Reason out Emergence from Cellular Automata Modeling

Qi Leilei[1] and Zhang Huaxia[2]

[1] School of Public Administration, South China Normal University, Guangzhou, P. R. China
`qillei@126.com`
[2] Department of Philosophy, Sun Yat-sen University, Guangzhou, P. R. China
`hsszhx@sysu.edu.cn`

Summary. In this paper, we first construct a descriptive definition for emergence based on multilevel ontology, and then use Cellular Automata Modeling to simulate some classical emergent processes, such as Conway's game of life and virtual ants building highway, which shows how the emergent phenomena arise, how the emergence of system at higher levels is derived from the simply basic interaction rules of system elements and their initial conditions at lower-levels. Although those inferences are deducible, they are not analytic. They are "bottom-up" synthetic methods based on computer simulation. There are three conditions that must be met when an emergent phenomenon can be reasoned out from its low-level elements and their interaction rules: (1) They must be simulatable, namely, that the elements and their operation rules can be constructed. (2) They must be computable, at least computable in principle. (3) They are necessary configuration function at the high level and auxiliary hypotheses. These indicate the limitation of the method of "derived from simulation" for understanding emergence. Moreover, most of system emergent properties cannot be definitely predicted because of complexity, hierarchy, uncertainty and adaptability in the development of systems. That is to say, in fact, we are using a new reducible method to prove that it is insufficient to understand emergent phenomena only with reducible method.

1 Emergence

In both natural and social world, surprising new matter emerges in an endless way, such as the appearance of life with its own metabolism and self-reproduction, nervous systems with their own intelligent nature, mind with marvelous thoughts and the function of imagination and social culture with its diversity and cohesive force. It is unimaginable and incredible that these phenomena would appear, thinking of its lower levels objects and processes. We regard them as common things only after we have encountered them repeatedly. Emergence does not refer to the appearance of any kind of new matter, but it is an interlevel new phenomenon, referring to new phenomena properties, functions and laws of the whole complex system comprised of its parts. Therefore, we conclude there are four emergent features as follows:

(1) Wholeness. Wholeness is the idea that the whole possesses some kinds of properties which can not been provided with by and even no meaningful of its pre-existing components. Namely, the properties and system behaviors as a whole cannot be explained by behaviors of the parts. For example, atoms are lifeless, molecules are inelasticity and elemental particles are non-color. Wholeness presupposes different levels and it is "a gold medal" trademark of emergence.

(2) Novelty. The simple rules can cause surprising new phenomena, and thus generates numerous and complicate complex systems via continuous iteration and diachronic evolution. This characteristic is remarkable.

(3) Unpredictability. Even if initial conditions and evolved rules are known, future behaviors and structures of the system still cannot be predicted, as complex systems are complicated and non-linear. This "unpredictability" refers to "definite unpredictability", namely, actual states of emergence cannot be predicted definitely.

(4) Downward Causation. The whole at the higher level is comprised of elements at the lower level. While the lower level elements in turn are constrained by the higher level whole, and they act according to the rules which the higher level whole obeys. In other words, the system emergent properties have downward causation to its component elements. We can see the characteristic as a powerful weapon to argue against traditional reductionism. If the whole is reduced to its elements interaction at the lower level, the downward causation is neglected, and that is incomplete to understand the whole.

It is thus clear from the above characteristics that we should make observation from different aspects and levels when we try to understand the emergent properties. The world is of different levels with each level having its own specific entities, phenomena, properties and laws. That is the multilevel ontology. The existence of the different hierarchy as well as points of observation is the original concept, from which we can formally define emergence:

Let $\{S_i\}$ $(i = 1, 2, 3, \ldots, n)$ being a set of the elements or "agents" , I_{ij}^1 being the interaction rules between elements. Where ij means S_j acts on S_i or the input of S_j to S_i, while 1 denotes at the first level.

Then let's assume F^1 being the representation of the configuration function. $F^1(S_i^1)$ is the characteristics obtained from observation or measurement of the element S_i at the first level.

If the interaction between elements in $\{S_i\}$ can form a new mutually stable structure or system S^2, S^2 is called emergent structure or entity, which is,

$$S^2 = R(S_i^1, F^1, I_{ij}^1)$$

And if there is $P \in F^2(S^2)$, but $P \notin F^1(S_i^1)$, this P is regarded as P_e^2, which means the emergent properties at the second level can be defined as:

$$P_e^2 = \{P \mid P \in F^2(S^2) \wedge P \notin F^1(S_i^1)\}$$

Please note that the emergent structure R and emergent properties P defined in this way is a unified definition from an ontological and epistemological point of view. The configuration function $F(\ )$ is epistemological, but its independent variable, namely object of observation itself and S in $F(S)$ is ontological commitment. It indicates the existence of an observed complex system S.

The emergent concepts can date back to ancient Greek philosopher Aristotle who says: "The whole is more than the sum of its parts", while the concept is recounted by British Emergentists in the 20th century, such as S. Alexander [1], C. L. Morgan [5] and C. D. Broad [3] who think that the emergent properties at higher-level B "cannot be deduced from (lower-level) A-properties and the structure of the B-complex by law of composition which has manifested itself at lower levels.", and it characterized "non-deducible properties" [3, pp. 77–78].

Morgan [5] believes that the emergent properties are unpredictable before their appearance. He says: "What is it that you claim to be emergent? The brief reply is: Some new kind of relation... It may still be asked in what distinctive sense the relations are new. The reply is that their specific nature could not be predicted before they appear in the evidence, or prior to their occurrence... In like manner we think that, on the level of physicochemical events, there could be no knowledge on the basis of which vital relatedness could be foreseen before it came. And so, too, at a later stage with mind as an emergent quality which expresses new relatedness of the conscious order" [5, pp. 64–65]. Alexander [1] sums up the ideas of British Emergentism. He says: "The higher quality emerges from the lower level of existence and has its roots therein, but it emerges therefrom, and it does not belong to that level, but constitutes its possessor a new order of existent with its special laws of behavior. The existence of emergent qualities thus described is something to be noted, as some would say, under the compulsion of brute empirical fact, or, as I should prefer to say in less harsh terms, to be accepted with the 'natural piety' of the investigator. It admits no explanation" [1, pp. 46–47].

Our understandings of emergence are similar to the British Emergentists', namely we all think emergence is the phenomenon produced at different levels. We cannot understand it completely at certain level. But the British Emergentists were too absolute and think it cannot be deduced, predicted, or even explained. They think that the appearance of emergence is a fact, but how it appears is a black box. We can only see the input at the lower-level and the output at higher-level, while the internal mechanism is undetectable. However, in the last 10 to 20 years of the 20th century, due to the development of complex sciences and computer science, basing on the high-speed computer, new mathematical methods, especially discrete dynamics and chaos dynamics, it is possible for us to open the black-box and explore the emergent process, mechanisms and structures based on interactions between agents of the low-level, and to derive emergence from simulation.

2 Simulation and Emergence Derived from Simulation

Generally speaking, the model in mathematics is a kind of homomorphic mapping. For example, the map (map or model) system is a homomorphic mapping of real physiognomy (prototype systems), and differential equation is the homomorphic mapping of electromagnetic movement of physical circuit. And that simulation in this paper is an iterated homomorphic mapping. It emphasizes two points: (1) the simulated (prototype) and the simulation system (model) are both dynamical systems. The simulation system describes dynamic process of the simulated system through updating their state. (2) Unlike traditional mathematical models, a simulation system does not have analytical solutions. The equation cannot be worked out with a solution by means of algorithm, such as addition, subtraction multiplication, division and extraction.

Thus, it requires the following three essential factors to simulate a natural or social real complex system:

(1) Elements

$$S_i = S_i(t),\ i = 1, 2, 3, \ldots, n \tag{1}$$

S_i is a model of system component elements. Where S_i can be viewed as ith element or the internal state of object and it changes with the change of time. Therefore, J. H. Holland described it as a dynamic mechanism in his famous book Emergence.

(2) Interaction Rules

$$I_{ij} = I(S_i, S_j) \tag{2}$$

Where I_{ij} is the representation of interaction rules or laws between elements. To be concrete, it is the jth object's action rules on the ith objects and it is the simulations to interactions between elements.

(3) Update Function U. An object update functional U is the state transition of systems and their elements state which transit in term of time sequence t.

$$S_i(t+1) = U(S_i(t), I_{ij}(t)),\ i = 1, 2, 3, \ldots, n \tag{3}$$

According to these three elements, we can build the system simulation $\sum s$, It refers to iterated update collection of all the objects state during calculation to evolution on time and aggregation on space. Namely,

$$\sum s = \sum_{t=1}^{m} \sum_{i=1}^{n} (S_i(t+1)) = \sum_{t} \sum_{i} (U(S_i(t), I_{ij}(t))),$$
$$\text{here } t = 1, 2, 3, \ldots, m;\ i = 1, 2, 3, \ldots, n \tag{4}$$

On basis of system simulation, if there are appropriate initial conditions C in place of "i" and "t", we can infer emergent conditions:

When S^2, namely $R(S_i^1, F^1, I_{ij}^1) \in \sum_s^1$, we say there is emergent structure derived from system modeling $\sum_s^1(S_i^1, F^1, I_{ij}^1)$ at level L_1, i.e. $\sum_s^1 \& C \vdash S^2$, iff $S^2 = R(S_i^1, F^1, I_{ij}^1) \in \sum_s^1$; when $P \in \sum_s^1(S_i^1, F^1, I_{ij}^1)$, we think there is emergent property P^2 derived from $\sum_s^1$, but here is an additional condition F^2, namely $P^2 = F^2(S^2)$, which is to say there must be a configuration function F^2 at the higher-level, and then P_e^2 can be derived.

$$\sum_s^1 \& F^2 \& C \vdash P_e^2 \tag{5}$$

Note that the computer-based simulation we use must have physically implemental mechanisms, i.e. the machine of iteration procedure which implements $\sum s$, $\sum s$ is normally some kind of physically digital computers.

Next, we take the computer as the platform, using cellular automata as simulation methods of discrete dynamic systems to analyze several typical cases of life emergence.

3 Reason out "Living Organism" Emergence from Game of Life

"Game of Life" was invented by the mathematician John Conway in 1970. He first presents his grid construction (cell spaces) which is divided into many smaller lattices, as chessboard. Each lattice is called a cell and any adjoining cell is considered as its "neighbor", including diagonals. Because there are many ways to design neighbor, the way he used is called Moore neighborhood. In game of life, "life organisms" are "mobile" on the grid. Every cell of "life organisms" is element $S_i = S_i(t)$, which is described in the previous section. Each of these cells could have two states: 0 or 1, (it can be live or dead, active or quiescent) and they change states with time. Each cell updates simultaneously and independently in discrete time. The new state of a cell only depends on its own actual state itself and on its eight closest neighbors' state. Its update rules I_{ij} mentioned above is described as the following three transition rules or life rules:

1. A dead cell (0) with exactly three live neighbors becomes a live cell ("it's born").
2. A live cell with two or three live neighbors remains alive (survival).
3. In any other case, a cell dies or remains dead (overcrowding or loneliness).

If each grid is filled with 0 or 1 at random and allows this configuration to iterate according to the life rules, in other words, acting an update function

U on all cells, after a period of time, the cell on grid may be dead, disappeared, quiescent as a stable pattern (stability), or becoming an oscillating pattern (blinker). But what interests Conway most is that it generates another particular pattern, R-pentomino (Figure 1(a)) [9].

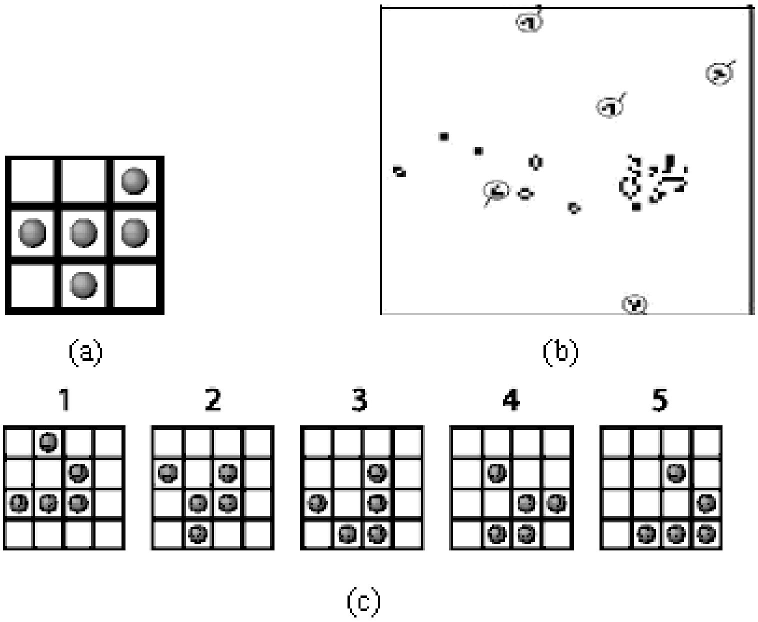

Fig. 1. The glider and its historical precursor. (a) R-pentomino. (b) Configuration developed from the R-pentomino, at the 224th step of evolution; the gliders are encircled, and their velocity vectors are sketched. (c) The glider.

The R-pentomino, its figure looks just like the letter "r", consisting of five non-zero states. It is thought to be a minimal initial configuration that generates unpredictably developing patterns. In the R-pentomino's evolution, it generates a cluster of small stable or oscillating patterns, and the most important one is called glider.

The behavior of each cell on cellular Spaces looks very simple, but it produces drastic activity and all kinds of unpredictably developing patterns (Figure 2) [8]. Figure 2 shows the configuration developed from R-pentomino's initial state after running 92 time step. Clicking "Start", the R-pentomino on grid is changing with time and gradually generates stability (encircled with a Pentagon), blinker (encircled with a rectangular), glider (encircled with an oval) and other life forms. These forms of life are at the edge in certain way, at the edge of growth and degeneration and at the edge of order and chaos. Sometimes, some stable and oscillating patterns are activated when they are influenced by other cells, and then they are likely to evolve into another form of life. The whole cellular spaces stabilize eventually only after 1103 generations. At this time, the other cells stay stable state, except for a blinker.

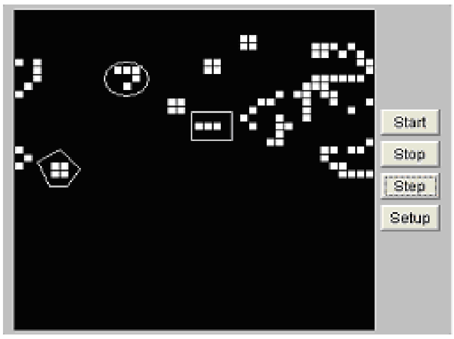

Fig. 2. The life forms.

It is seen that the interesting game of life is the same as life. Simple component elements and rules create rather complex life forms in an unpredictable way. Therefore Conway has always firmly believed that "life organisms" in Game of Life look like the ones in the real world, moving, growing, reproducing, evolving, and even thinking. According to this, he designs life glider (in Figure 2, the glider is encircled with an oval) and other "life forms" in order to reveal the truth of life.

At present because the computer screen is not long or wide enough, the movement of certain kind of configurations is displayed only with the limited grid like checkerboard. When the glider and other "life forms" move to one side of the lattice, they are required to crawl out from the opposite side. The designing idea is just the same as Einstein's finite but boundless cosmological model. After clicking "Start", they are moving uniformly along the diagonal just like a reptile, and its velocity equal to 1/4 of c, here c denotes the velocity from one cell to the next one in the grid . That is their macroscopic behavior rules and configuration behaviors that we can explore. In other words, it is a kind of phenomenon that we have seen at the higher-level, namely emergent hierarchy. That is $F^2(S^2))$ that we have seen after using the configuration function F^2 mentioned above. Note that if we click "step" to decompose the whole evolution, just like doing "everything in one action" in military training, we will clearly see that the transition rules of "Game of Life" only limit each cell to be "dead" or "alive", but do not limit the moving direction of every "life-organisms" and do not show how different forms of gliders shift across the screen. In other words, the position of each cell on the lattice is actually unchanged, and they do not "glide" themselves.

The process of glider "gliding" we see is merely the process of cells appearing and disappearing, being dead or alive, which only shows the cells updating and is a phenomenon observed to use configuration function F^1 at low-level.

It is these phenomena that make us think of our lives. Macroscopically our life is persistent, but in fact, it is only the process of delivery or transition of atoms and molecules as well as metabolism of cells in our body. A glider is not just a set of cells. The cells of each generation are replaced completely. The process that the component atoms of your body is updating all the time after you were born and the component parts of a glider work in the same way. In view of dynamics, the application of game of life to simple rules causes "life glider" which is the dynamic, coherent and independent structure of mysterious phenomenon. This shows how the simple rules results in the complex structure similar to life and behavior. The phenomenon of life emergence is deduced from component elements at the low-level, simple rules and additional function $F^2(\ )$.

Game of life which belongs to the artificial system can perfectly explain how the complexity of life emerges from simple configurations and rules via cellular automaton simulation. The following is an example of the simulation of natural phenomena via cellular automata, which can further explain the emergent process of complexity.

4 Virtual Ants Building Highway

Ants are complex in physiology, whereas we will consider a virtual ant created by Chris Langdon as a simple agent S_i. It is able to produce a different type of complex behaviors according to simple rules I_{ij}. First, this ant is located on one of the grids (cells) that are painted either black or white. These grids are theoretically finite but boundless, which we have talked about in the last section. At each time step, the ant is always facing in one of four directions: north, south, east or west and acts according to the following rules ("reptilian rules"):

1. If the ant is now standing on a white cell, it paints the cell black and turns 90 degrees clockwise. Then the ant moves onto the cell.
2. If the ant gets onto a black cell, it paints the cell white and turns 90 degrees counterclockwise. Then the ant moves onto the cell.

According to "reptilian rules", Figure 3 [4, p. 265] illustrates eight steps that an ant would take starting from an initially blank grid. At each time step, the pictures are arranged from left to right and from top to down.

Based on Figure 3 for another 10,000 time steps or so, the ant will indeed form a chaotic-looking mess that has little or no structure. But after another 250 or more steps, the ant will start to build its "highway systems" (This phenomenon was discovered by James Propp and he calls a highway) (Figure 4) [4, p. 266].

Figure 4 shows an ant highway created from an initially blank configuration. The ant takes a step forward strictly obeying the "reptilian rules". After extremely "chaotic" state, it finally builds a highway stretching towards the

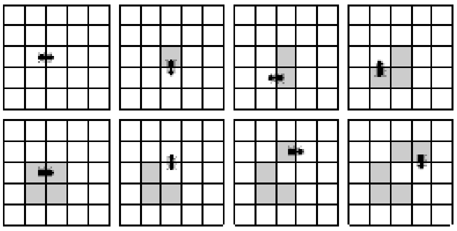

Fig. 3. Langton's virtual ant.

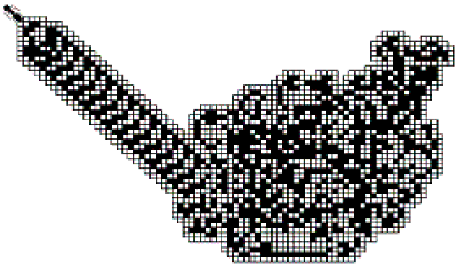

Fig. 4. A virtual ants building a highway.

north-west. As an ant can build a highway that flows in any of the four diagonal directions, this one is only a chance in necessity. That is to say, an ant builds a highway ultimately according to "reptilian rules", but what is different is the direction. Clearly, if the ant's universe consists of an infinite cellular space that is initially blank, then the ant will happily build the highway forever. If it is in wraparound universe, the highway must eventually intersect a place in the ant's grid space where the ant has been before. As shown by Gary William Flake [4] such a "bump in the road" will usually force the ant back into a chaotic-like behavior, but it will often spontaneously start to build another highway, and so on. It is a process from chaos to order and then chaos out of order, which is explained by the simulation to the virtual ant.

Virtual ants building highway proves not only the iteration of simple rules which can cause suddenly new phenomenon, but also the fact that the new phenomenon cannot be explained by the sum of its component parts alone, i.e. what Aristotle (350B.C.) said: "The whole is more than the sum of its parts". Undoubtedly, it is an emergent phenomenon for an ant to build a highway, and that emphasis "the gold medal" feature of emergence-system wholeness, i.e. an emergent structure is a whole one. Note that it is unwitting for the ant to build the highway. Maybe there is no concept of "highway" in ants' world and they only obey inherent laws to take a step forward. Contrastively, the reason why we define such an outspread structure of the grid as a highway is our possession of knowledge of highway. In other words, we cannot discover such an emergent phenomenon until we have configuration F^2 at the higher-level.

Ian Stewart [4, p. 270] has made an interesting observation regarding our lack of knowledge about the long-term behavior of some of these virtual ants. To paraphrase, for any of these ants we know their Theory of Everything, in that all of the "physical" laws that govern the ant's universe are simple and known to us. We also know the initial configuration of the ant's universe. Yet we are helpless to answer a simple question in an analytical way: how does the ant build the highway? Does it ever build a highway? But with using cellular automata iterative modeling, it is possible to answer that. In the simulation of the virtual ant, we have seen it extremely simple that the virtual ant takes each step forward, but it produces unpredictable complex configuration step by step through iteration or recursion which is exactly the arising process of the emergence of complexity. And we have known, "building highway" or something like that is not virtual ant's exclusive feature. Similar patterns have been found in other fields. This shows it is feasible to use virtual ant and other multiagents-based simulation to illustrate the emergence of complexity.

5 Features and Conditions Derived from Simulation

According to the mathematical analysis of emergence above and the simulated case using cellular automata, we can see it is inappropriate for the British Proto-Emergentists, especially Alexander to claim emergence is completely unexplainable. So long as some simple elements and interaction rules could be found, various strange emergent phenomena will arise unpredictably from the higher-level. That is like playing chess – with simple rules, you can have quite different results. Moreover, the cellular automaton modeling shows how emergence springs up step by step and explains the emergent process and mechanism.

The so-called explanation of a kind of phenomenon is to clarify the mechanism of its generation and to illustrate how it is logically deduced. We have arrived at a conclusion through the above analysis: emergent phenomena are explainable and can be deduced from simulation. Let us first account for the methodological characteristics that the simulation of emergence can be deduced.

(1) It is not analytically deduced but synthetically simulated. So called analytical solution is a classical mathematical analysis, in which methods is used in order to solve a problem or to explain a phenomenon. First, it is necessary to set a mathematical equation and add some initial condition and certain parameters, then work it out via limited computation of the limited numbers by means of such algorithms as addition, subtraction multiplication, division and extraction. In this way, the known phenomenon can be explained exactly and the future behavior of system can be predicted. But it cannot solve the problems as complexity, emergence and nonlinearity. The so-called

"derived from simulation" does not mean to get an analytical solution from decomposing the whole system to establish the equation. Its main purpose is to demonstrate and compute how local casual effects make use of aggregation in space and iteration in time to show the whole phenomenon of emergence, after modeling the system. And thereby, it is not a reasoning process from the whole to the parts, but from the parts to the whole. In fact, it is a synthetic method. Methodologically, R. K. Sawyer [7] regards it as a conversion from equation-based modeling to agent-based modeling [7, p. 263].

(2)It is partially rather than wholly reducible. Emergence arises from the high-level system. Moreover, its characters are observed and measured by configuration function F^2 and expressed by proposition P^2 and theory T^2 of the high-level. There are no F^2, P^2 or T^2 in configuration function F^1, proposition P^1 and theory T^1 of lower level. For example, in chemistry, such phenomena and concepts as freezing point, boiling point, heat of solution, heat of sublimation of water are at the higher level, but they cannot be reasoned out completely by the quantum physics, because there are no such notions in quantum physics. Certainly, when the heat of sublimation of ice is 12.2 kilocalories, three-quarters of energy can be considered to destabilize the hydrogen bond between molecules. However, there is no term of hydrogen bond in quantum mechanics. Thus it can not be fully derived from quantum mechanics of the low-level. Only by adding the hypothesis about hydrogen bond which does not exist in quantum mechanics, can the heat of sublimation of ice be deduced. This is reasoning of partial reduction.

Similarly, life emergence is derived from cellular automata modeling, which is also partially deduced. The configuration, moving direction and speed of life gliders as well as glider guns and so on cannot be deduced from the initial conditions of cells state and their interaction rules. As long as we add function of configuration of the higher level to the reasoning, the emergent phenomenon of that level can be derived by simulation of interaction of elements at the lower level, namely, that the formula $\sum_s^1(S_i^1, F^1, I_{ij}^1)\&C \vdash P^2$ is not tenable, while, $\sum_s^1 \&F^2\&C \vdash P^2$ is tenable, where $F^2()$ is a function of configuration of the high-level. If the method of "reasoned out emergence by simulation" is considered as reductionism, it is only partial reduction. This method itself indicates some emergent properties themselves are irreducible, or at least partially irreducible. Therefore, the reductive method of "reasoned out emergence by simulation" just proves that it is not enough to understand emergence only by means of reductionism. At least two (high and low) kinds of configuration functions would be used at the same time. Emergence can be fully understood by the analysis and the synthesis of both the higher level and the lower level. In addition, the macroscopic emergent phenomenon may be composed of multiple emergent microscopic mechanisms. Certain microscopic mechanism is only an example among many explanations or illustrations to explain the macro-emergence, and this mechanism is always incomplete, especially in the case that multiple emergences are not meaningfully related.

In this way, if an emergent property can be derived from simulation, it requires to meet the following three conditions at least:

(1) It must be simulatable. Emergent phenomena and properties are sometimes very complicated. It is quite difficult or even impossible to simulate a system when their subsystems are indistinguishable or subsystems' interrelations entangled. Therefore, a system is simulatable if and only if two conditions are met. Condition I: There exists $S_i \in M$, where M is a model, namely some crucial features of the simulated system were abstracted as elements of the simulation system. Condition II: Some update functions U exist and they distribute all over S_i. These two conditions are not generally available, due to the existence of inseparable systems and system properties. So the existence of these conditions themselves also indicated the limitation of reasoning out emergence by simulation. When you abstract elements S_i from a system, whether this S_i reflects the real properties of the system, or whether the update function U you define can show the development of systems is still a problem. In the best case, S_i is always simplified and that indicates inaccuracy and uncertainty.

(2) It must be computable. For example, in fractal theory, recursion or mapping of Mandelbrot Sets $Z_{n+1} = Z_n^2 + C$ is simulatable, but both Z and C are complex numbers. Because most C is approximate to Julia Sets, this recursion is instable equilibrium, and then this mapping is incomputable because computation is defined in real number field. Similarly, it is inevitable to extend to complex number when we use iteration of various functions to work out imaginary number. We should say it is of non-calculability in principle. As for computable emergent process in principle, if the amount of S_i is too large or the update function U too complicated, it will go beyond the really computable confine, it is actually incomputable and does not satisfy the conditions of simulation.

(3) It must possess necessary configuration function F^2 of the high-level. As is mentioned above, if the necessary function F^2 is not given, emergence can not be derived from simulation. The reasoned out emergent conditions from simulation show us that because of the complexity, uncertainty, sensitivity to initial conditions emerging in complex systems as well as the limitation of simulation, computation and observational states, the emergence of systems cannot be predicted accurately. Because of the lack of F^2, some real instances such as financial crisis, political situation abruptness and earthquake are considered unpredictable, especially when we saw them for the first time. On the other hand, we also notice that the emergence of system is not completely unpredictable. When we possess F^2, it is likely to infer the result by means of simulation. It is a good example that the warning and serving system of disasters has been putting into practice in our country. It can be seen that our emergence theory of complex systems is a farewell to that of British Emergentists, but in some extent a kind of return to the idea of the British Emergentists.

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