An Argument for 4D Blockworld from a Geometric Interpretation of Non-relativistic Quantum Mechanics

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1. Introduction

We use a new, distinctly "geometrical" interpretation of non-relativistic quantum mechanics (NRQM) to argue for the fundamentality of the 4D blockworld ontology. We argue for a geometrical interpretation whose fundamental ontology is one of spacetime relations as opposed to constructive entities whose time-dependent behavior is governed by dynamical laws. Our view rests on two formal results: Kaiser (1981 & 1990), Bohr & Ulfbeck (1995) and Anandan, (2003) showed independently that the Heisenberg commutation relations of NRQM *follow from* the relativity of simultaneity (RoS) per the Poincaré Lie algebra. And, Bohr, Ulfbeck & Mottelson (2004*a* & 2004*b*) showed that the density matrix for a particular NRQM experimental outcome may be obtained from the spacetime symmetry group of the experimental configuration. This shows how the blockworld view is not only consistent with NRQM, not only an *implication* of our geometrical interpretation of NRQM, but it is necessary in a non-trivial way for explaining quantum interference and "non-locality" from the spacetime perspective. Together the formal results imply that contrary to accepted wisdom, NRQM, the measurement problem and so-called quantum non-locality do not provide reasons to abandon the 4D blockworld implication of RoS. But rather, the deep non-commutative structure of the quantum and the deep structure of spacetime as given by the Minkowski interpretation of special relativity (STR) are deeply unified in a 4D spacetime regime that lies between Galilean spacetime (G4) and Minkowski spacetime ($\mathcal{M}^4$).

Taken together the aforementioned formal results allow us to model NRQM phenomena such as interference without the need for realism about 3N Hilbert space, establishing that the world is really 4D and that configuration space is nothing more than a calculational device. Our new geometrical interpretation of NRQM provides a geometric account of quantum entanglement and so-called non-locality free of conflict with STR and free of interpretative mystery.

In section 2 we discuss the various tensions between STR and NRQM with respect to the dimensionality of the world. Section 3 is devoted to an explication of the Kaiser *et al.* results and their philosophical implications. Likewise, the Bohr *et al.* results and their implications are the subject of section 4. In section 5, we present our geometric interpretation of quantum entanglement and "non-locality."

2. Motivating the Geometric Interpretation: STR versus NRQM on the Dimensionality of the World

In relativity theory, we have two physical postulates (relativity and light postulates) and we have a *geometric model* or "interpretation" of those postulates – Minkowski's hyperbolic 4-geometry that gives us a geometry of "light-cones." The "blockworld" (BW) view tries to establish a *metaphysical* interpretation of the Minkowski geometrical rendition of special relativity. It is a view that tries to establish the reality of all spacetime events (contra presentism), whose structure is given by the special relativistic metric. We shall not rehearse the familiar arguments for the BW implication from the relativity of simultaneity (see Stuckey *et al.* 2007), but only describe it herein:

> There is no dynamics within space-time itself: nothing ever moves therein; nothing happens; nothing changes. In particular, one does not think of particles as moving through space-time, or as following along their world-lines. Rather, particles are just in space-time, once and for all, and the world-line represents, all at once, the complete life history of the particle. Robert Geroch, *General Relativity from A to B* (University of Chicago Press, Chicago, 1978) p. 20-21.

When Geroch says that there is no dynamics within spacetime itself, he is not denying that the mosaic of the BW possesses patterns that can be described with dynamical laws. Nor is he denying the predictive and explanatory value of such laws. Rather his point is that in a BW (given the reality of all events) dynamics such as Schrödinger dynamics are not event factories that bring heretofore non-existent events (such as measurement outcomes) into being. Dynamical laws are not brute unexplained explainers that "produce" events. Geroch is advocating for what philosophers call Humeanism about laws. Namely, the claim is that dynamical laws are *descriptions of regularities* and not the *brute explanation* for such regularities. His point is that in a BW, Humeanism about laws is an obvious position to take because everything is just there.

Some have actually suggested that we ought to take the fact of BW seriously when doing physics and modeling reality. Huw Price (1996) for example calls it the "Archimedean view from nowhen" (260) and it has motivated him to take seriously the idea of a time-symmetric quantum mechanics. Price is primarily concerned to see if one can construct a local hidden-variables interpretation of NRQM that explains so-called quantum non-locality with purely time-like dynamics or backwards causation.

Not only is the BW strikingly at odds with NRQM dynamically conceived, but NRQM and STR appear to disagree about the very dimensionality of the world. For as David Albert says:

> the space in which any realistic interpretation of quantum mechanics is necessarily going to depict the history of the world as *playing itself out* … is *configuration*-space. And whatever impression we have to the contrary (whatever

> impression we have, say, of living in a three-dimensional space, or in a four-dimensional space) is somehow flatly illusory (1996, p. 277).

Is the world a 4D Minkowski spacetime BW as relativity tells us or is it a 3*N*-dimensional configuration space of possibly infinite dimensions as quantum mechanics tells us? How can we resolve this apparent conflict? If we assume that it is in fact a 4D BW as we do here, then what should we make of Hilbert space?

Most natural philosophers are inclined to accept that special relativity unadorned implies the blockworld view. Among those who might agree that special relativity unadorned implies a blockworld are those who think that quantum theory provides an excellent reason to so adorn it even apart from Hilbert space realism. That is, there are those who claim that quantum non-locality or some particular solution to the measurement problem (such as collapse interpretations) require the addition of, or imply the existence of, some variety of preferred frame (a preferred foliation of spacetime into space and time)[1] in order to render quantum mechanics covariant and resolve potential conflicts between observers in different frames of reference. This trick could be done in a number of ways and *need not* involve postulating something like the "luminiferous aether." For example, one could adopt the Newtonian or neo-Newtonian spacetime of Lorentz or one could *add* a physically preferred foliation to $\mathcal{M}^4$. With a constructive theory of STR in hand one might also attempt to block the blockworld interpretation. As Callender notes (2006, 3):

> In my opinion, by far the best way for the tenser to respond to Putnam *et al.* is to adopt the Lorentz 1915 interpretation of time dilation and Fitzgerald contraction. Lorentz attributed these effects (and hence the famous null results regarding an aether) to the Lorentz invariance of the dynamical laws governing matter and radiation, not to spacetime structure. On this view, Lorentz invariance is not a spacetime symmetry but a dynamical symmetry, and the special relativistic effects of dilation and contraction are not purely kinematical. The background spacetime is Newtonian or neo-Newtonian, not Minkowskian. Both Newtonian and neo-Newtonian spacetime include a global absolute simultaneity among their invariant structures (with Newtonian spacetime singling out one of neo-Newtonian spacetime's many preferred inertial frames as the rest frame). On this picture, there is no relativity of simultaneity and spacetime is uniquely decomposable into space and time. Nonetheless, because matter and radiation transform between different frames via the Lorentz transformations, the theory is empirically adequate. Putnam's argument has no purchase here because Lorentz invariance has no repercussions for the structure of space and time. Moreover, the theory shouldn't be viewed as a desperate attempt to save absolute simultaneity in the face of the phenomena, but it should rather be viewed as a natural extension of the well-known Lorentz invariance of the free Maxwell equations. The reason why some tensers have sought all manner of strange replacements for special relativity when this comparatively elegant theory exists is baffling.

[1] *See* Tooley (1997) ch. 11, for one example.

The task we have set for ourselves in this paper is to take up the charge of Archimedean physics in a way far more radical than even time-symmetric quantum mechanics suggests. Our account is a hidden-variables statistical interpretation of a sort, but unlike Price and others we are not primarily motivated by saving locality. Rather we are motivated by seeing how far we can take Archimedean physics. What follows is a purely geometric (acausal and adynamical) account of NRQM. Our view defends the surprising thesis that the relativity of simultaneity plays an *essential role* in the spacetime regime for which one can obtain the Heisenberg commutation relations of non-relativistic quantum mechanics – the cornerstone of the structure of quantum theory. This point bears repeating. While it is widely appreciated that special relativity and quantum theory are not necessarily incompatible, what is *not* widely appreciated are a collection of formal results showing that quantum theory and the relativity of simultaneity are not only compatible, but in fact are *intimately related*. More specifically, in the present paper we will draw on these results and clearly show that it is precisely this "nonabsolute nature of simultaneity[2]" which survives the $c \rightarrow \infty$ limit of the Poincaré group, and which *entails* the canonical commutation relations of *non-relativistic* quantum mechanics. These results lead us to formulate a new *geometric* account of NRQM that will be elucidated in later sections of the paper.

We will also show that this geometric interpretation of NRQM nicely resolves the standard conceptual problems with the theory: (i) *prior to* the invocation of any dynamical interpretation of quantum theory itself and (ii) *prior to* the issue of whether any interpretation of quantum theory – i.e., a *mechanics* of the quantum – can be rendered relativistically invariant/covariant. Namely, we will provide both a geometrical account of entanglement and so-called "non-locality" free of tribulations, *and* a novel version of the statistical interpretation that deflates the measurement problem. Our geometrical NRQM has the further advantage that it does not lead to the aforementioned problems that some *constructive* accounts of NRQM face when relativity is brought into the picture, such as Bohmian mechanics and collapse accounts like the wave-function interpretation of GRW. On the contrary, not only does our view require no preferred foliation but it also provides for a profound, though little-appreciated, *unity* between STR and NRQM *by way of the relativity of simultaneity*[3]. Our interpretation of NRQM can be characterized as follows:

(i) Realism about M4 and the BW but not Hilbert space.
(ii) We adopt the view that NRQM is a geometric theory in the following respects:
   a. it merely provides a probabilistic rule by which new trajectories are generated – i.e., we take NRQM *qua* to provide *constraints on the distribution of events in spacetime*;

[2] Kaiser (1981), p. 706.
[3] In this respect, our interpretation is close to that of Bohr and Ulfbeck. In their words, "quantal physics thus emerges as but an implication of relativistic invariance, liberated from a substance to be quantized and a formalism to be interpreted" (1995, 1).

b. it is not *fundamentally* a dynamical theory of the behavior of matter-in-motion. Our ontology does not accept matter-in-motion as *fundamental* (though such a view is phenomenologically/pragmatically useful);
c. quantum "entities" and their characteristic properties such as entanglement and non-locality are geometric features of the spacetime structure just as gravity is taken to be a feature of the geometry in general relativity (GR) and not ultimately explained by the "inner constitution" of material bodies themselves or dynamical forces. Though our view is more radically geometric than GR, even Einstein did not dream of geometrizing matter-energy itself in GR;
d. spatiotemporal relations are the means by which all physical phenomena (including both quantum and classical "entities") are modeled, allowing for a natural transition from quantum to classical mechanics (including the transition from quantum to classical probabilities) as simply the transition from rarefied to dense collections of spacetime relations;

(iii) we adopt an explanatory strategy that is faithful to our methodological and ontological commitments: we take the view that the determination of events, properties, experimental outcomes, etc., in spacetime is made with spacetime symmetries both *globally* and a*causally/adynamically*. That is, we will invoke an acausal global determination relation that respects *neither* past nor future common cause principles.

Many will assume that a geometric interpretation such as ours is impossible because quantum wave-functions live in Hilbert space and contain much more information than can be represented in a classical space of three dimensions. The existence of entangled quantum systems provides one obvious example of the fact that more information is contained in the structure of quantum mechanics than can be represented completely in spacetime. As Peter Lewis says, "the inescapable conclusion for the wavefunction realist seems to be that the world has 3*N* dimensions; and the immediate problem this raises is explaining how this conclusion is consistent with our experience of a three-dimensional world" (2004, 717). On the contrary, the existence of the non-commutativity of quantum mechanics is deeply related to the structure of *spacetime* itself, without having to invoke the geometry of Hilbert space. Surprisingly, as will be demonstrated in the following section, it is a spacetime structure for which the relativity of simultaneity is upheld, and not challenged.

3. The Relativity of Simultaneity and Non-relativistic Quantum Mechanics

Lorentz boosts (changes to moving frames of reference according to the Poincaré group of STR) do not commute with spatial translations since different results obtain when the order of these two operations is reversed. Specifically, this difference is a temporal displacement which is key to generating a BW. This is distinct from Newtonian mechanics whereby time and simultaneity are absolute per Galilean invariance. If spacetime was Galilean invariant, observers would agree as to which events were simultaneous and presentism could be true. In such a spacetime, it would not matter if you Galilean boosted then spatially translated, or spatially translated then Galilean boosted. *Prima facie*, one might suspect that *non-relativistic* quantum mechanics would be in accord with Galilean spacetime. And indeed, the linear dynamics – the Schrödinger equation – is Galilean invariant (Brown and Holland 1999). However, as we will show, while it is indeed true that the Schrödinger dynamics is Galilean invariant, the appropriate spacetime structure for which one can obtain the Heisenberg commutation relations is *not* a Galilean spacetime! Surprisingly, it is a spacetime structure "between" Galilean spacetime and Minkowski spacetime, but one for which the relativity of simultaneity is upheld, unlike in Galilean spacetime.

Inevitably, the very means by which we can establish a determinate position in spacetime – or a determinate momentum (mass times velocity) – is going to have to speak to the quantum theory, a theory which places strictures on such questions. Now, a position can be given by an "axis of rotation" in a spacetime (just imagine a line around which some reference frame is spinning, or around which every other coordinate system is contracting if we are talking about Lorentz boosting from one frame to another). Such a thing can be picked out by "boost" operators, to use the language of the spacetime symmetry group. Given a Lorentz boost, one effectively picks out a position in spacetime (since the new coordinate systems given by the boost operator *all* share exactly their *origin* in common – thus uniquely picking out *one point in 2D spacetime and a line in 3D spacetime, etc.*). That is, the axis of rotation yields a spacetime trajectory which would yield a point in 'space' at any given time. Similarly, we might think about "momentum" as nothing but (speaking again in terms of spacetime groups) the generators of spatial translations. That is, spatially translating is simply "moving" from one position to another (albeit into a new frame); and this is something like a velocity (i.e., a time-derivative of position).

Now, if we *define* a commutator between position and momentum *in terms of* the generators of boosts and spatial translations respectively – and note that they *do not commute* when simultaneity is nonabsolute (relative) – is it possible to show that one can arrive at the quantum-mechanical commutator *of position and momentum*, and have it *equal* to the quantum mechanically well-known quantity $-i\hbar$? This is equivalent to asking "what is the spacetime structure such that, if simultaneity is non-absolute, the Heisenberg commutator can be deduced?"[4]

Quite surprisingly, it turns out that *because* boosts do not commute with spatial translations *given that simultaneity is relative*, one can indeed deduce the quantum mechanical Heisenberg commutator (in the appropriate *"weakly"* relativistic spacetime regime). This shows that some interpretation exists for both non-relativistic quantum mechanics and any relativistic quantum mechanical theory, where there is a *single, unified spacetime arena* from which either theory can be obtained in the appropriate asymptotic limit. More specifically, what the formal results in the following sections will show is that classical mechanics "lives in" G4, surprisingly NRQM "lives in" a spacetime regime that is between G4 and $\mathcal{M}^4$ (we can call it K4 after Kaiser) and RQFT "lives in" $\mathcal{M}^4$. It will also become clear that NRQM is truly "baby" RQFT in that it also is about

[4] And since quantum theory is already well-established empirically, we essentially know what needs to be derived, we just as-yet have not found the right spacetime structure. This is, admittedly, flipping the order of discovery somewhat, and asking an entirely new question regarding the "origin" of quantum theory (looking to spacetime structure, and not to the structure of matter per se, which is how the theory of the quantum was arrived at historically).

new trajectories—or particle creation to use dynamical lingo. All of this makes for a great deal more unity between spacetime structures and quantum structures than is generally appreciated.

3.1 NRQM: Spacetime structure for commutation relations. Kaiser[5] has shown that the non-commutivity of Lorentz boosts with spatial translations is *responsible for* the non-commutivity of the quantum mechanical position operator with the quantum mechanical momentum operator. He writes[6],

> For had we begun with Newtonian spacetime, we would have the Galilean group instead of [the restricted Poincaré group]. Since Galilean boosts commute with spatial translations (time being absolute), the brackets between the corresponding generators vanish, hence no canonical commutation relations (CCR)! In the [c → ∞ limit of the Poincaré algebra], *the CCR are a remnant of relativistic invariance where, due to the nonabsolute nature of simultaneity, spatial translations do not commute with pure Lorentz transformations*. [Italics in original].

Bohr & Ulfbeck[7] also realized that the "Galilean transformation in the weakly relativistic regime" is needed to construct a position operator for NRQM, and this transformation "includes the departure from simultaneity, which is part of relativistic invariance." Specifically, they note that the commutator between a "weakly relativistic" boost and a spatial translation results in "a time displacement," which is crucial to the relativity of simultaneity. Thus they write[8],

> "For ourselves, an important point that had for long been an obstacle, was the realization that the position of a particle, which is a basic element of nonrelativistic quantum mechanics, requires the link between space and time of relativistic invariance."

So, *the essence of non-relativistic quantum mechanics – its canonical commutation relations – is entailed by the relativity of simultaneity*.

If the transformation equations entailed by some spacetime structure necessitate a temporal displacement when boosting between frames, then the relativity of simultaneity is true of that spacetime structure. Given this temporal displacement between boosted frames, and given that this implies the relativity of simultaneity, our arguments supplied above show that BW is true of this spacetime structure. Furthermore, since the relativity of simultaneity, via the kind of temporal displacement necessitated by boosting between frames in this spacetime regime, is essential to the Heisenberg or canonical commutation relations, we find a heretofore unappreciated deep unity between STR and *non-relativistic* quantum mechanics.

To outline Kaiser's result, we take the limit $c \rightarrow \infty$ in the Lie algebra of the Poincaré group for which the non-zero brackets are:

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[5] Kaiser (1981 & 1990).
[6] Kaiser (1981), p. 706.
[7] Bohr & Ulfbeck (1995), section D of part IV, p. 28.
[8] *Ibid*., p. 24.

$[J_m, J_n] = iJ_k$
$[T_0, K_n] = iT_n$
$[K_m, K_n] = \frac{-i}{c^2} J_k$
$[J_m, K_n] = iK_k$
$[J_m, T_n] = iT_k$
$[T_m, K_n] = \frac{-i}{c^2} \delta_{mn} T_0$

where expressions with subscripts m,n and k denote 1, 2 and 3 cyclic, J_m are the generators of spatial rotations, T_0 is the generator of time translations, T_m are the generators of spatial translations, K_m are the boost generators, $i^2 = -1$, and c is the speed of light. We obtain

$[J_m, J_n] = iJ_k$
$[M, K_n] = 0$
$[K_m, K_n] = 0$
$[J_m, K_n] = iK_k$
$[J_m, T_n] = iT_k$
$[T_m, K_n] = \frac{-i}{\hbar} \delta_{mn} M$

where M is obtained from the mass-squared operator in the $c \to \infty$ limit since

$$c^{-2}\hbar T_0 = c^{-2} P_0$$

and

$$\frac{P_o}{c^2} = \left(M^2 + c^{-2} P^2\right)^{1/2} = M + \frac{P^2}{2Mc^2} + O(c^{-4}) .$$

Thus, $c^{-2} T_0 \to \frac{M}{\hbar}$ in the limit $c \to \infty$. [$M \equiv mI$, where m is identified as "mass" by choice of 'scaling factor' ħ.] So, letting

$$P_m \equiv \hbar T_m$$

and

$$Q_n \equiv \frac{-\hbar}{m} K_n$$

we have:

$$\left[P_m, Q_n\right] = \frac{-\hbar^2}{m}\left[T_m, K_n\right] = \left(\frac{-\hbar^2}{m}\right)\left(\frac{i}{\hbar}\right)\delta_{mn} mI = -i\hbar\delta_{mn} I \qquad (3.1)$$

Bohr & Ulfbeck (1995) point out that in this "weakly relativistic regime" the coordinate transformations now look like:

$$\begin{aligned} X &= x - vt \\ T &= t - \frac{vx}{c^2} \end{aligned} \qquad (3.2)$$

These transformations differ from Lorentz transformations because they lack the factor

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

which is responsible for time dilation and length contraction. And, these transformations differ from Galilean transformations by the temporal displacement vx/c^2 which is responsible for the relativity of simultaneity, i.e., in a Galilean transformation time is absolute so T = t. Therefore, the spacetime structure of Kaiser *et al.* lies between Galilean spacetime and Minkowski spacetime and we see that the Heisenberg commutation relations are not the result of Galilean invariance, where spatial translations commute with boosts, but rather they result from the relativity of simultaneity per Lorentz invariance.

3.2 Heterodoxy: NRQM Does Not Live In Galilean Spacetime. The received view has it that Schrödinger's equation is Galilean invariant, so it is generally understood that NRQM resides in Galilean spacetime and therefore respects absolute simultaneity[9]. However, as we have seen above, Kaiser (1981), Bohr & Ulfbeck (1995) and Anandan (2003) have shown independently that the Heisenberg commutation relations of NRQM follow from the relativity of simultaneity[10]. *Prima facie* these results seem incompatible with the received view, so to demonstrate that these results are indeed compatible, we now show that these results do not effect the Schrödinger dynamics[11].

Why is it that the *dynamics* of NRQM, given by the Schrödinger equation, are Galilean invariant? That is, why are the dynamics of NRQM unaffected by the relativity of simultaneity reflected in the geometry of Eq. 3.1?

To answer this question we operate on $|\psi\rangle$ first with the spatial translation operator then the boost operator and compare that outcome to the reverse order of operations. The spatial translation (by *a*) and boost (by v) operators in x are:

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[9] *See* Brown and Holland (1999).

[10] Of course, all other commutation relations in NRQM follow from those of position and momentum – with the exception of spin. Since, operationally, spin measurements are simply binary outcomes in space related to, for example, the spatial orientation of a Stern-Gerlach apparatus, our model encompasses such properties as spin to the extent that we model all outcomes in space and time as *irreducible relations* between the spatiotemporal regions corresponding to source and detector.

[11] See also Lepore (1960) who also realizes that this time-shift between frames is without effect on the dynamics of Schrödinger evolution.

$$U_T = e^{-iaT_x} \text{ and } U_K = e^{-ivK_x} \tag{3.3}$$

respectively. These yield:

$$U_K U_T |\psi\rangle = U_T U_K e^{iavmI/\hbar} |\psi\rangle \tag{3.4}$$

Thus, we see that the geometric structure of Eq. 3.1 introduces a mere phase to |ψ> and is therefore without consequence in the computation of expectation values. And in fact, this phase is consistent with that under which the Schrödinger equation is shown to be Galilean invariant[12].

Therefore, we realize that the spacetime structure for NRQM, while not $\mathcal{M}^4$ in that it lacks time dilation and length contraction, nonetheless contains a "footprint of relativity"[13] due to the relativity of simultaneity. Thus, there is an unexpected and unexplored connection between the relativity of simultaneity and the non-commutativity of NRQM. In light of this result, it should be clear that there is no metaphysical tension between STR and NRQM. This formal result gives us motivation for believing that NRQM is intimately connected to the geometry of (a suitable) spacetime[14].

3.3 Philosophical significance. One important point should be brought out, which reveals how we understand the relationship between spacetime structure (given by relativity) and the theory of quantum mechanics (in a non-Minkowskian, but non-Galilean, spacetime regime, i.e., K4). Most natural philosophers agree that STR just constrains the set of possible dynamical theories to those which satisfy the light and relativity postulates. It is often worried, as we have pointed out, that somehow quantum theory *violates* those constraints. The view we adopt here is importantly different, in that we distinguish between:

(a) the question of how to relate the *structures of* quantum theory and relativity
(b) the question of the compatibility of constructive interpretations of quantum theory and whether they violate relativistic constraints.

Using a collection of formal results, we show that the spacetime structure for which one can obtain the Heisenberg commutation relations is one where the relativity of simultaneity is *upheld* – a fact often not appreciated in most interpretations of quantum theory. Furthermore, with an ontology of spacetime relations, we show how to construct a quantum density operator from the spacetime symmetry group of any quantum experimental configuration, and how one can use this to deduce and then explain the phenomenon of quantum interference – all by appealing to nothing more than a spacetime structure for which one can obtain the Heisenberg commutator while obeying the relativity of simultaneity.

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[12] *See* Eq. 6 in Brown and Holland (1999). A derivation of Eq. 3.1, assuming the acceptability of a phase difference such as that in Eq. 3.4, is in Ballentine (1990), p. 49 – 58.

[13] This phrase was used by Harvey Brown in a conversation with the authors while describing his work with Peter Holland (Brown and Holland, 1999).

[14] The Bohr *et al.* result of section 5 below shows how to relate this spacetime geometry to non-relativistic quantum mechanics by showing how a quantum density operator can be constructed from the spacetime symmetry group of the quantum mechanical experiment.

We take the deepest significance of the Kaiser *et al.* results to be that, given the asymptotic relationship between the spacetime structure of special relativity and the "weakly relativistic" spacetime structure of quantum theory, non-relativistic quantum mechanics is something like a relativity theory in an "embryonic" stage. It is "embryonic" in that it is yet without the Lorentz-contraction factor γ that appears in the familiar Lorentz transformation equations of special relativity[15].

Having identified the appropriate spacetime structure for the Heisenberg commutation relations, and having discovered that this structure upholds the relativity of simultaneity, we have provided a geometric explanation for the quantum. A natural question now arises: what would the appropriate description of NRQM and quantum mechanical phenomena such as interference be like in light of the asymptotic relationship between relativity and quantum theory? Our "geometric" interpretation of NRQM elaborated below is one answer to this question, an answer grounded in our fundamental ontology of spacetime relations.

4. Density Matrix Obtained via Symmetry Group

Having found which *spacetime* structure is appropriate for the Heisenberg commutation relations (whose empirical manifestation is quantum interference), we now seek to address the question of how to model – *in spacetime and not in Hilbert space* – any quantum system which manifests quantum interference. That is, we are asking:

> *how can we describe a quantum system with nothing more than the geometry of spacetime, where the relativity of simultaneity and the non-commutivity of position and momentum obtain?*

The following formal results provide us with an answer to this question.

4.1 Formalism. We present a pedagogical version of the appendix to Bohr, Mottelson and Ulfbeck (2004a) wherein they show the density matrix can be derived using only the irreducible representations of the symmetry group elements, $g \in G$. We begin with two theorems from Georgi

> The matrix elements of the unitary, irreducible representations of G are a complete orthonormal set for the vector space of the regular representation, or alternatively, for functions of $g \in G$ (1999, 14)

which gives

> If a hermitian operator, H, commutes with all the elements, D(g), of a representation of the group G, then you can choose the eigenstates of H to transform according to irreducible representations of G. If an irreducible representation appears only once in the Hilbert space, every state in the irreducible representation is an eigenstate of H with the same eigenvalue (*ibid.*, p. 25).

[15] And given that it is the contraction/dilation phenomena, characteristic of relativity, that motivates the introduction of the "field" as a unifying structural device, non-relativistic quantum mechanics in light of this new spacetime structure is simply relativity minus the "field."

What we mean by "the symmetry group" is precisely that group G with which some observable H commutes (although, these elements may be identified without actually constructing H). Thus, the mean value of our hermitian operator H can be calculated using the density matrix obtained wholly by D(g) and <D(g)> for all $g \in G$. Observables such as H are simply 'along for the ride' so to speak.

To show how, in general, one may obtain the density matrix using only the irreducible representations[16] D(g) and their averages <D(g)>, we start with eqn. 1.68 of Georgi (*ibid.,*18)

$$\sum_g \frac{n_a}{N}\left[D_a(g^{-1})\right]_{kj}\left[D_b(g)\right]_{lm} = \delta_{ab}\delta_{jl}\delta_{km}$$

where n_a is the dimensionality of the irrep, D_a, and N is the group order. If we consider but one particular irrep, D, this reduces to the orthogonality relation (eqn. 1) of Bohr *et al.*

$$\sum_g \frac{n}{N}\left[D(g^{-1})\right]_{kj}\left[D(g)\right]_{lm} = \delta_{jl}\delta_{km} \tag{4.1}$$

where n is the dimension of the irrep. Now multiply by $[D(g')]_{jk}$ and sum over k and j to obtain

$$\sum_j \sum_k \sum_g \frac{n}{N}\left[D(g^{-1})\right]_{kj}\left[D(g)\right]_{lm}\left[D(g')\right]_{jk} = \sum_j \sum_k \delta_{jl}\delta_{km}\left[D(g')\right]_{jk} = \left[D(g')\right]_{lm}$$

The first sum on the LHS gives:

$$\sum_j \left[D(g^{-1})\right]_{kj}\left[D(g')\right]_{jk} = [D(g^{-1})D(g')]_{kk}$$

The sum over k then gives the trace of $D(g^{-1})D(g')$, so we have:

$$\frac{n}{N}\sum_g \left[D(g)\right]_{lm} Tr\left\{D(g^{-1})D(g')\right\} = [D(g')]_{lm}$$

Dropping the subscripts we have eqn. 2 of Bohr *et al*:

$$\frac{n}{N}\sum_g D(g)\ \ Tr\left\{D(g^{-1})D(g')\right\} = D(g')\,. \tag{4.2}$$

If, in a particular experiment, we measure directly the click distributions associated with the various eigenvalues of a symmetry D(g), we obtain its average outcome, <D(g)>, i.e., eqn. 3 of Bohr *et al*:

$$\left\langle D(g)\right\rangle = \sum_i \lambda_i\, p(\lambda_i) \tag{4.3}$$

where λ_i are the eigenvalues of D(g) and $p(\lambda_i)$ are the distribution frequencies for the observations of the various eigenvalues/outcomes.

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[16] Hereafter, "irreps."

In terms of averages, Bohr *et al.* eqn. 2 becomes:

$$\frac{n}{N}\sum_{g}\langle D(g)\rangle Tr\{D(g^{-1})D(g')\} = \langle D(g')\rangle \tag{4.4}$$

which they number eqn. 4. Since we want the density matrix to satisfy the standard relation (Bohr *et al.* eqn. 5):

$$Tr\{\rho D(g')\} = \langle D(g')\rangle \tag{4.5}$$

it must be the case that (Bohr *et al.* eqn 6):

$$\rho \equiv \frac{n}{N}\sum_{g} D(g^{-1})\langle D(g)\rangle \tag{4.6}$$

That this density operator is hermitian follows from the fact that the symmetry operators are unitary. That is, $D(g^{-1}) = D^{\dagger}(g)$ implies $\langle D(g^{-1})\rangle = \langle D(g)\rangle^*$, thus:

$$\rho^{+} = \frac{n}{N}\sum_{g} D^{+}(g^{-1})\langle D(g)\rangle^{*} = \frac{n}{N}\sum_{g} D(g)\langle D(g^{-1})\rangle = \frac{n}{N}\sum_{g} D(g^{-1})\langle D(g)\rangle = \rho\ .$$

[The second-to-last equality holds because we are summing over all g and for each g there exists g^{-1}.] So, the density operator of eqn. 4.6 will be hermitian and, therefore, its eigenvalues (probabilities) are guaranteed to be real. This is not necessarily the case for D(g), since we know only that they are unitary. However, *we need only associate detector clicks with the eigenvalues of D(g) and in this perspective one does not attribute an eigenvalue of D(g) to a property of some 'click-causing particle'*. Therefore, whether or not the eigenvalues of any particular D(g) are real or imaginary is of no ontological or empirical concern.

4.2 Philosophical significance. With the above formal result in hand, we can now provide a clear answer to the question posed at the beginning of this section:

> *the spacetime symmetry group of the quantum mechanical experiment will yield the quantum mechanical density matrix.*

The methodological significance of the Bohr *et al.* formal result is that any NRQM system may be described with the appropriate *spacetime* symmetry group. But the philosophical significance of this proof is more interesting, and one rooted in our ontological spacetime relationalism.

Our view is a form of ontological structural realism which holds that the features of our world picked out by STR and NRQM are structures; moreover, we think that the structures picked out by our most successful theories to date – spacetime theories – are geometrical structures. And those structures, if taken seriously, are, we posit, structures of spacetime *relations*. Furthermore, we see the quantum theory as providing a *further structural constraint* on the distribution of spacetime events. Isolated to an idealized model of "sources," "detectors," "mirrors," etc. (see figure 5 for an idealized interferometer), our ontology is that each and every "click" or "measurement event" observed in the detector region *is itself evidence of a spacetime relation between the source and detector.* So, while the "click" itself maybe regarded as a transtemporal or classical object, it is not "caused by" a structural entity such as a particle that is independent from the physical spacetime geometry of this entire measurement process and experimental set-up, rather, the *click itself* is a manifestation of spatiotemporal relations between elements of the experimental set-up. It is in this way, via our radical ontology of spacetime relations[17], that the essential features of quantum systems with interference can be described with features of the spacetime geometry *without appealing to features of the usual Hilbert space of quantum mechanical states* [18].

Secondly, as will be demonstrated below, the Bohr *et al.* proof will allow us to show that the posit of a blockworld – the reality of all spacetime events, and hence in our ontology, of all spacetime relations constituting those events – does real explanatory work. While one can imagine quite trivial explanations of EPR-Bell correlations invoking the blockworld[19], the Bohr *et al.* result will allow us to provide a non-trivial, geometric explanation for such quantum correlations.

Thirdly, as demonstrated below, the Bohr *et al.* result provides the foundation for our distinctly *geometrical* ontological structuralist[20] interpretation of NRQM. This ontology is an ontology of spatiotemporal relations which are the means by which all physical phenomena (including both quantum and classical "entities") are modeled. Our relationalism allows for a natural transition from quantum to classical mechanics (including the transition from quantum to classical probabilities) as simply the transition from rarefied to dense collections of spacetime relations[21].

5. The Geometric Interpretation of NRQM

In order to motivate our relational approach to physical reality, consider first a rival interpretation of NRQM which is antithetical to the view we are developing here, Bohmian mechanics. Bohmian mechanics provides us with a classical-like picture of reality[22]. It begins by modeling the behavior of a classical-like particle whose velocity is determined, via "Bohm's equation" (i.e., the "guiding field"), by a wave-function; the wave-function evolves according to Schrödinger's equation (Maudlin 1994, 118). Such particles always have well-defined locations in spacetime, and their total Hamiltonian is constructed from both a non-classical quantum potential and classical potential fields. In a basic twin-slit experiment, a simple picture of the mechanism behind the interference pattern is provided: a particle is directed deterministically by the guiding field to a particular location and registered as a "click" in a detector. Measurement on Bohm's theory is just like any other physical interaction. A constructive account of measurement,

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[17] Which, if you want to speak constructively, "constitute" the spacetime geometry.
[18] A Hilbert space is not analogous to spacetime geometry, but rather to phase-space geometry. Anandan (1991) for example adopts the view that the geometry of Hilbert space is appropriate for a geometric interpretation of quantum theory.
[19] *E.g.* Barrett (2004) critiques one such trivial explanatory model, which he calls a "teleological spacetime map."
[20] *See* French & Ladyman 2003*a* for an account of ontological strucuturalism in the context of quantum theory.
[21] Though a full explication and defense of this view is unfortunately beyond the scope of this paper.
[22] *See* Holland (1993) p. 26 and 81ff.; Barrett (1999) sections 5.2 – 5.6; and Maudlin (1994) p. 116ff. for the sense in which Bohm is classical-like.

from particle to "click" registration, is provided by breaking down the whole process into particles and wave-functions. A "click" is clearly the result of a causal process (however non-classical/non-local that process might be), and evidences a particle trajectory in spacetime.

Given our geometrical interpretation of NRQM, it should be clear that we do not take detector events to be indicators of the trajectories of classical-like particles and wave-functions, propagating from the source to the detector as in Bohm's mechanics or even, as it turns out, like disturbances in a field per RQFT. In RQFT for a scalar field without scattering or sources we have for the transition amplitude (Zee 2003, 18)

$$Z = \int D\varphi e^{i\int d^4x\left[\frac{1}{2}(d\varphi)^2 - V(\varphi)\right]} \quad 5.1$$

According to Zee, NRQM then obtains in (0+1) dimensions. In Zee's derivation of eqn. 5.1 from NRQM, the field φ is obtained in the continuum limit of a discrete set of oscillators q_a distributed in a spatial lattice. Any *one* of these q_a is supposed to replace φ in eqn. 5.1 to reduce to NRQM. However, each q_a is fixed in space so the notion that we're integrating over all possible paths *in space* (standard treatment) from a source to a detector when we compute Z is not ontologically consistent with the fact that we integrate over all values of q but *not* over all values of the index 'a' in q_a. We rather suggest that the method for reducing RQFT to NRQM is to associate sources J(x) with elements in the experimental set up while assuming the q's are distributed discretely therein. Thus, we want to obtain NRQM from

$$Z = \int D\varphi e^{i\int d^4x\left[\frac{1}{2}(d\varphi)^2 - V(\varphi) + J(x)\varphi(x)\right]} \quad 5.2$$

rather than eqn. 5.1. This leaves us to compute

Consider for example the twin-slit experiment, which "has in it the heart of quantum mechanics. In reality, it contains the *only* mystery" (Feynman *et al*, 1965, italics theirs).

5.1 Interpretive consequences of our geometrical NRQM.

The Measurement Problem. According to the account developed here, we offer a deflation of the measurement problem with a novel form of a hidden-variables "statistical interpretation." The fundamental difference between our version of this view and the usual understanding of it is the following: whereas on the usual view the state description refers to an "ensemble" which is an ideal collection of similarly prepared quantum particles, "ensemble" according to our view is just an ideal collection of spacetime regions D_i "prepared" with the same spatiotemporal boundary conditions per the experimental configuration itself. The union of the click events in each D_i, as $i \rightarrow \infty$, produces the characteristic Born distribution[23]. Accordingly, probability on our geometrical NRQM is interpreted per relative frequencies. It should be clear, also, that probabilities are understood as the likelihood that a particular relation between source-detector in spacetime is realized, from among a set of all equally likely relations between source-detector.

On our view, the wave-function description of a quantum system can be interpreted statistically because we now understand that, as far as measurement outcomes are concerned, the Born distribution has a basis in the spacetime symmetries of experimental configurations. Each "click," which some would say corresponds to the impingement of a particle onto a measurement device and whose probability is computed from the wave-function, corresponds to a spacetime relation in the context of the experimental configuration. The measurement problem *exploits* the possibility of extending the wave-function description from the quantum system to the whole measurement apparatus, whereas the spacetime description according to our geometrical quantum mechanics *already includes* the apparatus via the spacetime symmetries instantiated by the *entire* experimental configuration. The measurement problem is therefore a non-starter on our view.

Entanglement & Non-locality. On our geometric view of NRQM we explain entanglement as a feature of the spacetime geometry[24] as follows. Each detection event, which evidences a spacetime relation, selects a trajectory from a family of possible trajectories (one family per entangled 'particle'). In the language of detection events *qua* relations, it follows that correlations are correlations between the members of the *families* of trajectories and these correlations are the result of the relevant spacetime symmetries for the experimental configuration. And, since an experiment's spacetime symmetries are manifested in the Hamilton-Jacobi families of trajectories throughout the relevant spacetime region D, there is no reason to expect entanglement to diminish with distance from the source. Thus, the entanglement of families of trajectories is spatiotemporally global, i.e., non-local. That is, there is no reason to expect entanglement geometrically construed to respect any kind of common cause principle. Obviously, on our geometric interpretation there is no non-locality in the odious sense we find in Bohm for example, that is, there are no instantaneous causal connections (construed dynamically or in terms

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[23] There would be N first events in trials with N entangled particles, since each "particle" would correspond to a family of possible trajectories.

[24] Established in section 2 as one which is "weakly" relativistic in that it lacks the Lorentz contraction factor.

of production—bringing new states of affairs into being) between space-like separated events—no action at a distance. However our view is non-local in the sense that it violates the locality principle. The locality principle states: the result of a measurement is probabilistically independent of actions performed at space-like separation from the measurement. Keep in mind that in our BW setting, talk of "actions performed" gets only a purely logical-counterfactual meaning—the entire experimental EPR set-up, its past, present and future if you will and the spacetime symmetries of that set-up are all just there—no one could really perform some alternative measurement on the other wing of the experiment.

We understand quantum facts to be facts about the spatiotemporal relations of a given physical system, not facts about the behavior of particles, or the interactions of measurement devices with wave-functions, or the like. Entanglement and non-locality are built into the structure of spacetime itself via relations. Correlations between space-like separated events that violate Bell's inequalities are of no concern as long as spacetime symmetries instantiated by the experimental apparatus warrant the correlated spacetime relations. Since the non-local correlations derive from the spatiotemporal relations per the spacetime symmetries of the experiment, satisfaction of any common-cause principle is superfluous. To sloganize: ours is a *purely* geometric/spacetime interpretation of non-relativistic quantum mechanics.

That the density matrix may be obtained from the spacetime symmetries of the Hamiltonian is consistent with the notion that $\psi^*\psi$ provides the distribution for detector events in single-event trials for each family of trajectories obtained via the Hamilton-Jacobi formalism. Our view exploits this correspondence to infer the existence of a spacetime relation between source and detector for each detector event.

Subsequent detector events in close spatiotemporal proximity to the first tend to fall along a trajectory of the family consistent with the first event thereby allowing for the inference of a "particle." In this sense, what constitutes a "rarefied" distribution of spacetime relations is but one relation per "particle," i.e., family of trajectories, since subsequent events tend to trace out classical trajectories (scattering and particle decay events aside). It is a collection of these single-event trials that will evidence quantum interference in, for example, the twin-slit experiment.

Our account provides a clear description, in terms of fundamental spacetime relations, of quantum phenomena *that does not suggest the need for a "deeper"* causal or dynamical *explanation*. If explanation is simply determination, then our view explains the structure of quantum correlations by invoking what can be called *acausal global determination relations*. These global determination relations are given by the spacetime symmetries which underlie a particular experimental set-up. Not objects and dynamical laws, but rather acausal spacetime relations per the relevant spacetime symmetries do the fundamental explanatory work according to our version of geometrical quantum mechanics. We can invoke the *entire* spacetime configuration of the experiment so as to predict, and explain, the EPR-Bell correlations. Indeed, it has been the purport of this paper that the spacetime symmetries of the quantum experiment can be used to construct its quantum density operator, that such a spacetime is one for which simultaneity is relative, and that events in the detector regions evidence spatiotemporal relations.

This constitutes an acausal and *non-dynamical* characterization and explanation of entanglement. According to our view, the *structure of EPR correlations* are *determined* *by* the spacetime relations instantiated by the experiment, understood as a spatiotemporal whole. This determination is obtained by systematically *describing* the spatiotemporal symmetry structure of the Hamiltonian for the experimental arrangement[25]. Since

(i) the explanation lies in the spacetime symmetries as evidenced, for example, in the family of trajectories per the Hamilton-Jacobi formalism,
(ii) each family of trajectories characterizes the distribution of spacetime *relations,*
(iii) we take those relations to be a timeless "block,"
(iv) these relations collapse the matter-geometry dualism,
therefore,
(v) our geometrical quantum mechanics provides for an *acausal*, *global* and *non-dynamical* understanding of quantum phenomena.

6. Conclusion.

Can one do justice to the non-commutative structure of NRQM without being a realist about Hilbert space? Our geometric interpretation constitutes an affirmative answer to this question. The trick is to appreciate that while everything "transpires" or rather resides in a 4D spacetime and nowhere else, nonetheless, some phenomena, namely quantum phenomena, cannot be modeled with worldlines if one is to do justice to its non-commutative structure. Thus while clicks in detectors are perfectly classical events, the clicks are not evidence of constructive quantum entities such as particles with worldlines, rather, the clicks are manifestations of spacetime relations between elements of the experimental configuration—distributions per the spacetime symmetries. Thus on our view there is no "Dedukind cut" between the quantum and the classical as some versions of the Copenhagen interpretation would have it. After all, we can explain asymptotically the transition from the quantum to the classical in terms of density of "events." And there is also no "Einstein separability" between the system being measured and the system doing the measuring on our interpretation. Our view respects the causal structure of Minkowski spacetime in the sense that there are no faster than light "influences" or "productive" causes between space-like separated events as there are in Bohm for example. So our view is not non-local in any robustly dynamical sense. However our view does violate Einstein separability and it does have static "correlations" outside the lightcone as determined acausally and globally by the spacetime symmetries.

Such acausal global determination relations do not respect any common cause principle. This fact should not bother anyone who has truly transcended the idea that the dynamical or causal perspective is the most fundamental one. We are providing a model of an irreducibly relational blockworld, which is what realism about the quantum structure and the 4D spacetime structure yields once one accepts the implication therein of Hilbert space anti-realism.

[25] The experimental apparatus itself providing the particular initial and final "boundary conditions" needed for a prediction unique to the apparatus.

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