_3(l/2) + P_4(b + l/2) \quad (4)$$

$$F_1 = P_1, \quad (5)$$

$$M_1 = P_1x \quad (6)$$

$$F_2 = P_1 + P_2, \quad (7)$$

$$M_2 = P_1x + P_2(x - a) \quad (8)$$

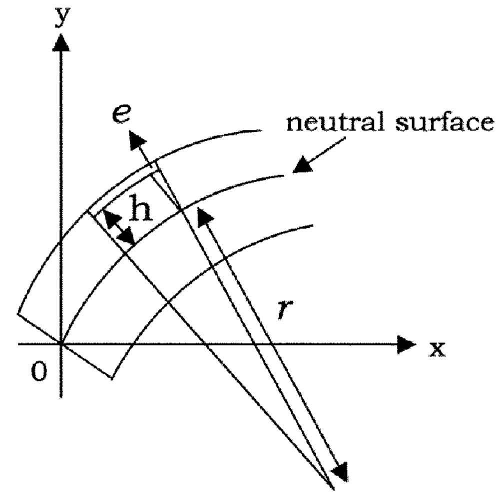


**FIGURE 2.** The strain (*e*) in the longitudinal section of the beam. *r* is the radius of curvature of the neutral surface, and h is an arbitrary distance from the neutral surface.

$$F_3 = P_1 + P_2 + P_3, \quad (9)$$

$$M_3 = P_1x + P_2(x - a) + P_3(x - a - l) \quad (10)$$

Next, the following relation between the radius of curvature ρ of the orthodontic wire that produced bending and deflection curve y = y (x) was derived:

$$\frac{1}{\rho} = \frac{\pm d^2y/dx^2}{[1 + (dy/dx)^2]^{3/2}} \quad (11)$$

Generally, for the bending of an orthodontic wire that does not obey Hooke's law, the longitudinal expansion or shrinking at any position because of bending is proportional to the distance from a neutral plane to that position.[15] Therefore, the strain *e* in a longitudinal section of the wire was determined from the neutral surface position and the radius of curvature of the neutral surface *r* (Figure 2).

$$e = h/r \quad (12)$$

where h is an arbitrary distance from the neutral surface.

Next, we consider how to determine the neutral surface for a wire with a round cross section. The relationship between an arbitrary distance from a neutral surface and the cross-sectional width can be expressed as

$$b = 2\sqrt{(d/2)^2 - (h + a)^2} \quad (13)$$

where a is the distance from the centerline of the cross section to the neutral surface, b is the cross-sectional width, and d is the maximum width of the cross section (Figure 3). In this study, we assumed that a = 0. By substituting equation (12) with equation (13), the following equation was obtained;

$$b = 2\sqrt{(d/2 + re)(d/2 - re)}. \quad (14)$$

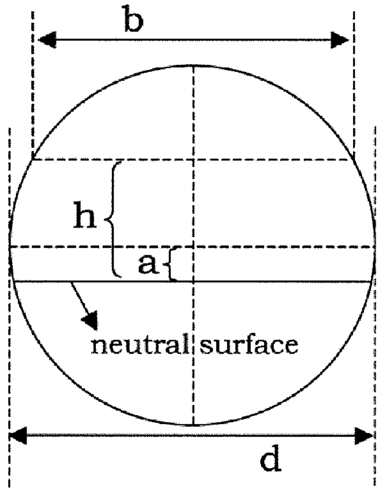


**FIGURE 3.** The relationship between an arbitrary distance from the neutral surface and the cross-sectional width for a round cross section. *a* is the distance from the centerline of the cross section to the neutral surface, *b* is the cross-sectional width, d is the maximum width of the cross section, and h is an arbitrary distance from the neutral surface.

By considering a condition in which the moment is in equilibrium, the following equation was obtained,

$$\int_{-d/2}^{d/2} b\sigma h \, dy = r^2 \int_{e_2}^{e_1} b\sigma e \, de = M \qquad (15)$$

where σ is stress, $e_1 = (d/2)/r$, $e_2 = -(d/2)/r$, and de is infinitesimal strain.

To analyze orthodontic wires with nonlinear tensile properties, it is necessary to clarify the relationship between the bending moment and the deflection curve. Considering equation (14), $e_1$ and $e_2$, equation (15) can be written as

$$M = \frac{2}{(1/r)^3} \int_{-(2/d)/r}^{(2/d)/r} \sqrt{-\left(e - \frac{2/d}{r}\right)\left(e + \frac{2/d}{r}\right)} \sigma e \, de. \qquad (16)$$

Therefore, calculating M by substituting the value of 1/r, the relationship between the curvature and the moment was obtained. For a rectangular cross-section, the following equation was used:

$$M = kr^2 \int_{-d/2}^{d/2} \sigma e \, de \qquad (17)$$

where k is the width of the cross section. The stress-strain curve obtained from the tensile test was approximated by a cubic polynomial, which was used as the stress, σ, in equations (16 and 17). In this study, we assumed that the stress-strain curve for the compression side was symmetric to that for the tensile side about the origin. The resulting relationship between the curvature 1/r and moment M for each orthodontic wire also was approximated by a cubic polynomial. In the bending of a beam, the bending moment $f_M(M)$ was considered to be a function of x.

$$f_M(M) = g(x) = m_0 + m_1 M + m_2 M^2 + m_3 M^3$$

Generally, the relationship between the curvature and deflection curve was expressed as

$$1/r = d^2y/dx^2 = g(x).$$

Therefore, the following equations were obtained,

$$\frac{dy}{dx} = \int g(x)\, dx + C_1$$

$$y = \int \left[\int g(x)\, dx + C_1\right] dx + C_2$$

$$= \int\int g(x)\, dx\, dx + C_1 x + C_2 \qquad (18)$$

where $C_1$ and $C_2$ are integral constants. Equation (18) was applied to each span length (Figure 1) of orthodontic wire, each of which has a boundary condition. As a result, simultaneous equations with 10 unknowns were deduced. The Newton-Raphson method was used for calculations, and the initial value was obtained from the small-deflection linear theory.

Nonlinear characteristics of the orthodontic wires were evaluated in terms of the difference in 10 unknowns between the small-deflection linear theory and the large-deflection nonlinear theory. Calculations were carried out for right and left bracket angles that were changed in steps of one degree from zero° to eight ° in the same direction, respectively. The four other parameters remained fixed (bracket slot size = 0.46 mm, bracket width-right = 4.00 mm, bracket width-left = 4.00 mm, interbracket distance = 7.00 mm).

## RESULTS

Figure 4 shows the stress-strain curves for 0.016-inch round stainless steel wire and 0.016 × 0.022–inch rectangular stainless steel wire obtained from the tensile test. The yield strength of 0.016-inch wire was $1.3 \times 10^9$ Pa ($N/m^2$) and that for 0.016 × 0.022–inch wire was $1.4 \times 10^9$ Pa ($N/m^2$). The elastic modulus of 0.016-inch wire was $154.5 \times 10^9$ Pa ($N/m^2$) and that for 0.016 × 0.022–inch wire was $157.6 \times 10^9$ Pa ($N/m^2$). The breaking point of 0.016-inch wire was $2.2 \times 10^9$ Pa ($N/m^2$) and that for 0.016 × 0.022-inch wire was $2.4 \times 10^9$ Pa ($N/m^2$). Thus, there were no remarkable differences in tensile properties between the two wires. Figure 5 shows the differences in nonlinear characteristics between the above wires that were obtained from the simulation of the force system produced by orthodontic wires based on the large-deflection nonlinear theory. As the inclination angle of the brackets was increased from zero° to eight°, the differences in the results of the simulation between small-deflection linear theory and large-deflection nonlinear theory increased. However, the nonlinearity of

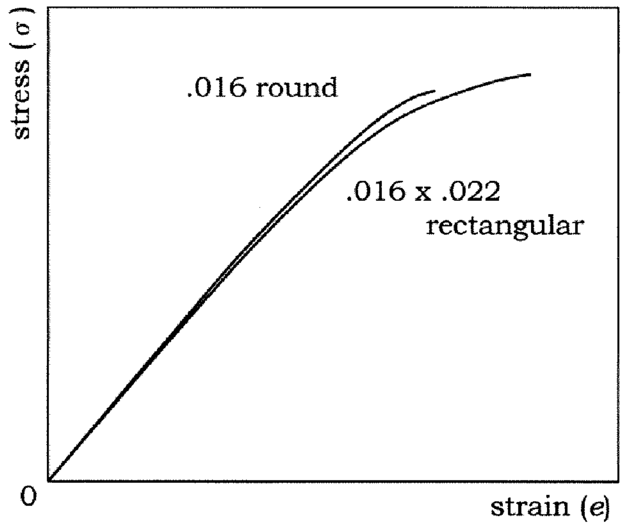


**FIGURE 4.** Stress-strain curves for 0.016-inch round and 0.016 × 0.022 –inch rectangular stainless steel wire.

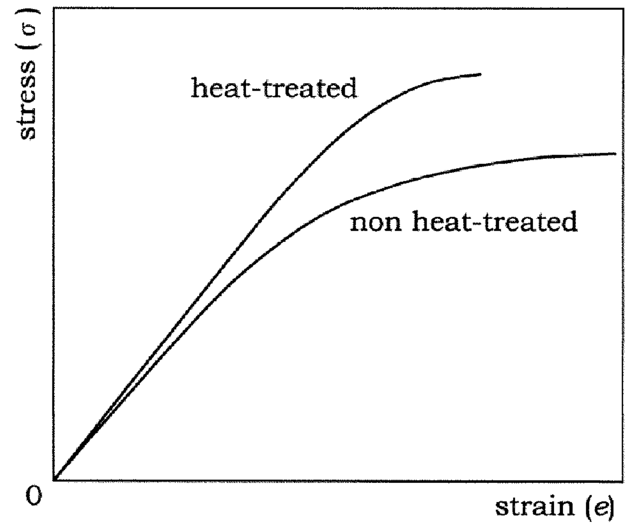


**FIGURE 6.** Stress-strain curves for 0.016-inch round cobalt-chromium wire and 0.016-inch round heat-treated cobalt-chromium wire.

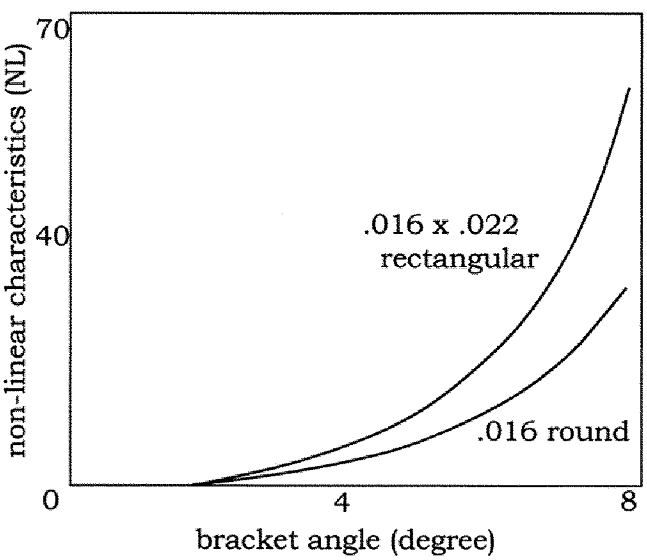


**FIGURE 5.** The difference in nonlinear characteristics between 0.016-inch round and 0.016 × 0.022–inch rectangular stainless steel wires.

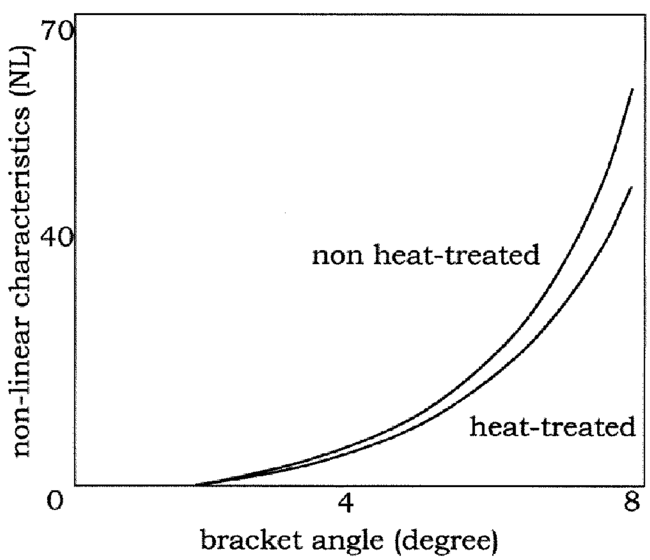


**FIGURE 7.** The difference in nonlinear characteristics between 0.016-inch round cobalt-chromium wire and 0.016-inch round heat-treated cobalt-chromium wire.

0.016 × 0.022–inch stainless steel wire increased more than that of 0.016-inch stainless steel wire.

Figure 6 shows the stress-strain curves for 0.016-inch round cobalt-chromium wire and 0.016-inch round heat-treated cobalt-chromium wire obtained from the tensile test. The yield strength of non–heat-treated wire was $1.2 \times 10^9$ Pa ($N/m^2$) and that for heat-treated wire was $1.6 \times 10^9$ Pa ($N/m^2$). The elastic modulus of non–heat-treated wire was $141.5 \times 10^9$ Pa ($N/m^2$) and that for heat-treated wire was $162.0 \times 10^9$ Pa ($N/m^2$). The breaking point of non–heat-treated wire was $1.9 \times 10^9$ Pa ($N/m^2$) and that for heat-treated wire was $2.2 \times 10^9$ Pa ($N/m^2$). Thus, there was a remarkable difference in tensile properties between these wires. Figure 7 shows the differences in nonlinear characteristics between the above wires obtained from the simulation of the force system produced by orthodontic wires based on the large-deflection nonlinear theory. As the inclination angle of the brackets was increased from zero° to eight°, the differences in the results of the simulation between the small-deflection linear theory and large-deflection nonlinear theory also increased. However, the nonlinearity of non–heat-treated wire increased more than that of heat-treated wire.

## DISCUSSION

### Validity of the assumptions used

In the present study, we established a method for extending a mathematical model for simulating the force sys-

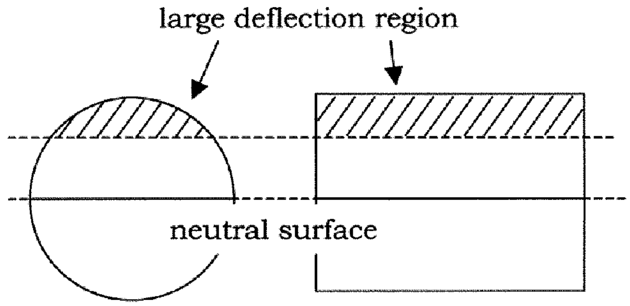


**FIGURE 8.** The difference in large-deflection areas between 0.016-inch round wire and 0.016 × 0.022–inch rectangular wire.

tem produced by orthodontic wires based on the small-deflection linear theory to the large-deflection nonlinear theory. This simulation method was based on two assumptions: (1) the neutral surface passed through the center of the cross section of orthodontic wires and (2) the stress-strain curve for the compression side was symmetric to that for the tensile side about the origin. In a strict sense, the neutral surface should be determined from the relation between the diameter of the cross section, the radius of curvature, and the equilibrium of the force, the sum of the maximum strains on the tensile and compression sides. Based on the obtained neutral surface, the tensile and compression properties should be considered. In addition, the use of orthodontic wire in the real world is associated with various phenomena (ligation method, wire sliding within the brackets, etc), which were not taken into account in the present model. However, it is almost impossible to address all these conditions analytically. Therefore, a numerical solution with some assumptions using a regression formula was appropriate. On the other hand, it is also necessary to consider the limitations of the analytical method in this study.

### Nonlinear characteristics of orthodontic wires

Figure 5 shows the difference in nonlinear characteristics between cross sections obtained from the simulation of the force system produced by orthodontic wires based on the large-deflection nonlinear theory. As the inclination angle of the brackets was increased from zero° to eight°, the nonlinearity of 0.016 × 0.022–inch rectangular stainless steel wire increased more than that of 0.016-inch round stainless steel wire. This result was due to the difference between the areas over which large deflection occurred. The area of large deflection in the arbitrary cross section of 0.016 × 0.022–inch rectangular stainless steel wire was greater than that of 0.016-inch round stainless steel wire, whereas the wire deflection angles were the same (Figure 8).

Figure 7 shows the difference in nonlinear characteristics between mechanical properties obtained from the simulation of the force system produced by orthodontic wires based on the large-deflection nonlinear theory. In heat-treated wire, nonlinearity decreased because the proportional limit and the elastic modulus increased with heat treatment. However, the difference in nonlinearity between wires was smaller than that in the comparison of the cross-sectional shape.

These findings suggest some significant conclusions. For each wire, nonlinear characteristics were apparent with two-degree tilting of the right and left brackets. This means that the dangerous cross section (the cross-section with the maximum bending moment) of the wire exceeded the elastic limit with slight bending because we consider play between the bracket and archwire. In addition, the results showed that the nonlinearity of orthodontic wires was affected more by the cross-sectional shape than by mechanical properties. Therefore, a nonlinear large-deflection analysis of the force system may become an important approach for the theoretical prediction of the forces and moments produced by an orthodontic appliance.

## CONCLUSION

In the present study, we established a method for extending a mathematical model for simulating the force system produced by orthodontic wires based on the small-deflection linear theory to the large-deflection nonlinear theory, which can give a clear view of the true nature of orthodontic wires. Furthermore, our results demonstrated that the nonlinearity of orthodontic wires was affected more by the cross-sectional shape than by mechanical properties.

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