CAN CLASSICAL DESCRIPTION OF PHYSICAL REALITY BE CONSIDERED COMPLETE?

GABRIEL CATREN

CREA, UMR 7656, ÉCOLE POLYTECHNIQUE/CNRS, 1 RUE DESCARTES, 75005, PARIS, FRANCE

Abstract. We propose a definition of physical objects that aims to clarify some interpretational issues in quantum mechanics. We claim that the transformations generated by the objective properties of a physical system must be strictly interpreted as gauge transformations. We will argue that the uncertainty principle is a consequence of the mutual intertwining between objective properties and gauge-dependant properties. The proposed definition implies that in classical mechanics gauge-dependant properties are wrongly considered objective. We will conclude that, unlike classical mechanics, quantum mechanics provides a complete objective description of physical systems.

I. Introduction

According to Einstein, quantum mechanical description of physical reality cannot be considered complete.[1] In other words, there would be 'elements of physical reality' that do not 'have a counterpart in the physical theory'. In classical mechanics, both the exact position and the exact momentum of a particle can be simultaneously predicted for all times from a given set of initial conditions. In quantum mechanics, on the other hand, the momentum of a system characterized by a well-defined position cannot be predicted by the theory (and vice versa). More generally, these quantities can be simultaneously predicted up to some inversely correlated uncertainties. Indeed, this is the crux of Heisenberg's celebrated uncertainty principle. The conceptual content

[1]This is the conclusion of the seminal Einstein-Podolsky-Rosen article (Einstein *et al.* [1935]). An historical account can be found in (Mittelsteadt [2006]).

of this principle has been the object of a heated debate that remains unresolved today.[2]

In this paper, we will argue that quantum mechanics can be understood as the formalization of a rigorous definition of physical objects. According to the standard characterization, the objective properties of a physical object are the invariants under a certain set of symmetry transformations (Auyang [1995]; Born [1998]; Nozick [1998]; Weyl [1952]). However, it is never clearly stated which transformations need to be considered. We will argue that these transformations are generated by the objective properties themselves. In other words, we claim that the transformations generated by the objective properties of a physical system must be strictly interpreted as gauge transformations. This definition imposes a compatibility condition on the set of objective properties that characterizes a certain object. This condition requires that an objective property be invariant under the transformations generated by the other objective properties of the same object. The significant result is that this compatibility condition is not consistent with classical mechanics, but rather with quantum mechanics.

According to our definition, the uncertainty principle would be the formal translation of the intertwining between objective properties and gauge-dependent properties. Roughly speaking, to ask which position is objective in a quantum particle with a well-defined momentum would be as nonsensical as asking which side of a die is the objective one. In other words, in classical mechanics non-objective elements of physical reality are considered objective. On the contrary, quantum mechanics provides a complete description of all the objective elements of physical reality. It follows that the quantum description of a physical system is not incomplete, but rather that classical states are specified by means of too many variables. This explains why the quantum mechanical

[2]Many interpretations were proposed for the uncertainty principle. It was alternatively interpreted as a consequence of the unpredictable perturbations in experimental measures of physical quantities, as a result of the mutual incompatibility of certain experimental contexts, in terms of a subjective lack of knowledge of well-defined objective states, as a description of the statistical spread in an ensemble of similarly prepared systems, as the manifestation of an ontological indeterminateness in the definition of physical quantities, etc. (see for example Hilgevoord and Uffink [2006]).

description of a physical system depends on half of the variables as the classical description.

This article presents a conceptual version of the results discussed in (Catren [unpublished]).[3] In translating these technical arguments into conceptual terms, we hope not only to make them intelligible to a wider audience, but more importantly to contribute to an understanding of the rational necessity of quantum mechanics from a philosophical perspective. In Section II, we propose a definition of physical objects. In Section III, we consider the dynamics of physical systems. Section IV compares the quantum mechanical description of physical objects with the classical description. Finally, the last Section provides a summary of the obtained results.

II. Physical objects, objective properties and profiles

A physical object will be defined as a physical configuration that can be completely characterized by a set of objective properties. Such a set will be called the *eidos* ε of the physical object.[4] In order to unpack this definition, it is necessary to specify what we understand by "objective property". In doing so, we will try to keep our reflections as close to common sense as possible.

In order to describe a physical object completely, it is first necessary to observe it. In general, there are different kinds of observations that can be performed. For example, we could rotate the object and register its different profiles, we can observe the object at different times, etc.[5] An objective property will be defined as a generator of a particular kind of transformation of the object's profiles. This means that an objective property specifies how the object's profiles change when the object is acted upon by a certain operation. For example, there is an objective property that specifies how the observed profiles change when the object is rotated, there is another objective property that

[3]These results were obtained by means of a conceptual analysis of the symplectic formulation of mechanics (Abraham and Marsden [1978]; Libermann and Marle [1987]; Marsden and Ratiu [1999]; Souriau [1997]) and the geometric quantization formalism (Brylinski [1993]; Kostant [1970]; Souriau [1997]; Woodhouse [1992]).

[4]This terminology is borrowed from (Heelan [2004]).

[5]In principle, there is no difference between rotating an object and modifying the angle of observation. Hence, there is an equivalence between active transformations of the object and passive transformations of the observer's position. In what follows, we will use both descriptions indistinctly.

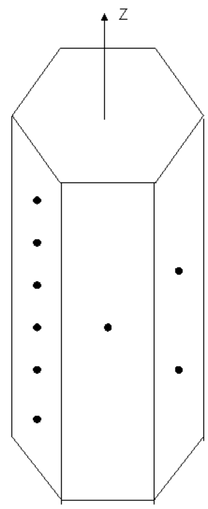

FIG. 1

specifies how the observed profiles change when the object is observed at different times, and so on. We will say that each objective property specifies the particular way in which the physical object realizes a universal operation, such as a rotation or a temporal evolution.[6] In this way, the *eidos* defines the identity of the object by generating the transformations between all its possible profiles.

By way of example, it will be useful to point to a physical object as common as a die. In order to simplify the exposition, let us suppose that the die is composed of a hexagonal base and a hexagonal top that define six rectangular sides (see Fig.1). Each side of the die can have a different natural number $n \in \mathbb{N}$. Let's also suppose that the die is very long, such that the die can be assumed to land on one of its six lateral sides when thrown.

Let us now consider the set of all possible distinct dice. Each die in this set is a different physical object with a different sequence of numbers on its six sides. When we perform an observation of a particular die from this set, the result is a particular die's profile (that is to say a die's side with a natural number). However, if we rotate the die around the axe z, the observed profile will change. According to the previous definition, this change is prescribed by an objective property

[6]In (Catren [unpublished]), it was argued that this assertion is encoded in the so-called momentum map (see for example Marsden and Ratiu [1999]).

p_z which specifies how the die rotates around the axe z. Now, let us compile a catalogue of all the die's profiles when it is rotated 360° in intervals of 60°. This catalogue composes a discrete sequence of six natural numbers $\{n_{\theta_1}, n_{\theta_2}, ..., n_{\theta_6}\}$. Such a sequence will be called an orbit of profiles. The orbit of profiles contains all the possible results of an observation. By definition, an objective property is the generator of an orbit of profiles for a certain kind of operation. For each kind of operation, there is a different objective property and a different orbit of profiles of the same object. In particular, the objective property p_z generates the die's orbit of profiles under rotations around the axe z. We can thus restate our definition as follows. An object is a physical configuration that can be completely identified by specifying the set of generators of its orbits of profiles.

Our definition establishes a difference between two possible kinds of predicates that can be asserted about physical objects, namely the objective properties and the profiles. Since a particular object's profile changes when we modify our perspective on the object, the profile is not objective. Reciprocally, an objective property cannot change when we modify the relative position between the object and the observer. In other words, a property is objective if it is invariant under the operations that interchange the object's profiles. In this way, we recover the idea that an object can be defined by means of the invariants under the transformations that connect its different representations or projections (see for example Auyang [1995]; Born [1998]; Nozick [1998]). The transformations that leave the object (i.e. the set of its objective properties) invariant will be called *automorphisms* of the object. Following H. Weyl, we can thus state that '[...] objectivity means invariance with respect to the group of automorphisms' (Weyl [1952]). Nevertheless this standard characterization is not sufficient for defining objectivity. This problem was clearly stated by R. Nozick ([1998]): 'The notion of invariance under transformations cannot (without further supplementation) be a *complete* criterion of the objectivity of facts, for its application depends upon a selection of *which* transformations something is to be invariant under.'[7] Our definition provides this "further supplementation" by stating that the transformations that must be

[7]Analogously, H. Weyl continues the preceding quotation as follows: 'Reality may not always give a clear answer to the question what the actual group of automorphisms is [...]' (Weyl [1952]).

considered in order to define these invariants are generated by the objective properties themselves. Hence, the object itself prescribes those transformations which define its automorphisms. Hence, not only is an objective property invariant under all the object's automorphisms, but it also generates one of these automorphisms.[8]

One important consequence of this definition is that the operation generated by an objective property in the object's *eidos* cannot modify the other objective properties in the *eidos*. Objective properties therefore must be invariant under operations generated by other objective properties. Let's consider for example an object defined by the *eidos* $\varepsilon = \{p_1, p_2, ..., p_n\}$, where each p_i is an objective property of the object. The standard definition of objectivity requires that each objective property p_i be invariant under a certain group of appropriate transformations. Nevertheless, it is not clearly stated which transformations have to be considered. Our definition overpasses this flaw by stating that each objective property p_i is the generator of a particular kind of transformation that interchanges the different profiles in a certain orbit. By combining both prescriptions, we arrive at the conclusion that each objective property has to be invariant under the transformations generated by all the other properties in the same *eidos*. This fact imposes a restrictive condition on the *eidos* of an object. The *eidos* is not just a collection of unrelated objective properties. Each property has to satisfy the condition of being invariant under the transformations

[8]This link between objective property and generators of automorphisms is the conceptual kernel of the correspondence between observables and operators. In classical mechanics, an observable $f \in \mathcal{C}^{\infty}(M)$ defines a Hamiltonian vector field v_f by means of the expression $i_{v_f}\omega = df$, where ω is the symplectic form of the phase space M. The Hamiltonian vector field v_f is the generator of a symplectic diffeomorphism $\phi^f_\lambda : M \to M$, that is to say of the canonical transformation induced by the observable f (Abraham and Marsden [1978]; Libermann and Marle [1987]; Marsden and Ratiu [1999]). The problem in classical mechanics is that the correspondence between the Poisson algebra of classical observables $f \in \mathcal{C}^{\infty}(M)$ and the Lie algebra of classical operators v_f (under the Lie bracket of vector fields) is not an isomorphism of Lie algebras (since the map $f \mapsto v_f$ is not injective). The conceptual consequences of this fact were analyzed in (Catren [unpublished]). The guiding idea of geometric quantization is that a Lie algebra isomorphism between observables and operators can be forced by properly extending classical operators to quantum operators (see Brylinski [1993]; Kostant [1970]; Souriau [1997]; Woodhouse [1992]).

generated by all the others. If a property p_1 is invariant under the action generated by p_2, we will say that they are compatible. Therefore, the *eidos* is characterized by an internal structure that guarantees the mutual compatibility between the objective properties that define the object.[9] The object will be completely determined if the *eidos* contains the maximum number of mutually compatible properties. In particular, a predicate q that is modified by the action generated by an objective property p cannot also belong to the *eidos*. In other words, q and p are not compatible. In our previous example, the numbers in the different sides are not objective properties of the die, but rather its possible profiles. In other words, since the property p_z belongs to the die's *eidos*, the predicates modified by the action that p_z generates cannot be objective properties, but only possible profiles that change when the die is rotated. In general, we will say that the predicates modified by the action of an objective property are *gauged out* by this action. The action generated by an objective property will be called *gauge transformation*.

We claim that the following statement can be considered the conceptual translation of the uncertainty principle: if a predicate q is modified by a gauge transformation generated by an objective property p in the object's *eidos*, then the predicate q cannot be an objective property of the object, but only its profile. In particular, the momentum p of a physical object is the generator of a transformation that modifies the object's position q (and vice versa).[10] If the momentum p is an objective property in the object's *eidos*, then the position q cannot be an objective property, but rather is a profile that changes when the object is acted upon by a transformation generated by p. As we have already stated, asking which position is objective in an object with a well-defined momentum is as nonsensical as looking for the objective face of a die. Nevertheless, even if the die has no privileged side, it will show a particular side when thrown. Analogously, even if a physical system with a well-defined momentum has no objective position, it will appear in a particular position if a position observation is performed.

[9]In technical terms, the *eidos* defines a *commutative algebra*.

[10]The Poisson bracket $\{q,p\} \doteq v_p(q) = \mathcal{L}ie_{v_p} q = 1$ means that the symplectic diffeomorphism generated by the Hamiltonian vector field v_p induced by the momentum p transforms the position q. This means that the momentum p is the generator of the infinitesimal canonical transformations of the position q.

This does not mean that the observed position is the objective position of the system.

For the sake of simplicity we have only considered the case of an object with a well-defined momentum and a completely undetermined position. The reciprocal case (a well-defined position with an undetermined momentum) is completely analogous. In the general case, both the position and the momentum are subject to certain indeterminacies. In fact, the flexibility of quantum mechanics' formalism makes possible the definition of intermediate physical states characterized by properties which are neither objective properties nor profiles, but rather a mixture of both. In these cases, neither q nor p are sharp objective properties of the object. If, for example, q is an unsharp objective property of a physical state, the conjugated momentum p is not completely gauged out. Hence, p is in turn an unsharp objective property that partially gauges the coordinate q. This means that, for a given physical state, a certain predicate can be partially considered an unsharp objective property (that partially gauges the conjugated variable) and partially a profile. It follows that the sheer distinction between invariants and gauge-dependent properties does not suffice for treating generic cases. The resulting subtle equilibrium between unsharp objective properties and induced unsharp non-objective profiles is formally governed by the uncertainty principle.

III. Dynamics

The analysis presented in the previous section makes no reference to temporal processes. Since physics, as it is usually understood, studies the temporal evolution of physical systems, we will now introduce a temporal parameter. A consideration of temporal processes will allow us to shift the discussion from momenta (whose definition does not make any reference to a temporal parameter) to velocities (temporal variation of positions).

We will begin by noting that a physical system moving with a well-defined velocity lacks, by definition, a well-defined position. Analogously, one may think of this as a nomad who necessarily lacks a fixed address. We claim that this trivial fact contains the conceptual kernel of the uncertainty principle for positions and velocities. One might argue that it is necessary to distinguish between the definition of the physical system as an object and the specification of its instantaneous states. Even though its state of motion makes it impossible to assign it a well-defined constant position, it might still be possible to define its instantaneous position at any time. To the object's definition given by its *eidos* we could add the information concerning its instantaneous state. We will now analyze whether this strategy can be consistently pursued in the framework of our definition of physical objects.

In our previous example, we could imagine that the die is uniformly rotating such that a different side is visible to us at each time of a given sequence. If we observe the die at any time belonging to this sequence, we will observe a die's profile, that is to say a single side of the die with a natural number. Even if the die does not change as a physical object, the observed side changes. We could therefore define an instantaneous state at time t given by the objective property p_z and the side observed at t. While the observed side is not an objective property of the die (since it is a profile acted upon by the gauge transformation generated by p_z), it might nevertheless be an instantaneous objective property. If this were the case, then we could define the die's instantaneous objective states as in classical mechanics. However, there is a fundamental objection to this strategy. In order to make sense of the notion of instantaneous states, it is necessary to introduce an auxiliary parameter to index the different observations. As usual, this parameter is called time. Just as the die can be observed from different angles, in principle it can also be observed from different times. Nevertheless, the transformation of the temporal perspective on the object will be an admissible transformation only if the property p_t that generates "time evolution" is included in the object's *eidos*. In other words, in order to define a uniformly rotating die, it is necessary to assume that the property p_t that generates the die's orbit of profiles parameterized by the variable t is also an objective property of the die.[11] If p_t belongs to

[11]This argument supposes that it is possible to treat time and energy as if they were another pair of conjugated canonical variables. In fact, this is possible in the framework of the so-called parameterized systems (see for example Lanczos [1986] and Castagnino *et al.* [2002]). In a parameterized system, the pair $(t, p_t = -h)$ (where h is the Hamiltonian) is added to the original set of canonical variables (q^i, p_i). To do so, it is necessary to incorporate the definition of p_t within the action by means of the so-called Hamiltonian constraint $H = p_t + h = 0$ and the corresponding Lagrange multiplier. The variable of integration of the resulting parameterized action is a parameter τ which has no physical meaning (i.e. the theory is invariant under reparameterizations of τ).

the object's *eidos*, then different times are just different perspectives from which the physical object can be observed.[12] Since time evolution is only a particular kind of modification of the perspectives on the same object, the object cannot change objectively in time. It can only appear differently, that is to say it can only show different temporal profiles. Reciprocally, properties that change in time cannot be objective. It follows that instantaneous states cannot change objectively in time. However, the notion of an instantaneous objective state is meaningful only if it can become another instantaneous objective state as time passes. Therefore, it is not possible to define instantaneous objective states for a physical object whose *eidos* contains the property p_t. Because the instantaneous states are not objective (i.e. they have no invariant meaning), we cannot describe the rotating die in terms of a continuous sequence of instantaneous objective states. If instantaneous objective states cannot be defined, then a system moving with a well-defined constant velocity cannot objectively have a well-defined position, even at a certain instant. Hence, such a system should not be analyzed in terms of instantaneous objective states that evolve in time, but rather in terms of a non-temporal physical object with non-objective temporal profiles. The fact that the object "evolves" in time, that is to say that it can be observed at different times, demonstrates that it is a non-temporal object. These conclusions result from the assumption that the property p_t belongs to the object's *eidos*. Nevertheless, in principle it is also possible to define an instantaneous object such that its *eidos* contains the property t instead of p_t. In this case, the property p_t is no longer an objective property. Since the transformation that modifies the temporal profiles is no longer an automorphism, an instantaneous object cannot be observed at different times. Therefore, an instantaneous object cannot evolve. These arguments suggest that a satisfactory comprehension of the uncertainty principle for time and energy is an essential component of a consistent interpretation of quantum mechanics.

[12] This statement is a rigorous interpretation of the fact that '[...] the motion of a mechanical system corresponds to the continuous evolution or unfolding of a canonical transformation.' (Goldstein [1981]).

IV. Classical objects

The arguments proposed in the previous section rely on the assumption that the transformations generated by the objective properties of a physical system are gauge transformations. As we have shown, the uncertainty principle is a direct consequence of this assumption. If the momentum p is an objective property of a physical system, then the position q is completely gauged out by the gauge transformation generated by p. Since the classical definition of objective physical states comprises both its exact position and its exact momentum, this assumption cannot be consistently implemented in the framework of classical mechanics.

We can also argue differently. If both the position and the momentum are included in the object's *eidos*, then both the position and the momentum will be gauged out by the gauge transformations generated by the momentum and the position respectively. Therefore, both the position and the momentum will only be gauge-dependant properties and the physical system will have no objective properties at all. We can thus conclude that the classical definition of states by means of both q and p is incompatible with our definition of physical objects. The classical definition of a physical state is consistent only if we deny that the action generated by an objective property of the system is a gauge transformation. In fact, in classical mechanics the action generated by an objective property is not interpreted as a gauge transformation, but rather as a transformation between states that are objectively different. For example, the transformation generated by the Hamiltonian is interpreted as a temporal evolution between different objective states. By doing so, the definition of classical states becomes consistent. Nevertheless, objective properties can no longer be defined as the generators of the physical object's automorphisms. Hence, the classical definition of both objective properties and physical objects remains problematic. The situation has thus been happily reversed: the problem is no longer how to recover objectivity in quantum mechanics, but rather to understand how classical objects can be consistently defined.

V. Conclusion

We have defined a physical object as a set of mutually compatible objective properties. By definition, each objective property generates a gauge transformation, that is to say an object's automorphism. This relationship between objective properties and automorphisms is the conceptual counterpart of the formal correspondence between observables and operators. The compatibility condition guarantees that the objective properties are invariant under the automorphisms generated by all the others objective properties of the same object. The uncertainty principle is a direct consequence of the mutual intertwining between objective properties and gauge-dependant properties: if p is an objective property of an object, then the property q gauged out by p cannot also be an objective property.

We could restate Einstein's characterization by saying that a satisfactory physical theory has to provide a complete *objective* description of physical reality (Einstein *et al.* [1935]). Firstly, this means that every objective element of physical reality should have a counterpart in the theory. Secondly, gauge-dependant elements should not be taken as objective by the theory. The classical description of a physical system includes both its objective properties and its gauge-dependant properties. Unlike classical mechanics, quantum mechanics provides a complete objective description of physical systems.

References

[1] Abraham, R. and Marsden, J.E. [1978]: *Foundations of Mechanics*, second ed., Cambridge, MA: Addison-Wesley Publishing Company.

[2] Auyang, S.Y. [1995]: *How is Quantum Field Theory Possible?*, Oxford-New York: Oxford University Press.

[3] Born, M. [1998]: 'Physical Reality', in E. Castellani (ed.), 1998, *Interpreting Bodies. Classical and Quantum Objects in Modern Physics*, New Jersey: Princeton University Press.

[4] Brylinski, J.-L. [1993]: *Loop Spaces, Characteristic Classes, and Geometric Quantization*, Progr. Math. 107, Boston MA: Birkhäuser Boston Inc.

[5] Catren, G. [unpublished]: 'On Classical and Quantum Objectivity', http://philsci-archive.pitt.edu/archive/00003542/.

[6] Castagnino, M., Catren, G. and Ferraro, R. [2002]: 'Time Asymmetries in Quantum Cosmology and Boundary Conditions to the Wheeler-DeWitt Equation', *Class. Quant. Grav.* 19, No. 18, pp. 4729-4746.

[7] Einstein, A., Podolsky, B. and Rosen, N. [1935]: 'Can Quantum-Mechanical Description of Physical Reality be Considered Complete?', *Phys. Rev.* 47, pp. 777-780.

[8] Goldstein, H. [1981]: *Classical Mechanics*, Cambridge, MA: Addison-Wesley Publishing Company.

[9] Heelan, P. [2004]: 'The Phenomenological Role of Consciousness in Measurement', *Mind and Matter*, Vol. 2(1), pp. 61-84.

[10] Hilgevoord, J. and Uffink, J. [2006]: 'The Uncertainty Principle', in E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*.

[11] Kostant, B. [1970]: 'Quantization and Unitary representations', *Lecture Notes in Mathematics*, Vol. 170, Berlin-Heidelberg-New York: Springer-Verlag.

[12] Lanczos, C. [1986]: *The Variational Principles of Mechanics*, New York: Dover.

[13] Libermann, P. and Marle, C.-M. [1987]: *Symplectic Geometry and Analytical Mechanics*, Dordrecht, Holland: D. Reidel Publishing Company.

[14] Marsden, J.E. and Ratiu, T.S. [1999]: *Introduction to Mechanics and Symmetry*, second ed., New York: Springer-Verlag.

[15] Mittelsteadt, P. [2006]: 'Einstein's Objections against Quantum Mechanics', *Physics and Philosophy*, http://hdl.handle.net/2003/22995.

[16] Nozick, R. [1998]: 'Invariance and Objectivity', *Proceedings and Addresses of the American Philosophical Association*, Vol. 72, No. 2, pp. 21-48.

[17] Souriau, J.-M. [1997]: *Structure of Dynamical Systems. A Symplectic View of Physics*, Cambridge, MA: Birkhäuser Boston.

[18] Weyl, H. [1952]: *Symmetry*, Princeton: Princeton University Press.

[19] Woodhouse, N. [1992]: *Geometric Quantization*, second ed., Oxford: Oxford University Press.