Classical Bell's Inequalities

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Abstract

An example of a classical system violating Bell's inequalities is discussed. Existence of a classical system violating Bell's inequalities takes away the "mysterious" property usually called "non-locality" which according to some characterizes quantum-mechanical systems.

In recent years there has been an explosion of research on consequences from what initially had been an attempt by Einstein, Podolsky and Rosen (EPR)[1] to put quantum mechanics (QM) in question. Turning the tide is mainly due to the theoretical paper of Bell [2] discussing a two ½-spin particle system proposed by Bohm [3] as a special case of the system discussed in the initial critique by EPR [1].

Despite the initial questions posed by EPR the problems regarding QM are usually considered nowadays as settled and questioning it is considered highly unusual. Furthermore, claimed violation of Bell's inequalities only by QM systems makes many today to believe that QM systems are endowed with some special qualities, such as e.g. "non-locality", which they consider that classical systems lack.

It is shown below, however, using a simple example, that Bell's inequalities can be violated by a classical system as well. This puts into question the notion of "non-locality" which is the basis for the revolutionary solutions some assume QM seems to be offering.

The "Experiment"

Consider two large vessels of over 10L each. Transparent water of 20L volume is to be distributed among these two vessels. In a manner pointed out by Aerts [4] we will make simultaneous measurements on the water in the two vessels. Unlike Aerts [4], however, we will have these two vessels completely separate and placed at a large distance from each other.

Also, unlike Aerts [4] we will consider the outcome from the measurement of the quantity of water in A as "yes" when the water in A is more than that in B. If there is more than 10L in vessel A then inevitably in vessel B the water will be less than that in A – the experiment on vessel B will yield the result "no".

Denote by m(a) the measurement which determines whether vessel A contains more water than vessel B ("quantity measurement"). If the quantity of water in A happens to be more than the quantity in B the value of m(a) is "yes", otherwise it is "no". The corresponding measurement for vessel B is m(b). Denote by m(a') the measurement which determines whether the water in A is transparent ("transparency measurement"). To carry out transparency measurement one removes 1L of water from the vessel and makes the determination of whether the water is transparent. If the water in A happens to be transparent the outcome from the measurement m(a') is "yes", otherwise the outcome is "no". Similar transparency measurement on the water in vessel B is denoted by B(b').

The measurements in which we will be interested here in this discussion are coincidence measurements done on both vessel A and vessel B at the same time. There could be only four such outcomes whose values will be denoted by E(a,b), E(a',b), E(a,b') and E(a',b') – expectation values of the observables corresponding to the respective coincidence measurements m(a,b), m(a',b), m(a,b') and m(a',b'). The values of these quantities are +1 when measurements involving both arguments (coincidence measurements) yield either "yes,yes" or "no,no". When the two arguments in each expectation value have opposite meaning the value of the expectation value is –1.

Let us carry out the experiments:

First coincidence measurement: Let vessel A contain 10.1L. This means that vessel B contains 9.9L. This, according to Aert's notation [4] (under our condition) will give E(a,b) = –1. Note that above distribution of the 20L of transparent water is completely random. With the same probability we could have had 9.9L water in A and 10.1L in B. For simplicity we will observe the former case.

Second coincidence measurement: This measurement consists in an experiment to measure the transparency of the water in A and, together with it (simultaneously), an experiment to compare the volume of water of B with that in A. Following Aerts we take a 1L sample of the water in A and determine that the water is transparent (the result is "yes"). Removing of 1L water from A causes A to contain already 9.1L which is less than the volume of water in B (9.9L). Therefore, an experiment to compare the volume in B with that in A yields a result "yes". Thus, according to Aerts' notation this second coincidence measurement will yield a result E(a',b) = +1.

Third coincidence measurement: Now we take a sample of 1L from vessel B. Observation on the sample indicates that the water is transparent – the result is "yes". The act of taking the sample, however, leaves 8.9L water in vessel B which is less than the volume of water in vessel A (9.1L) – the result for A is "yes". Therefore, the result from the coincidence measurement in this case is E(a,b') = +1.

Fourth coincidence measurement: Evidently the result for this measurement will be E(a',b') = +1 because the water in both vessels is transparent.

Thus, we get [5]

$$|E(a,b) - E(a',b)| + |E(a,b') + E(a',b')| = 4 > 2$$

and Bell's inequality is violated.

Both in the classical and in the QM case the two parts of the system (the two vessels, respectively, the two particles) are not independent from the onset, as a result of the way the problem is construed. The fact that the two vessels, far removed from each other, are not independent (and this is the prerequisite for the violation of Bell's inequalities) is not something immediately evident. The dependence between the two vessels is ensured by the initial condition that the total volume of water in the system is 20L. In the same exact way, the fact that the two EPR particles are not independent is not something immediately evident. However, the dependence between these two particles is ensured from the beginning – their state is described by a common psi-function.

The paralel between the two classical vessels removed from each other at a great distance and the two EPR particles can continue also when measurements are considered. When a measurement is carried out of a given observable A, for instance on the first EPR particle, all the eigenvalues of the matrix A representing this observable are known *apriori*, without exception (although the very act of measurement "extracts" at random only one member of this set of eigenvalues). Thus, when we apply the matrix A on the psi-function, common for the two particles, we do not expect to create something that was not there in the first place, i.e. something that was not there by definition. Exactly because of this initial setup of the function, when we measure the momentum p of the first particle the momentum of the second particle must necessarily be –p, the concrete values of p being completely random (if we repeat the experiment the concrete value of p may be different).

It is to be noticed now, in connection with the above classical experiment, that if we like to wonder at various things, as some do in QM, we can do it here too. For instance, we may be puzzled by the fact that a measurement m(a') which we carry out on vessel A and which gives the result "yes", in some "mysterious" way causes m(b) to be necessarily "yes" and not anything else (if someone cares to check that). Therefore, we may continue if we consider this path of thought fruitful, information between A and B has passed at a speed greater than the speed of light which contradicts STR (another questions is whether STR has indeed anything to do with the speed of information transfer). We can even write that the common probability P(a*b) does not equal any more the product P(a)P(b) and conclude all kinds of other things.

It seems more reasonable, however, to admit that it is hardly possible to maintain an argument claiming that in all cases whenever there is a connection between two systems Bell's inequality is violated and this violation is entirely plausible, while, on the contrary, in QM Bell's inequality is violated although there is no connection between the two particles and that makes QM something very special.

The above indicates that the notion of two entirely isolated particles in QM which somehow exchange information among themselves loses content. This is, of course, if we are not willing to accept (not likely !) that also the two classical vessels exchange information among themselves.

This discussion is done without assuming the validity of the Special Theory of Relativity (STR). Not assuming validity of STR seems to be the methodologically correct approach. Especially we have ignored an often mentioned requirement that the speed of information exchange cannot be infinite. Provided that assumption, probably one may use the above results to even explore the validity of STR itself.

References

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