Are quantum particles objects?
Simon Saunders

It is widely believed that particles in quantum mechanics are metaphysically strange; they are not individuals (the view of Cassirer 1956), in some sense of the term, and perhaps they are not even objects at all, a suspicion raised by Quine(1976a, 1990). In parallel it is thought that this difference, and especially the status of quantum particles as indistinguishable, accounts for the difference between classical and quantum statistics - a view with long historical credentials.[1]

'Indistinguishable' here mean permutable; that states of affairs differing only in permutations of particles are the same - which, satisfyingly, are described by quantum entanglements, so clearly in a way that is conceptually new. And, indeed, distinguishable particles in quantum mechanics, for which permutations yield distinct states, do obey classical statistics, so there is something to this connection.

But it cannot be the whole story if, as I will argue, at least in one notable tradition, classical particle descriptions may *also* be permutable (so classical particles may also be counted as indistinguishable); and if, in that same tradition, albeit with certain exceptions, quantum particles are *bona fide* objects.

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I will follow Quine in a number of respects, first, with respect to the formal, metaphysically thin notion of objecthood encapsulated in the use of singular terms, identity, and quantification theory; second (Quine 1970), in the application of this apparatus in a first-order language L, and preferably one with only a finite non-logical alphabet; and third (Quine 1960), in the use of a weak version of the Principle of Identity of Indiscernibles (PII). Applying the latter requires a listing of the allowable predicates (the non-logical vocabulary of L); for our present purposes this should be dictated on theoretical and experimental grounds, grounds internal to the physics - for example, that only predications of measurable properties and relations should be allowed. Our minimal, logical question is then: whether indistinguishable quantum particles are L-discernible by their measurable properties and relations.

But as a criterion for membership in L, measurability may be somewhat too restrictive; it threatens to settle our question, negatively, solely on the basis that quantum particles are unobservable. Better is a condition that is both precise and more general, namely that *only predicates invariant under the symmetries of the theory qualify.* This condition implicitly or explicitly underlines a good many recent debates in the philosophy of physics over symmetry principles,

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[1] It has been called the 'received view' by French and Rickles (2003).

and in important cases (for a number of space-time symmetries) it is physically uncontroversial.[2]

In our case the symmetry is the permutation group. Our criterion, then, is that L-predicates should be invariant under permutations (I shall then call them *symmetric* or *symmetrized*). Whatever the metaphysical questions that accompany the idea of a 'loss of identity' in quantum mechanics (for indistinguishable particles), its sole mathematical signature is permutation symmetry, the syntactical expression of which (in terms of a regimented formal language) is surely that predicates be symmetrized. So if it is true that in the words of an early contributor to quantum statistics 'the conception of atoms as particles losing their identity cannot be introduced into the classical theory without contradiction' (Stern 1949: 535) - and if the difficulty does not concern the details of the classical theory but its basic concepts - one would expect it to show up in our elementary framework of a finitary language restricted to totally symmetrized predicates. Bach (1997) indeed takes it as self-evident that a description of particles having definite coordinates can only be permutation invariant in so far as it is incomplete (specifying only the *statistical* properties of a particle ensemble, not the microscopic details).

But is there really any such conceptual impediment? If not - and from what follows it seems not - the case for metaphysical novelty following on from particle indistinguishability in quantum mechanics remains unmotivated. Add to this the argument (Huggett 1999) that classical statistics is every bit as consistent with permutation symmetry as is quantum statistics, and the claim that indistinguishability explains quantum statistics looks threadbare indeed.

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Let F be an $n-$ary predicate; the symmetrized language L_S that we envisage must be such that if $F \in L_S$, then in any valuation $Fx_1...x_n$ can be replaced by $Fx_{\pi(1)}...x_{\pi(n)}$ without change of truth value, for any permutation π of $\{1,...,n\}$.

We imagine this as our procedure: we start from some language L, based in part on other physical theories, which lacks permutation symmetry; and we examine the effects of implementing it, defining totally symmetrized L_S-predicates as complex predicates in L (e.g. $\bigvee_{\pi} Fx_{\pi(1)}...x_{\pi(n)}$, call F_{dis}; of course there are plenty of other constructions too - we shall spell out a general one shortly). Clearly $L_S \subseteq L$.

How limited is L_S? The answer depends on L. Of particular interest is the case when L has no names, so that singular reference is by means of bound quantification only (paraphrasing contexts involving names by Russellian definite descriptions). In fact, restricted to descriptions of particle distributions in space - as used in specifying the coordinates of particles (or initial or final data more generally) - it would seem that indefinite descriptions are enough to be going on with, of the form 'particles of such-and-such a kind have such and such properties and relations'. It is not at all clear that in giving such descriptions

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[2]For a review see (Saunders 2003b).

one must single out one arrangement of particles, and no permutation of it; for one has the logical equivalence:

$$\exists x_1...\exists x_n F x_1...x_n \equiv \exists x_1...\exists x_n F_{dis} x_1...x_n. \tag{1}$$

It is unlikely that L_S and L can differ very much for uses like this: what more does one want to say in L, after listing all the relations among the mentioned objects, in the form of a purely existential statement? - other that is than which object (value of variable), for each k, is a_k (an illegitimate question, if L has no names).

In fact when L is devoid of names use of L_S involves no restriction at all, at least in the case of sentences with only finite models of a fixed cardinality (say N). Given any such $L-$sentence T, one can construct a logically equivalent L_S-sentence T_S, true in all and only the same models.

The claim is sufficiently surprising to warrant at least a sketch of the proof. We may suppose, with no loss of generality, that T is given in prenex normal form (all the quantifiers, say n in number, to the left). Now construct a sentence T_1 in a language L^+, which is L supplemented by N names $a_1,..,a_N$, by replacing each rightmost quantifier and the complex predicate that follows in T by a disjunction (in the case of $\exists$) or conjunction (in the case of $\forall$) of formulas in each of which x_n is replaced by a name (yielding $\bigvee\limits_{k=1}^{N} F x_1...x_{n-1}a_k$ or $\bigwedge\limits_{k=1}^{N} F x_1...x_{n-1}a_k$ respectively).). Repeat, removing each innermost quantifier, obtaining at each step a complex predicate completely symmetric in the N names (ensured only because T has no names); on the final step one obtains a sentence T_2, in which the x_ks do not occur. Now replace every occurrence of a_k by x_k to obtain a totally symmetrized $N-$ary L-predicate, which is therefore in L_S . Prefacing with N existential quantifiers (and conjoining with a cardinality-fixing sentence) one obtains the sentence T_S; by construction it has the same truth conditions as T.

The two languages, insofar as they are used to describe a finite collection of objects, are in this sense strictly equivalent; under this condition, symmetrization makes no difference to truth values of sentences. I suggest this is evidence enough that indistinguishability in itself indicates nothing metaphysically untoward, or otherwise strange.

Why might one have thought any otherwise? But the constraint is certainly prohibitive applied to ordinary language; take the predicate '... is in the kitchen, not....', as in:

(i) Bob is in the kitchen, not Alice.

To symmetrize and say instead:

(ii) Bob is in the kitchen and not Alice, or Alice is in the kitchen and not Bob

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doesn't tell us what we want to know. But in a language sufficiently rich in predicates to replace 'Bob' and 'Alice' by definite descriptions, the situation is quite different. We then have a sentence like:

(iii) There is someone Bob-shaped who is in the kitchen, and someone Alice-shaped who is not

(where 'Bob-shaped' etc. is shorthand for some purely geometric, anatomical description). Symmetrizing, in L_S we say instead:

(iv) There is x_1 and x_2, where x_1 is Bob-shaped and in the kitchen and x_2 is Alice-shaped and not, or x_2 is Bob-shaped and in the kitchen and x_1 is Alice-shaped and not.

Unlike the passage from (i) to (ii), there is no difference between (iii) and (iv); they are an instance of the equivalence (1)). One might of course reintroduce the question of which of x_1 and x_2 is which (say, which of two persons, specified independent of their appearance), but that is only to invite further definite descriptions, whereupon we will be back to the same equivalence.

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This argument would all by itself settle the matter, were it not for the worry that the objects that we end up with - the values of x_1 and x_2, that only contingently have the bodies or personalities (or what have you) that they do - are themselves rather strange. It may be they are just as problematic, when it comes to questions of identity, as quantum particles.

We should face this challenge head on. The account of identity that follows applies to any first-order language L without equality, for any finite non-logical alphabet, whether or not symmetrized.

Any such L effectively comes with identity, a point that Quine has often emphasized. We get for free the defined sign:

$$s = t \underset{def}{=} \bigwedge_{\text{all primitive } L\text{-predicates}} Fs \leftrightarrow Ft \tag{2}$$

(here s and t are L-terms, occupying the same predicate position in F). Unpacking this schematic definition, and temporarily introducing the notation F_k^n for the k-th $n-$ary predicate symbol of L, we obtain on universal generalization over free variables not in s, t:

$$s = t \underset{def}{=} \bigwedge_n \bigwedge_k \bigwedge_{j=1}^n \forall x_1...\forall x_n (F_k^n x_1..x_{j-1}sx_{j+1}..x_n \leftrightarrow F_k^n x_1..x_{j-1}tx_{j+1}..x_n). \tag{3}$$

The RHS is of the form

$$\bigwedge \forall\forall...\forall(Fs \leftrightarrow Ft) \tag{4}$$

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and not:

$$\bigwedge (\forall\forall...\forall(Fs) \leftrightarrow \forall\forall...\forall(Ft)) \tag{5}$$

the point that so often goes unstated.[3] By construction the schemes variously written as (2),(3),(4) (but *not* (5)) imply the usual axiom scheme for identity:

$$s = s, \ s = t \rightarrow (\Phi s \leftrightarrow \Phi t)$$

(where Φ can be replaced by any L-predicate, primitive or otherwise); moreover any other scheme with implies the latter yields an equality sign coextensive with the one defined, so identity as given by (2), (3), (4) is essentially unique.

But isn't this just the familiar PII? - yes, or it should be familiar. In fact it has received surprisingly little attention, despite its endorsement by Quine. A correct formal classification of L-discernibles, according to this scheme, was only given quite recently.[4] And Quine made no applications of the principle to physically problematic cases (nor, so far as I know, has anyone since).

Quine's amended classification is (I follow his earlier terminology for the first two cases):

Two objects are

- *absolutely discernible* in L if there is an L-formula in one free variable that applies to one of them only

- *relatively discernible* in L if there is an L-formula in two free variables that applies to them in one order only

- *weakly discernible* in L if they satisfy an irreflexive L-formula in two free variables.

As stated each category contains the one before it (here I follow Quine), but they are exhaustive: values of variables not even weakly discernible are counted (in L) as the same.

The interesting cases are mere relative or weak discernibility. For example, let the only non-logical symbol in L be an irreflexive and symmetric dyadic F; then from the definition (3):

$$x = y \leftrightarrow \forall z((Fxz \leftrightarrow Fyz).$$

On any valuation in which Fxy is true, $\forall z(Fxz \leftrightarrow Fyz)$ is false (as $Fxy \wedge \neg Fyy$ is true); it follows that $x \neq y$. Thus, to take Black's famous example of two spheres of iron, positioned in an otherwise empty universe, one mile apart in space; they are weakly discerned by the symmetric and irreflexive relation 'one mile apart in space'; but they are neither absolutely nor relatively discernible.[5]

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[3] It is discussed at length by Quine (1976b).

[4] By Quine in 1976, amending, without comment, the classification he gave in *Word and Object* in 1960.

[5] For other physical examples, see Saunders (2003a) and, in mathematics, Ladyman (2005).

What now of names? Let L contain names, and the categories of relative and weak discernibility seem to be obliterated. For named objects, if discernible at all, are absolutely discernible.[6] However, everything turns on the proviso, *if discernible*; whether or not the names do in fact name different objects will still depend, just as before, on predicates alone. In this sense, then, named objects can still be classified as absolutely, relatively, or only weakly discernible, just as before.

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We are finally ready to answer our question: Are permutable particles discernible? The answer depends, evidently, on L_S, and specifically its non-logical vocabulary. In quantum mechanics the state of a particle is specified by a vector φ, up to a complex multiplicative constant, or *phase*, in the state-space of the system (Hilbert space) . An $N-$particle state is a sequence of 1-particle vectors, or - this the essential difference from classical theory - a *sum* of such sequences. A state of a collection of quantum particles, if the particles are indistinguishable, must be invariant under the permutation group. Among these are expressions of the form (for a 3$-$particle state):

$$const.(\varphi\psi\chi + \varphi\chi\psi + \psi\chi\varphi + \psi\varphi\chi + \chi\varphi\psi + \chi\psi\varphi) \tag{6}$$

where φ, ψ, χ are 1$-$particle vectors. Pretty evidently, it does not specify which particle is in which state - there is no such determinate rule here. It is like the symmetrized triadic 'the first particle is in the state 'const. φ' the second in the state 'const. ψ', the third in the state 'const. χ', or the first particle is in the state 'const. φ', the second in the state 'const. χ', the third in the state 'const. ψ', or ....' (evidently sequence positions are here functioning as names). Such states (permuting and summing) are also called *symmetrized*; the particles described by them are *bosons*.

But what are we to make of the allegedly 3-particle (and manifestly symmetrized) state 'const. $\varphi\varphi\varphi$'? Evidently, that there are three particles each in exactly the same 1-particle state, and therefore exactly alike in every respect. They are surely not absolutely discernible, hence, since relative discernibles require non-symmetric predicates, they are at most weakly discernible. But are they even that? What (physical, invariant) relation does a boson enter into with a boson in exactly the same state, supposed to be a complete description, that it does not enter into with itself?

If the answer is none (as it appears), or none that can be sanctioned by the physical theory, then either the PII, or the objectual status of quantum particles, is in question. Were that the end of the story, either way our total system would be in trouble. In fact there is another possibility - another prescription under which the state is invariant under permutations: vectors may instead be *antisymmetric*, changing sign on any odd number of interchanges of particles

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[6] If discernible at all, then they are at least weakly discernible; so there is a totally symmetric and totally irreflexive $N-$ary predicate F such that the sentence $Fa_1...a_N$ is true. But then $Fa_1..a_{k-1}xa_{k+1}...a_N$ absolutely discerns a_k.

(the state itself - the vector up to phase - remaining unchanged). The particles of antisymmetrized states are *fermions*. In place of (6) we have

$$const.(\varphi\psi\chi - \varphi\chi\psi + \psi\chi\varphi - \psi\varphi\chi + \chi\varphi\psi - \chi\psi\varphi).$$

Evidently antisymmetric states cannot assign two different particles to exactly the same 1-particle state; antisymmetrizing '$\varphi\varphi\varphi$' produces the zero. The problem we encountered with bosons does not arise.

Antisymmetrization ensures Pauli's exclusion principle (the principle that fermions cannot have all their quantum numbers in common). The latter was indeed early on considered, by Weyl among others, to vindicate the PII,[7] but the suggestion was squashed by Margenau in 1944 (and seems to have been hardly advocated since). Margenau came up with a new argument to show that fermions are indiscernible, namely, that all the *1-particle expectation values* (which may be taken as exhausting the 1-place predications) of any fermion in an antisymmetrized state must be the same. This was thought to show that the PII cannot apply.[8]

But discernibility does not require absolute discernibility; and if one considers the remaining candidates, relative or weak discernibility, it seems that Margenau was wrong and Weyl was right all along. For even in a situation of maximal symmetry, for example in the singlet state of spin

$$\Psi = \text{const.} \; (\varphi_\uparrow\varphi_\downarrow - \varphi_\downarrow\varphi_\uparrow) \tag{7}$$

the two particles are still weakly discernible. Here $\varphi_\uparrow\varphi_\downarrow$ correspond to the two opposite possible values (parallel or antiparallel) of the spin of the particle along a given direction $\uparrow$. Here any direction can be chosen, without change of the state - it is in this sense that (7) is a specially symmetric state, invariant under rotations as well as permutations - still the two particles satisfy the symmetric but irreflexive predicate '... has opposite $\uparrow$ −component of spin to...'.[9]

Why was this simple observation missed? The answer, presumably, is that it would then seem that the particles must each have a definite and opposite value for the $\uparrow$ −component of spin, implying some kind of hidden-variable interpretation of quantum mechanics (contentious in itself, for entirely unrelated reasons). But this is to fall back on our old habit of turning discernment on the basis of relations into discernment by differences in properties ('relational properties'); it is to miss the logical categories of relative and weak discernibility.

Consider again Black's two iron spheres, each exactly alike, but one mile apart in space. They are weakly discerned by the irreflexive relation '...one mile apart from...', but - on pain of begging the question against relationism - it does not follow, because the spheres bear spatial relations to each other, that they each have a particular position in space. Neither, if two lines are weakly discerned by the irreflexive relation 'at right-angles', does it follow that each line has a particular direction in space. Two particles can have opposite $\uparrow$

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[7] It was called the 'Leibniz-Pauli' principle by Weyl (1949: 247).

[8] Similar arguments have since been given by French and Redhead (1988) and Dieks (1990).

[9] For more formal details, see Saunders (2003a, 2006).

—component of spin (they are anticorrelated as regards spin in the $\uparrow$ −direction) without each having a particular value for the $\uparrow$-component of spin.

On the strength of this we can see, I think, the truth of the general case: so long as the state of an N−fermion collective is antisymmetrized, there will be some totally irreflexive and symmetric N-ary predicate that they satisfy. Fermions are therefore invariably weakly discernible.

Not only are fermions secured; so too, concerning the atomic constituents of ordinary matter, are bosons. For all but one of the stable bosons are composites of fermions (the exception is the photon). In all these cases, the bosonic wave-function (with its symmetrization properties) is an incomplete description, and at a level of finer detail - irrelevant, to be sure, to the statistical properties of a gas of such composites - we have a collection of weakly discernible particles. By reference to the internal structure of atoms, if nothing else, we are assured that atoms will be at least weakly discernible.

The only cases in which the status of quantum particles as objects is seriously in question are therefore *elementary* bosons - bosons (supposedly) with no internal fermionic structure. The examples in physics (according to the Standard Model) of truly elementary bosons are photons and the other gauge bosons (the W and Z particles and gluons) and the conjectured (but yet to be observed) Higgs boson. But in *these* cases there is a ready alternative to hand for object position in sentences: the mode of the corresponding quantum field. We went wrong in thinking the excitation numbers of the mode, because differing by integers, represented a count of things; the real things are the modes.[10]

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The answer to Quine's question - Are quantum particles objects? - is therefore: Yes, except for the elementary bosons.

Similar conclusions follow in the classical case. If indistinguishable, and permutations are symmetries, we should speak of them using only symmetrized predicates. If impenetrable they will be at least weakly discerned by the irreflexive relation '...non-zero distance from...'; but even if one relaxes this assumption, and allows classical particles to occupy the same points of space, they may still be (relatively) discerned by their relative velocities. Problems only arise if relative distances and velocities are zero, in which case, if no more refined description is available, they will remain structureless and forever combined, and we would do better to say there is only a single particle present (with proportionately greater mass). This, a classical counterpart to elementary bosons, makes the similarities in the status of particles in classical and quantum mechanics only the closer.

What of the more metaphysical question, of whether quantum particles are *individuals*? But here it is not clear what more is required of an object if it is to count as an individual: perhaps that it is not permutable, or that it is always absolutely discernible, or discernible by intrinsic (state-independent) properties and relations alone. But in all cases, one is no closer to an explanation, in

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[10] A suggestion first made by Erwin Schrödinger in 1924.

logical terms, of the difference between classical and quantum statistics, for none of these distinctions cut along lines that demarcate the two.

The facts about statistics are these:[11]

- Distinguishable classical particles obey classical statistics.

- Indistinguishable classical particles obey classical statistics.

- Distinguishable quantum particles obey classical statistics.

- Indistinguishable quantum particles obey quantum statistics (Bose-Einstein or Fermi-Dirac statistics[12]).

Distinguishable particles in physics we may take to be absolutely discernible, and in all cases they obey classical statistics; but indistinguishable particles, particles ensured only to be weakly discernible, may or may not obey classical or quantum statistics. No more does the discernible/indiscernible distinction line up with the classical/quantum divide; it only serves to distinguish between certain classical and quantum particles, on the one hand, and the elementary bosons (and their classical analogues) on the other. And finally, names do not capture the distinction; given the restriction to totally symmetrized predicates, the presence or absence of names is irrelevant.

Is there some other dimension along which one might mark out a distinctive status for indistinguishable quantum particles? Perhaps - say, in whether or not quantum particles are re-identifiable over time (as argued by Feynman (1965)). But this takes us away from permutation symmetry *per se*, and there are many classical objects (shadows, droplets of water, patches of colour) that likewise may not be identifiable over time. In the weakly interacting case, taking the 1-particle states that enter into a symmetrized or antisymmetrized state as objects instead, one may or may not have things reidentifiable over time,[13] and yet the statistics remain the same. But the overriding objection, in the present context, is that in considerations like these we seem to be getting away from the purely logical notion of identity.

It seems the only remaining alternative, if indistinguishability is to have the explanatory significance normally accorded it, is to deny that permutability is intelligible at all as a classical symmetry - that it is simply a metaphysical *mistake*, on a traditional conception of objects, to think that particles can be really indistinguishable. One would then be left with the clean equation: permutable if and only if quantum mechanical.

But the claim is implausible, as we are in a position to see. Finitary, categorical descriptions in L_S, that are restricted to totally symmetrized predicates,

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[11] In all cases the entropy is extensive (even for closed systems) if and only if the particles are indistinguishable. For arguments that extensivity (and hence indistinguishability) is strictly required for closed systems, even in classical thermodynamics, see Pniower 2006, Saunders 2006.

[12] There is also the possibility of *parastatistics* (not so far experimentally detected), involving mixed boson and fermion transformations.

[13] Depending on whether or not the total state is a superposition of states of definite occupation numbers.

are logically equivalent to those in L that omit only names. Descriptions of the latter sort, whatever their philosophical inadequacies, can hardly be called *unintelligible*.

In the face of this, our conclusion is rather that indistinguishability has nothing at all to do with the quantum and classical divide, and that the reason for quantum statistics, in the face of permutability, must be sought elsewhere.[14]

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[14] For further discussion, see Saunders (2006).

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