FORTHCOMING IN BJPS

Dust, Time, and Symmetry

Gordon Belot

ABSTRACT

Two symmetry arguments are discussed, each purporting to show that there is no more room in general relativistic cosmology than in Minkowski spacetime for a preferred division of spacetime into instants of time. The first argument is due to Gödel, and concerns the symmetries of his famous rotating cosmologies. The second turns upon the symmetries of a certain space of relativistic possibilities. Both arguments are found wanting.

1 Introduction

In Newtonian physics there is a preferred notion of time: spacetime falls in a natural way into a one dimensional family of three dimensional subsets that we identify as the instants of time. There is, of course, no such preferred division of spacetime into instants of time in the Minkowski spacetime of special relativity. But if we introduce into Minkowski spacetime a preferred inertial frame—a family of freely falling observers, exactly one passing through each point of spacetime—then we *do* get a preferred division of Minkowski spacetime into instants of time—the one dimensional family of three dimensional subsets orthogonal to our family of observers.

What happens in general relativity? On the one hand, general relativity is a generalization of special relativity—and intuitively, it ought not to re-establish an absolute division of spacetime into instants of time. On the other hand, in central cosmological applications of the theory, one often models the material contents of the universe as *dust*—a family of freely falling objects, exactly one passing through each point of spacetime. And it is natural to wonder whether, as in Minkowski spacetime, such a family suffices to determine a preferred notion of time.

In this paper I discuss two arguments, each aiming to show that there is no more room for a preferred notion of time in general relativistic cosmology than in the Minkowski spacetime of special relativity.

The first argument is due to Gödel, and can be found in his brief and enigmatic 'A Remark about the Relationship between Relativity Theory and Idealistic Philosophy' ([1949*a*]). Section 3 of the present paper offers a commentary on Gödel's argument.

Although the focus is somewhat different from that of most philosophical discussions of Gödel's writings on time—I do not discuss time travel—my conclusion is the usual one: that Gödel's argument teaches us less than it was supposed to about the nature of time in general relativity. In brief: Gödel shows that there exist cosmological spacetimes whose symmetry properties are incompatible with a preferred slicing into instants of time; and he offers some reason to think that this establishes a sense in which the laws of general relativistic cosmology are incompatible with a preferred notion of time; but a comparison of this case with other cases in which we take symmetry arguments to establish structural features of laws shows that Gödel's conclusion is disappointingly weak.

A discussion of the force and limitations of Gödel's argument naturally suggests a second sort of argument, which is the topic of §4 of this paper. Roughly speaking, one can show that just as in ordinary classical mechanics so in dust cosmology, the theory is indifferent to the identity of instants of time—in the sense that each theory is invariant under changes in the instant labeled as the origin. In the general relativistic context, this means that the theory is invariant under changes of slicing, as well as under changes in parameterization of a given slicing. And this seems to establish that the laws of dust cosmology are no more compatible with the choice of a preferred slicing of solutions into instants than the familiar laws of classical mechanics are compatible with a preferred choice of origin for the time coordinate. But we will see that this argument, although in some respects more promising than the first, also has a major shortcoming—it presupposes a formulation of general relativity which is unlikely to be acceptable to anyone for whom the status of time in the theory is a genuinely open question.

The two arguments to be considered turn upon structurally similar symmetry considerations—although in one case the symmetries in question are the symmetries of certain relativistic spacetimes, while in the other they are the symmetries of a certain space of relativistic possibilities. Before turning to these arguments, I begin in the next section with a discussion of the form such symmetry considerations, which I hope will prove helpful by setting up later discussion.

2 Symmetry Arguments

Here is an example from the older cosmological literature:

> It is entirely wrong to suppose that there are by nature two opposite regions dividing the universe between them, one 'below,' towards which all things sink that have bodily bulk, the other 'above,' towards which everything is reluctant to rise. For since the whole heaven is spherical in shape, all the points which are extreme in virtue of being equally distant from the centre, must be extremities in just the same manner; while the centre, being distant by the same measure from all the extremes, must be regarded as at the point 'opposite' to them all. Such being the nature of the ordered world, which of the points mentioned could one call either 'above' or 'below' without being justly censured for using a quite unsuitable term? … When a thing is uniform in every direction, what pair of contrary terms can be applied to it and in what sense could they be properly used? If we further suppose that there is a solid body poised at the centre of it all, this body will not move towards any of the points on the extremity, because in every direction they are all alike…. (*Timaeus* 62*c*–63*a*. Translation of Cornford [1997], pp. 262 f.)

This is an elegant instance of a type of symmetry argument that has played a prominent role in natural philosophical investigations of space, time, and motion. The argument form can be characterized as follows.

- In the background is the supposition that we are interested in a problem set in the context of a particular subject matter. In our example, Plato is interested in the definition of *up* and *down* in the cosmological context.
- The point of departure of the argument is the presentation of a structure which is claimed to provide a perspicuous representation of the subject matter at hand. Here a *structure* is a set of individuals instantiating certain properties and standing in certain

relations to one another. We will suppose that the properties and relations are qualitative.[1] In our example, Plato asks us to model the cosmos as a solid sphere, with the distinguished central point representing the Earth—we can take our structure to be a subset of the points of Euclidean space, subject to the usual relations of betweenness and congruence.

- Our analysis turns upon the symmetries of the given structure. Here a *symmetry* is an automorphism of the structure—a permutation of the individuals that fixes each of the given properties and relations.[2] The symmetries of a structure form an algebraically well-behaved set—a group.[3] If two individuals are related by a symmetry, then these individuals play identical roles in the system of properties and relations encoded in the structure.[4] The structure in our example is invariant under reflections in planes through the center—and hence under the result of successive reflections, including rotations of the sphere about its center. It follows that each point on the surface of the cosmic sphere is qualitatively identical to each other, since for any pair of such points we can find a rotation that maps the first to the second.
- An approach to solving the given problem is on the table. We consider an instance of this approach, assumed to take the form of a proposed enrichment of the given structure by new properties and relations, which is to lead directly to a solution of the problem at hand. Thus, Plato supposes that any definition of *down* would involve the choice of a distinguished normal to the surface of the cosmic sphere—directed from a distinguished point on the sphere towards the center.
- We now ask whether every symmetry of the original structure is a symmetry of the extended structure. Suppose that the answer is 'No.'[5] Then the new properties and

relations fail to respect the symmetries of the original structure—they draw invidious distinctions between (sets of) individuals which play equivalent roles in the original structure. To the extent that we are confident that our original structure does indeed present a perspicuous view of our subject matter, we have reason to reject the proposed problem solution.[6] In our example, every extension of the sort Plato considers is inadmissible, as the symmetries of the cosmos require us to treat each point on the surface of the cosmic sphere equivalently.

The point of vulnerability of such an arguments is its point of departure—Does the proffered structure really represent a perspicuous representation of the subject matter, whose symmetries any problem solution must respect? It might be objected to Plato that the heavens are *not* uniform—the distribution of the stars is utterly asymmetric. So why can't we define *down* by privileging, say, Polaris? Similarly: Why shouldn't we take advantage of any asymmetries in the shape of our central body in determining the direction of its motion? Plato can only respond that such suggestions miss the point of the sort of cosmological investigation he is engaged in—for his purposes, the description of the heavens as an undifferentiated sphere is the appropriate one, and any further structure defined at this level must respect the symmetries of this description.

Note that, thus far, arguments of this form give us reason to reject individual proposed solutions. But if we are very confident that the approach considered is the only sensible approach to the problem, and if all instances of this approach can be shown to run afoul of our symmetry requirement, then we have reason to conclude that the problem is insoluble. Thus, Plato concludes that if the world is spherically symmetric, there can be no such definition of *up* or *down*.[7] Aristotle rejects a presupposition of the approach Plato considers, and concludes that *down* points from *each* point of the cosmic sphere towards the center (*De Caelo,* 308*a*).

3 Gödel's Argument

Let us now turn to the argument of Gödel's 'A Remark about the Relationship between Relativity Theory and Idealistic Philosophy.'[8] For our purposes, it is helpful to note that this paper can be broken into the following divisions:

(i) The first two paragraphs note that the structure of time according to the special theory of relativity is very different from what is suggested by commonsense and experience.

(ii) The third and fourth paragraphs discuss a difficulty that stands in the way of establishing a corresponding thesis about the nature of time in general relativity, deriving from a certain feature present in many relativistic cosmologies.

(iii) In the fifth paragraph, it is shown that this difficulty is not insuperable, for there exist solutions of the equations of relativistic cosmology that lack this feature. A symmetry argument shows that time has the desired structure in those solutions.

(iii*) In the next three paragraphs it is noted that those same solutions possess a strange causal structure which appears to render time travel possible. It follows that in such solutions: the problem discussed under (ii) does not arise; and time has the desired structure.

(iv) In the final paragraph Gödel argues that the structure of time in the cosmological models discussed under (iii) and (iii*) has consequences for the nature of time in *all* general relativistic cosmologies—notwithstanding the obstacle noted in (ii).

Gödel's famous paper has, of course, been widely discussed. (iii*) has received the lion's share of attention, with (iv) attracting somewhat less.[9] I mean to bypass (iii*) entirely—and with it all of the distractions surrounding time travel—and focus on the line of argument that leads from (i) and (ii) through (iii) to (iv) (henceforth '(i)–(iv)'). This is possible because (iii) and (iii*) are redundant with respect to one another in the structure of Gödel's argument. Indeed, (i)–(iv) is a freestanding argument, which appears to have been worked out by Gödel prior to his discovery of the causal pathologies inherent in his solutions (see Stein [1995], §§2 and 3).

I discuss (i)–(iv) sequentially in the next four subsections, before turning to the task of evaluating the success of Gödel's argument. I aim to provide a rational reconstruction of Gödel's reasoning, making explicit some of the background that he presupposes. Textual and historical remarks will, for the most part, be relegated to the notes.

3.1 Time in Special Relativity

Gödel opens his paper by remarking that 'One of the most interesting aspects of relativity theory for the philosophical-minded consists in the fact that it gave new and surprising insights into the nature of time, of that mysterious and seemingly self-contradictory being which, on the other hand, seems to form the basis of the world's and our own existence' ([1949*a*], p. 557). Indeed, consideration of the structure of Minkowski spacetime leads Gödel to conclude that 'one obtains an unequivocal proof for the view of those philosophers who, like Parmenides, Kant, and the modern idealists, deny the objectivity of change and consider change as an illusion or an appearance due to our special mode of perception' ([1949*a*], p. 557).

The argument works as follows. According to Gödel, the existence of an objective lapse of time is a necessary condition for the existence of genuine change; and the division of reality into a one dimensional family of three dimensional instants *prima facie* capable of being successively realized is in turn a necessary condition for an objective lapse of time.[10] But the familiar relativity of simultaneity in Minkowski spacetime shows that such a decomposition into instants is not possible absolutely, but only relative to a choice of inertial observer. Gödel insists that a merely relative decomposition cannot found an objective lapse of time: 'Each observer has his own set of "nows", and none of these various systems of layers can claim the prerogative of representing the objective lapse of time.'[11]

The crucial point is that there is no invariant means of decomposing Minkowski spacetime into subsets worth calling instants—this is part of what Minkowski had in mind in announcing that 'Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality' ([1952], 75). For our purposes, it is helpful to stipulate that an *instant* in Minkowski spacetime is a connected, three dimensional, spacelike submanifold. We will call a partition of a manifold by (embedded) submanifolds of equal dimension a *foliation*.[12] Then the present point can be reformulated as: There is no extension of the structure of Minkowski spacetime by an invariant equivalence relation which foliates Minkowski spacetime by instants.[13] (Here is a sketch of a proof; it will come in handy later, but can be omitted without loss of continuity. Suppose that we have an invariant equivalence relation of the desired sort. Choose an instant, T, from among its equivalence classes, and choose a point, x, lying on T. There is a unique inertial observer passing through x whose worldline is orthogonal to T at x. Choose a boost relative to this observer that fixes x. This boost does not fix T as a set. So the image of T under this Lorentz symmetry is distinct from T but has non-empty intersection with it—hence it cannot be an equivalence class of our equivalence relation. So the relation is not invariant under Lorentz symmetries after all.)

Before moving on to consider the next phase of Gödel's argument, it is helpful to recall some facts about the definability of simultaneity relations in structures related to Minkowski spacetime.

We can employ Einstein's construction to associate to a privileged inertial observer a simultaneity relation whose equivalence classes foliate Minkowski spacetime by instants. The observer carries a clock, and employs the following rule: if a light signal sent by the observer at time t and reflected by the event in question would return to the observer at time $t+2\varepsilon$, then the event is assigned time coordinate $t+\varepsilon$; events assigned the same time coordinate are counted as simultaneous by the observer. The corresponding equivalence relation is not, of course, invariant under the full set of symmetries of Minkowski spacetime. But it *is* invariant under those spacetime symmetries that leave invariant the worldline of the privileged observer—i.e., under the symmetries of the structure that results when Minkowski spacetime is supplemented by the privileged worldline.[14]

A similar picture emerges if we proceed instead by supplementing the structure of Minkowski spacetime by the choice of a field of inertial observers at relative rest (this gives us a *congruence* of inertial worldlines—exactly one privileged inertial worldline through each point of spacetime). Let us call the resulting structure *augmented Minkowski* *spacetime*. We can proceed geometrically: the family of hypersurfaces everywhere orthogonal to our set of worldlines forms a family of spacelike instants, which is *transverse* to the privileged worldlines (i.e., each worldline intersects each member of the family exactly once). Note: (i) that the same family of instants results when any of the comoving inertial observers applies the Einstein simultaneity construction; and (ii) that the resulting family of orthogonal hypersurfaces is the unique transverse foliation invariant under the symmetries of augmented Minkowski spacetime (Giulini [2001], Theorem 5).

The symmetries of augmented Minkowski spacetime are just the symmetries of Newtonian spacetime, absolute space and all—every symmetry is a product of a temporal symmetry (which shifts or reflects events along each privileged worldline) and a spatial symmetry (which acts on each instant via the same Euclidean isometry; the temporal translations determining the requisite notion of sameness). This suggests the following way of describing our situation. Minkowski spacetime itself does not decompose in a natural way into space and time—it does not support invariant relations that deserve to be called 'at the same place as' or 'at the same time as.' But if we supplement its structure with a privileged congruence of inertial observers—which can be viewed as introducing a notion of 'at the same place as'—, this brings along with it a notion of 'at the same time as'—namely, the equivalence relation whose equivalence classes are the instants orthogonal to the privileged observers. Thus, adding a certain sort of notion of space to Minkowski spacetime automatically yields a sort of notion of time.

3.2 Time in the Standard Cosmological Models

The observation that the structure of Minkowski spacetime is incompatible with the lapse of time and the existence of genuine change would be of limited interest if similar conclusions did not follow in more fundamental contexts. It is the principal aim of Gödel's paper to show that such conclusions can indeed be established in general relativistic cosmology.

In relativistic cosmology, one is concerned with solutions to the field equations of general relativity that model a universe filled with some sort of (more or less realistic) matter. Gödel focuses on dust cosmology, in which spacetime is everywhere filled with freely-falling dust motes (representing galaxies), whose interaction with one another is mediated via their joint role in determining the curvature of spacetime. This is thought to provide a reasonable idealization of the dynamics of the large-scale structure of the universe, except at early (and, possibly, late) times at which non-gravitational interactions cannot be neglected. A solution to the equations of dust cosmology consists of: a spacetime geometry; a congruence of timelike geodesics (i.e., a family of such curves, exactly one passing through each point of spacetime), representing the worldlines of the dust motes; and a positive real-valued function on spacetime, describing the matter density at each point (this varies as the dust worldlines converge and diverge).[15] In fact: the dust world-lines and matter density can be reconstructed from knowledge of the spacetime metric alone (see, e.g., Sachs and Wu [1977], §3.14). So, in effect, the geometry of such a world itself determines a distinguished set of co-moving observers.

Within this framework, we can introduce some helpful terminological stipulations (these will be in effect for the remainder of this paper). In a dust cosmological spacetime, an *instant* is a three dimensional (embedded) submanifold which intersects each dust worldline exactly once.[16] A choice of *time* is foliation of spacetime by a (necessarily one dimensional) family of instants (equivalently—the choice of an equivalence relation on spacetime with instants as equivalence classes). Finally, a time is called *absolute* if it is invariant under the symmetry group of the cosmology under consideration.[17] This last stipulation accords well with Gödel's usage.[18]

Now, a typical general relativistic spacetime admits no symmetry.[19] So in such a solution, *any* time is an absolute time—and any decent solution of the equations of dust cosmology admits infinitely many times.[20] So there is not, in the generic case, any difficulty in constructing an absolute time—the challenge is rather to provide a generally applicable technique for singling out physically interesting times.[21]

Gödel pursues a different question: Does every dust solution admit an absolute time—or do there exist symmetric solutions in which it is possible to run an analog of the argument that shows that there is no invariant foliation of Minkowski spacetime by instants?

Einstein's static universe is the most highly symmetric dust solution, and was the first to be discovered.[22] As in the Minkowski case, the group of spacetime symmetries is *transitive*—any pair of points is related by some isometry (we also say that a spacetime with such symmetry group is *spacetime homogeneous*). But unlike in the Minkowski case, there *is* a family of three dimensional spatial instants which is invariant under the symmetries of the spacetime—thus Gödel's argument for the nonexistence of change and a lapsing time will not go through in Einstein's static universe.

This should not be entirely surprising. The structure we are investigating includes a privileged congruence of freely falling worldlines. Such a congruence is the natural general relativistic analog of the congruence of inertial observers that underwrites the construction of the absolute time in augmented Minkowski spacetime. And, in fact, we find that, as in augmented Minkowski space time, so in the Einstein static universe: (i) there is a family of instants orthogonal to the privileged congruence of worldlines; (ii) this family is the unique family of instants invariant under the spacetime symmetries; and (iii) the symmetries can all be written as products of temporal translations and spatial isometries.[23]

Here is Eddington on the nature of time in Einstein's static universe:

> … we have already urged that the relativity theory is not concerned to deny the possibility of an absolute time, but to deny that that it is concerned in any experimental knowledge yet found; and it need not perturb us if the conception of absolute time turns up in a new form in a theory of phenomena on a cosmical scale, as to which no experimental knowledge is yet available. Just as each limited observer has his own particular separation of space and time, so a being coextensive with the world might well have a special separation of space and time natural to him.[24]

And here is Jeans:

> … Einstein tried to extend the theory of relativity so that it should cover the facts of astronomy and of gravitation in particular. The simplest explanation of the phenomena seemed to lie in supposing space to be curved…. It was natural to try in the first instance to retain the symmetry between space and time which had figured so prominently in [special relativity], but this was soon found to be impossible. If the theory of relativity was to be enlarged so as to cover the facts of astronomy, then the symmetry between space and time which had hitherto prevailed must be discarded. Thus time regained a real objective existence, although only on the astronomical scale, and with reference to astronomical phenomena. (Jeans [1936], pp. 21 f.).

By the end of the 1920's, it had been established that the spiral nebulae were: (i) located outside of our galaxy; and (ii) systematically red-shifted. As a result, cosmologists turned their attention to expanding solutions, and Big Bang models quickly became the new standard.[25] These solutions admit a unique absolute time, consisting of the instants orthogonal to the dust worldlines. Alternatively, these instants can be characterized as the spaces of constant spatial curvature, or the spaces of constant mass density—these quantities are preserved by the symmetries of the solutions, so no symmetry ever maps an event off of the instant upon which it lies.[26] These instants are homogeneous—the spacetime has a six dimensional symmetry group acting as a transitive group of isometries on each instant.

Speaking of these models, Jeans remarks:

> Now, the second property [in addition to expansion] which all the mathematical solutions have in common is that every one of them makes a real distinction between space and time. This gives us every justification for reverting to our old intuitional belief that past, present, and future have real objective meanings, and are not mere hallucinations of our individual minds—in brief we are free to believe that time is real. … we find a distinction between time and space, as soon as we abandon local physics and call the astronomy of the universe to our aid.[27]

This is the context of Gödel's project. Immediately after his discussion of the lack of an absolute time in the Minkowski case, Gödel remarks that some will object that this conclusion does not carry over to the cosmological regime, where the behavior of matter distinguishes an absolute time—at least in all solutions then known. Gödel takes Jeans as his foil: 'From this state of affairs, in view of the fact that [these solutions] seem to represent our world correctly, James Jeans has concluded that there is no reason to abandon the intuitive idea of an absolute time lapsing objectively' ([1949*a*], p. 559).

3.3 Time in Gödel's Stationary Rotating Solutions

Gödel opens phase (iii) of his paper by remarking that 'There exist cosmological solutions of another kind than those known at present,' and goes on to announce that he has discovered solutions which 'possess such properties of symmetry that for each possible concept of simultaneity and succession there exist others which cannot be distinguished from it by any intrinsic properties, but only by reference to individual objects, such as, e.g., a particular galactic system.'[28] In our present terminology, Gödel asserts that his newly discovered stationary rotating solutions do not support an absolute time.[29] This is an argument of the form discussed in §2 above.[30]

(The balance of this subsection outlines the context and structure of Gödel's proof, and may be omitted without loss of continuity.)

The examples discussed in the previous subsection suggest a couple of *sufficient* conditions for the existence of an absolute time.

- In Big Bang models, the subspaces of constant matter density form an absolute time. This will happen whenever a solution has the following two properties: (i) it *is dust-space homogeneous*, in the sense that for any two dust worldlines, there is a symmetry of the cosmology that maps the first onto the second (this ensures that the cosmology can be foliated by instants of constant matter density); (ii) the matter density is not everywhere constant (this ensures that the there will be no more than one way to foliate the cosmology by instants of constant matter density). Since matter density is invariant under symmetries, its level surfaces provide an absolute time for such solutions.
- In augmented Minkowski spacetime, Einstein's static universe, and Big Bang solutions, the family of instants everywhere orthogonal to the preferred freely falling congruence forms an absolute time. Now, in relativistic physics, as in Newtonian mechanics, we can at each spacetime point introduce a standard of rotation—*the compass of inertia*—by declaring that matter is rotating at that point if the surrounding matter rotates relative to a free gyroscope at that point.[31] It is possible to construct a family of instants orthogonal to the dust worldlines if and only if the solution is everywhere non-rotating.[32] The instants of orthogonality for any non-

rotating solution form an absolute time, since they are defined in terms of data (the metric and dust velocity at a point) invariant under any symmetry.[33]

These conditions are logically independent. The Einstein static universe is non-rotating, and hence falls into instants of orthogonality. But it is spacetime homogeneous—every pair of points is related by a symmetry—so the matter density is constant on the entire spacetime and does not provide a means of labeling instants. On the other hand, Gödel himself later discovered rotating solutions which are expanding and dust-space homogeneous—in these solutions the level sets of matter density form an absolute time, but there are no instants of orthogonality.[34] Further—although in the Big Bang case the two conditions pick out the same family of instants, there exist so-called *tilted homogeneous cosmologies* in which the both conditions obtain but lead to distinct foliations by instants.[35]

And neither condition is necessary: already in 1924, Lanczos discovered a solution describing the behavior of a rotating dust cylinder, which violates both of our conditions but which nevertheless admits an absolute time.[36]

Gödel appears to have sought a rotating dust solution in order to dispel the idea that absolute time, deposed by special relativity, is restored by cosmological considerations ([*1949*b*], p. 12). But while a solution which is atemporal in the desired sense must rotate, not every rotating solution is atemporal in the desired sense. What further conditions are required?

Gödel's approach is a variation on the strategy for the Minkowski case, sketched above in §3.1: he considers an arbitrary foliation by instants; then show that for any of these instants, there is a point on the instant which is fixed by a symmetry which does not fix the instant itself.

In the Minkowksi case, we observed that any boost relative to an inertial observer passing through the point along the normal to the instant had the desired effect. In the general relativistic case, a symmetry fixing a point will act on the tangent space of that point as a (homogeneous) Lorentz symmetry (differing from the identity iff the symmetry differs from the identity). In the realm of dust cosmology, a symmetry fixing a given spacetime point must also fix the dust velocity vector at that point—and hence acts, infinitesimally, as a product of reflections or rotations in the hypersurface orthogonal to the dust velocity vector. It follows that if a point is fixed by a continuous family of symmetries, this family will include a one parameter subgroup of symmetries acting infinitesimally as rotations at the fixed point. Call a dust solution *locally rotationally symmetric* if every point is fixed by such a group.

Gödel's stationary rotating solution is both locally rotationally symmetric and everywhere rotating.[37] No such solution admits an absolute time: given a foliation by instants, we select an arbitrary instant, *T*; this instant is not everywhere orthogonal to the dust worldlines, because the solution is everywhere rotating; so we can select a point, *x,* on *T* at which orthogonality fails; now act on *T* by a symmetry which fixes *x* and acts infinitesimally as a rotation; this symmetry does not fix *T*, since the instant is not orthogonal to the dust worldline passing through *x*; nor does it map *T* to another instant in the foliation, since the symmetry has *x* as a fixed point; so the foliation is not invariant under the symmetries of the spacetime.[38]

3.4 Gödel's Argument for the Significance of these Results

In the closing paragraph of his paper, Gödel confronts a crucial line of objection: 'It might, however, be asked: Of what use is it if such conditions prevail in certain *possible* worlds? Does that mean anything for the question interesting us whether in *our* world there exists an objective lapse of time?' These are pointed questions: Gödel himself grants that his solutions (lacking a red-shift) are far from physically realistic—and that more adequate models of our cosmos *do* support an absolute time.

In answer, Gödel offers two sorts of considerations.[39]

First, he remarks that while his stationary rotating solutions are not physically realistic, 'there exist however also *expanding* rotating solutions. In such universes an absolute time also might fail to exist, and it is not impossible that our world is a universe of this kind' ([1949*a*], p. 562). Presumably he has in mind already the family of expanding rotating solutions described in Gödel ([1952]). But those solutions *do* admit a well-behaved absolute time—or, rather, those with low rates of cosmic rotation do so.[40] And observation places a fantastically low upper bound on the rate of cosmic rotation for all known solutions (see Scherfner [1998]). So there is no empirical need for rotating solutions in cosmology, let alone for ones that fail to support an absolute time.

Gödel's second suggestion, with which he ends the paper, is more provocative:

> The mere compatibility with the laws of nature of worlds in which there is no distinguished absolute time … throws some light on the meaning of time also in those worlds in which an absolute time *can* be defined. For, if someone asserts that this absolute time is lapsing, he accepts as a consequence that whether or not an objective lapse of time exists (i.e., whether or not a time in the ordinary sense of the word exists) depends on the particular way in which matter and its motion are arranged in the world. This is not a straightforward contradiction; nevertheless, a philosophical view leading to such consequences can hardly be considered as satisfactory.[41]

Critics have baulked at accepting even this rather weak conclusion.[42] In the standard Big Bang account, the present matter density of the universe and the details of the relative motions of the galaxies together place constraints on the topology of space, and determine whether the universe will expand forever, or will begin to contract and end in a singularity after a finite time. If questions about the shape of space and the extent of time are allowed to depend on the arrangement of matter and motion, why should it be unsatisfactory to say the same of the lapse of time? Let us call this *Earman's challenge*.

The comparison underwriting Earman's challenge does indeed cast Gödel's conclusion in a rather unflattering light. In the remainder of this subsection, I lay out a second comparison, and use it to motivate a Gödelian response to this challenge. This line of thought will, however, lead to a negative assessment (in the following subsection) of the ultimate success of Gödel's argument.

Consider the question of the existence of a preferred parity in nature. This question, like the questions regarding the extent of time and the shape of space that the interlocutor urges us to consider, admits an interpretation under which it is to be settled by examining the distribution of matter and motion in our cosmos. That is, one asks whether there exist types of processes or objects which enjoy a predominance over their parity-reverses.[43]

But the question of a preferred parity also admits a *second* sort of reading, under which it is concerned with properties of the laws of nature rather than with features of any particular world. Under this second construal, the question is to be settled by determining whether the fundamental laws of nature are invariant under parity inversion.[44]

We face a similar choice when we come to the question of the structure and nature of time in general relativity—one construal leads to a focus on properties of particular realistic solutions, while another leads to a focus on features of the laws of the theory.

The former sort of approach appears to support the view of Jeans *et al.*: because realistic cosmological solutions feature privileged foliations by instants, the special relativistic argument against becoming does not apply; so the pre-relativistic division of spacetime into space and time is reinstated at the level of astronomy. Gödel urges that the latter sort of approach leads to a very different conclusion—since the laws permit cosmological solutions supporting an analog of the special relativistic argument, we are very far from having absolute time in general relativity.

Now, in the parity case, it is generally held that the interest of the law-structural construal of the question dwarfs that of the particular solution/empirical regularity construal. I suggest the following rationalization. The search for more fundamental theories involves making bets: isolating features of extant theories to be carried over (or generalized) from those to be left behind, in order to give form to the search for new theories. Because so much of the most creative and influential work in physics is directed towards creating these new theories, it is very natural that this activity should structure our attempts to understand our current theories. This provides the context in which it is natural to think that, in at least one very important sense, the investigation of properties of laws leads to deeper insights than does the natural-historical cataloguing of regularities in the world.[45]

What Gödel asserts in the closing sentence of his paper is that as when asking after a preferred parity, so when asking after the nature of time in general relativity—the interest of the law-structural construal dwarfs that of the particular solution/empirical regularity construal. I like to think of Gödel as motivated here by considerations running parallel to those sketched in the preceding paragraph.

I find some encouragement in this in the manuscript drafts ([*1946/9-*B2*]; [*1946/9-*C1*]). These are precursors to the paper we have been discussing, but have a different focus from that of the published paper—in them Gödel aims to extract from relativistic physics a substantial vindication of a thesis he finds in Kant, namely that 'time is neither "something existing in itself" (i.e., a separate entity besides the objects in it), nor "a characteristic or ordering inherent in the objects", but only a characteristic inherent in the relation of the objects to something else.'[46] Gödel remarks that he is not himself 'an adherent of Kantian philosophy in general' ([*1946/9-*B2*], fn. 1; [*1946/9-*C1*], fn. 1). But he expresses the hope that his discussion will show that the questions arising in a comparison of Kantian doctrine with relativistic physics 'are interesting and perhaps even fruitful for the future development of physics.'[47]

Now, Gödel thinks of general relativity as a theory very likely to be *false*.

In the present imperfect state of physics, however, it cannot be maintained with any reasonable degree of certainty that the space-time scheme of relativity theory really describes the objective structure of the material world. Perhaps it is to be considered as only one step beyond the appearances and towards the things (i.e., as one 'level of objectivation', to be followed by others).[48]

We are told more about these levels in a footnote:

each … is obtained from the preceding one by the elimination of certain subjective elements. The 'natural' world picture, i.e., Kant's world of appearances itself, also must of course be considered as one such level, in which a great many subjective elements of the 'world of sensations' are already eliminated.[49]

For Gödel, scientific progress consists in the attainment of greater 'objectivation,' a process that leads us towards knowledge of things as they are in themselves:

A real contradiction between relativity theory and Kantian philosophy seems to me to exist only in one point, namely, as to Kant's opinion that natural science in the description it gives of the world must necessarily retain the forms of our sense perception and can do nothing else but set up relations between appearances within this frame.

… at this point, it seems to me, Kant should be modified, if one wants to establish agreement between his doctrines and modern physics; i.e., it should be assumed that it is possible for scientific knowledge, at least partially and step by step, to go beyond the appearances and approach the world of things.

The abandoning of that 'natural' picture of the world which Kant calls the world of 'appearance' is exactly the main characteristic distinguishing modern physics from Newtonian physics.[50]

Gödel's makes some cursory remarks regarding the nature of the progress achieved by special relativity and quantum mechanics. In discussing the former, he focuses on the dissolution of the absolute distinction between space and time, and emphasizes that this required the application of something very like the principle of sufficient reason (to rule out Lorentzian approaches to special relativity).[51] Regarding quantum mechanics he mentions the disparity between the theoretical language of the new theory and the '"laboratory language"' (which he claims had until then sufficed for the purposes of theoretical physics) ([*1946/9-*B2*], p. 19; [*1946/9-*C1*], p. 28).

I sum this up as follows. Gödel hopes that his investigations will contribute towards scientific progress. That is, he hopes that his analysis of the nature of time will contribute to a deeper understanding of the respects in which general relativity represents an increase in 'objectivation' by yielding a world picture incrementally closer to the nature of things as they are in themselves. The examples he gives of achievements along these lines within the other revolutionary theories of the early 20th century suggest that he has in mind, above all else, high level features—regarding the structure of the conceptual framework—rather than the details of particular representations of the world.

If this characterization is accepted, I think we ought—as in the case of the consensus view regarding the question of a preferred parity—to grant that there is a sense in which the investigation of properties of laws leads to deeper insights than does the natural historical cataloguing of regularities in the world in considering questions like the nature of time in general relativity. This does *something* to shore up Gödel's argument in response to Earman's challenge—for such a concession appears to secure a sense in which it is unsatisfactory to look towards special features of physically realistic solutions in settling such questions.[52]

3.5 Is Gödel's Argument Successful?

Now, all parties will grant that we can hope to learn about the nature of time by attending to features of the laws of general relativity. It is natural to ask: To what extent does the argument of Gödel's paper advance this program?

The answer is, I think, *not very far*: the particular law-structural construal that Gödel opts for does not provide as much insight as he hoped. I offer two sorts of reason.

First sort of reason. Gödel asks whether the laws of dust cosmology guarantee the existence of an absolute time; and answers that they cannot do so, since there exists a dust solution that does not admit an absolute time. This is an argument by counter-example, and as such invites evasion by monster-barring—and in the case at hand, this stratagem would not by any means be entirely *ad hoc*. Gödel identifies two features of his solutions, each held to be incompatible with the ordinary notion of time: (i) they include closed timelike curves; and (ii) their symmetry prevents the construction of an absolute time.

> *Regarding* (i). While it is no longer as common as it once was to rule solutions containing closed timelike curves physically unrealistic on *a priori* grounds, such solutions are nonetheless routinely excluded from consideration. For it is quite standard among people working with general relativity as a dynamical theory—especially those looking for deeper quantum theories of gravity—to take the content of the theory to be demarcated, not by the class of solutions to the equations that Einstein wrote down, but by the class of solutions that can be recovered in a

Hamiltonian reformulation of the theory. For those interested in understanding the nature of time in the theory, this practice is quite reasonable: it would appear to be a promising strategy to focus on the relation between time as it is in general relativity and as it is in other theories—and the Hamiltonian approach provides an over-arching scheme in which to write down and compare dynamical theories. Now, in order to recast general relativity in Hamiltonian form, one must restrict attention to solutions that satisfy a certain causality condition (namely, that they can be foliated by spacelike instants which intersect every timelike and null curve exactly once)—and this means excluding from our attention solutions (such as Gödel's stationary rotating worlds) which include closed timelike curves.

Regarding (ii). For any set of differential equations, solutions with greater than minimal symmetry are freaks within the complete space of solutions.[53] It is not at all uncommon in contemporary work on general relativity in the Hamiltonian formalism to simply ignore symmetric solutions, which lead to technical complications in the study of the dynamics.[54] In any case, if one thinks of general relativity as a mere approximate theory, any property which is possessed only by very special solutions probably ought not to play a very prominent role in attempts to understand the theory.

Second sort of reason. More importantly: Gödel's argument, despite being a *tour de force* in many respects, does not seem to cut very deep in the direction he points us towards. We might in classical particle mechanics be interested in the question whether there is a distinguished origin for time. An argument analogous to Gödel's would proceed by observing that the existence of such a feature does not follow directly from the laws, since the laws allow a world containing only a single particle eternally at rest—in which what happens at one instant is indistinguishable from what happens at any other, so that a choice of a distinguished temporal origin would violate the symmetries of that world. But there is another, deeper, question that we are more likely to have in mind in asking whether time has a distinguished origin: namely, whether the equations of motion are invariant under time translation. If they are so invariant, then it seems that the dynamics treats all instants as being on a par with one another, and the laws of nature single out no preferred instant—although, of course, in all but those worlds in which the particles remain forever at rest, it will be possible to distinguish instants from one another while respecting the symmetries of the situation. This invariance property is deep: it is tied up with the conservation of energy, and is the sort of feature that one wants to carry over in constructing more fundamental theories. It is in analogous properties that we should be interested when we ask about the nature of time in general relativity.[55]

4 Another Argument

I pursue this line of thought a bit further, and consider a line of argument suggested by the above discussion. I begin in §4.1 with a more full discussion of the symmetry property that when present indicates that the dynamics of a classical mechanical system fails to select a preferred origin of time. When this feature is present, the laws of the theory are indifferent to the identity of instants. In classical mechanics, this implies that parameterizations of time differing only in their choice of origin are on all fours with one another. It is natural to think that if one could construct a parallel argument in the general relativistic case, identifying a sense in which the dynamics of the theory is indifferent to the identity of instants, this would show that in general relativity distinct times composed out of distinct sets of instants would be on all fours with respect to one another—and that this would secure a sense in which the laws of general relativity were incompatible with a preferred time. In §§4.2 and 4.3, respectively, I note that it is far from straightforward to establish anything like this for general relativity as a whole, and note that there exists a formulation of dust general relativity in which the desired symmetry property is present. In §4.4, however, I worry that this argument has little probative force—as in Minkowski spacetime, so in general relativistic cosmology: those attached to pre-relativistic notions of time will be happy to add the necessary structure by hand, thus spoiling any symmetry argument that departs from the more spare structure *prima facie* mandated by the physics as it is usually understood.

4.1 Time Translation Invariance in Classical Mechanics

Suppose that we are studying a system of classical particles. The natural dynamical variables for our theory of this system are the temporally evolving positions and momenta of the particles. Under one familiar approach, our theory is set in the *phase space* of the system, the space of possible positions and momenta of the particles. A point of the phase space specifies an instantaneous position and momentum for each particle; schematically, we denote such points (q,p). This space carries a natural geometric structure.[56] And this structure, together with information encoded in the Hamiltonian function of the theory—the real-valued function on the phase space that assigns to each possible state of the system its total energy—is enough to determine the dynamics of the theory, by determining for each point of the phase space a unique curve which passes through that point.[57] The points lying on a curve through a given point represent the dynamical past and future of the state represented by that point.

For our purposes it is helpful to work in a slightly different framework, in which the dynamics of the theory is formulated on a larger space, each of whose points corresponds to a point of phase space together with a real number—schematically, we write $(q,p;t)$. There is no standard name for this space; I will call it the *extended phase space*. The new variable should be thought of as labeling an instant of time, the instant of time at which the system is in the state (q,p). We can define our dynamics by giving this space a geometrical structure; in the construction of this structure, we employ both our system's Hamiltonian and the geometrical structure of its phase space.[58] In this formulation, the differential equations of classical mechanics serve to determine the geometrical structure of the extended phase space. And this structure singles out a set of curves on the extended phase space, exactly one through each point. For each value of t, each of these curves passes through at most one point of the form $(q,p;t)$. We interpret these dynamical trajectories as follows: the curve that passes through $(q_0,p_0;t_0)$ allows us to associate values of q and p to each value of t at which the state of the system is defined; thus such a curve tells us what state the system is in at each such instant if it is in the state (q_0,p_0) at the instant labeled by t_0. We write $(q',p';t')\rightarrow(q'',p'';t'')$ if $(q',p';t')$ and $(q'',p'';t'')$ lie on the same dynamical trajectory.

This framework treats instants in abstraction from their contents. There is a venerable tradition, deriving from Leibniz, according to which this strategy drastically over-counts the number of genuine possibilities:

Supposing anyone should ask, why God did not create everything *a year sooner*; and the same person should infer from thence, that God has done something, concerning which it is *not possible* there should be a *reason*, why he did it *so*, and not *otherwise*: the answer is, that his inference would be right, if *time* were any thing distinct from things existing in time. For it would be *impossible* there should be any *reason*, why things should be applied to such *particular instants*, rather than to *others*, their succession continuing the same. But then the same argument proves, that *instants*, considered without the things, are *nothing at all*….[59]

But even Leibnizeans may well want to adduce the invariance of the standard theory under time translation as part of an *argument* for their favored interpretative stance. So everyone should grant the conceptual interest of the technical question of the time translation invariance of classical mechanics. And the extended phase space provides a natural setting in which to address this question.[60]

To this end, we want to study the symmetries of theories defined on extended phase spaces. Such a symmetry is a permutation of the $(q,p;t)$'s which leaves invariant all of the structure we have defined on this space.[61] It follows that the set of dynamical trajectories is invariant under these symmetries—that the image of a dynamical trajectory under a symmetry is a dynamical trajectory—since the structure that determines these curves is itself invariant. This can be unpacked as follows. Let S be a symmetry of the extended phase space; we write $S(q,p;t)$ for the state which results when we apply S to the state $(q,p;t)$. Then the invariance of the dynamics means that for any symmetry S, $(q',p';t')\to(q'',p'';t'')$ implies $S(q',p';t')\to S(q'',p'';t'')$—in order to find the dynamical trajectory through $S(q',p';t')$, we do not need to solve the dynamics again, but simply to act upon the original dynamical trajectory by the same symmetry transformation that we apply to the initial data.

Suppose, for instance, that S is the symmetry which translates all of the particle positions some fixed amount in some direction, while leaving the momenta and t invariant. This is a symmetry any of reasonable isolated mechanical system in Euclidean space. If we know what happens when the initial data are set to be q and p at t and we want to know what happens if we alter our initial data by replacing q by the shifted initial positions, we can simply apply the same shift to the dynamical state which follows at time t' from the original initial data. The significance of this invariance is that we are unable to use the laws to single out a preferred origin in space: for the invariance of our dynamics tells us that no dynamical evidence could distinguish between the case where the system is located at the origin, and the case where its location differs from the origin by a translation.

Now consider the map, S_β, defined by $S_\beta(q,p;t)= (q,p;t+\beta)$ for some real number β. That is, S_β corresponds to a time translation. The physics is time translation invariant if, for each α and β, $(q',p';t')\rightarrow(q'',p'';t'+\alpha)$ implies $(q',p';t'+\beta)\rightarrow(q'',p'';(t'+\beta)+\alpha)$. In classical particle mechanics, this condition holds for a given system if and only if its energy function does not depend upon t (this fails, for instance, if our particles are constrained to move on a surface that changes shape in an externally prescribed manner). When time translation is a symmetry, the instants of time are dynamically identical—whether we set the positions and velocities to be q and p at t' or at t'', the subsequent evolution will look *exactly* the same.

Thus the situation is a familiar one. Let us consider the extended phase space of some (time-independent) system of particles as a structure in the sense of §2: its individuals are the instantaneous states of the particles; these states stand in a number of relations, encoding the fact that the space is a manifold carrying a certain geometric structure which determines the dynamics. We can consider extending this structure by introducing a new property whose intended extension is the set of initial data representing the system as being at the temporal or spatial origin. But there are no reasonable candidates for such extensions that respect the symmetries of the original structure.[62] The laws of classical mechanics are indifferent to the identity of points of space and instants of time.

4.2 Time Translation Invariance in General Relativity?

Now let us turn to the general relativistic case. There, speaking conveniently loosely, the situation can be described as follows.

The lessons of the dust case notwithstanding, it is natural to think that the distinction between space and time, dissolved in special relativity, remains unreinstated at the level of the dynamics of the theory. Natural, yes—but how can this thought be made precise?

One question is whether there is a method, generally if not universally applicable, for constructing privileged foliations by instants in general relativity.[63] As noted above (fn. 21) this remains an open question—even setting aside difficulties involving symmetric solutions, the most promising general methods are known to fail for certain dust solutions. But note that if such a technique were to exist, its availability would guarantee the existence of many competitors—for every proposed absolute time, there would be ever so many competing schmabsolute times whose recipes are tweakings of the given recipe.[64] And the prospects of singling one of these out on physical (as opposed to pragmatic or mathematical) grounds appear remote at this time.

So let us turn instead to a related question, which allows us to advance (a bit) further. One supposes that there is a sense in which the dynamics of general relativity is indifferent to the identity of instants—for otherwise it might well be possible to exploit dynamical differences to reinstate space and time. Let us look for an analog of the argument of the previous subsection that established the corresponding conclusion for classical mechanics. Now, the space of instants in any general relativistic spacetime is infinite dimensional. So if we are hoping for something along the lines of the time translation invariance of classical mechanics, we should look for an infinite dimensional symmetry group of the theory whose action permutes the identity of instants.

We are interested in formulations of general relativity in which the states of the theory form a space with an interesting geometrical structure, whose symmetry properties we aim to investigate. There are two main formulations along these lines, both prominently featuring the action of an infinite dimensional group of symmetries.

Under the first approach, the space of states is the space of spacetime solutions of the Einstein field equations, upon which the group of spacetime diffeomorphisms acts.[65] The principle of general covariance—the invariance of the theory under the action of the diffeomorphism group—tells us that any way of laying down on the spacetime manifold the tensor fields describing the geometrical and material behavior of a given solution is as good as any other. This has been taken by many to suggest that the theory will be mangled if any preferred split into space and time is introduced—but it remains difficult to establish a clear connection between this form of the principle of general covariance and our desired conclusion, since instants play no straightforward role in this formulation of general relativity.

Under the second major approach, the space of states of the theory is made up of instantaneous states—spatial initial data for the Einstein equations rather than solutions to them.[66] Here the group of *spatial* diffeomorphisms comes to the fore, permuting the roles of the points of the instant upon which the initial data are laid down. But the group of spatiotemporal diffeomorphisms no longer acts as a symmetry group (see, e.g., Kuchař [1986]). Although the physical content of the full symmetry group of the space of solutions must be encoded in *some* way in this new space, this is not achieved via the action of a symmetry group which permutes the identity of instants.

Nonetheless, there is a special case in which a group action of the sort we seek crops up—namely, dust coupled to gravity.

4.3 Time Translation Invariance in Dust Cosmology

In this subsection, I describe some results due to Brown and Kuchař ([1995]) concerning the remarkable symmetry properties of their formulation of dust general relativity.

It is helpful to begin by imposing some technical conditions. First, we restrict attention to globally hyperbolic solutions—those that can be foliated by spacelike instants which intersect every timelike and null curve exactly once. The spacetime manifold underlying any such solution is topologically of the form $\Sigma\times\mathbf{R}$, for some three manifold Σ (to be thought of as describing the topology of space). The space of solutions of the equations falls into disconnected components, one for each possible topology of Σ. It is common practice to fix some Σ from the beginning, and to study the corresponding set of solutions. We follow this practice, and further demand that our Σ be compact (so that our solution is spatially finite but without boundary, eliminating worries about boundary conditions at spatial infinity). It follows that every spacelike instant intersects each timelike and null curve exactly once and is topologically equivalent to Σ (see Budic *et al.* [1978], Theorem 1). Finally, we choose our Σ to be such that that no spacetime metric on $\Sigma\times\mathbf{R}$ admits any (continuous family of) symmetries.[67] This last assumption is in place to drive home that the symmetry considerations we come across below are different in kind from those discussed by Gödel.

For any dust solution satisfying these conditions, the set S of dust motes forms a three dimensional manifold topologically isomorphic to Σ. Each of these dust motes can be thought of as carrying a clock that measures proper time along the mote's worldline. So each event in such a spacetime can be named by mentioning the dust mote which occupies it, and the time which then shows on the dust mote's clock—the space of possible events of our theory is of the form $S\times\mathbf{R}$. We now require the origins for the clocks' times to be chosen so that the set of events where each of the clocks reads zero is a smooth submanifold of $S\times\mathbf{R}$. This set, which by construction includes exactly one event involving each dust mote, is a possible instant, which we call the *fiducial instant.* Once a fiducial instant is chosen, we can use real-valued functions S to specify possible instants: if $\boldsymbol{T}(z)$, $z\in S$, is such a function, then the corresponding instant is given by tracing $\boldsymbol{T}(z)$ units of proper time along the worldline of z from the event at which z lies on the fiducial instant, for each dust mote z.

This method of naming events is technically unremarkable—we are simply defining quantities on the state space of our theory. But it will seem misguided to many to talk about the identity of the events of a general relativistic world—let alone measurements of proper time!—in abstraction from the geometric and matter fields which describe what is going on at those events. My response is the same as in the classical mechanical case: the objection is an expression of an (entirely reasonable) interpretative tendency; but we are here pursuing a technical question which can be investigated without prejudice to interpretative questions.

Now, in their standard form the equations of dust general relativity have as solutions sets of tensors (describing the behavior of the geometry and dust) on a four dimensional spacetime manifold. Brown and Kuchař construct a formulation of dust general relativity in which solutions consist of tensors defined upon our space of possible events rather than upon a spacetime manifold. This may appear to be a trifling difference, since both are topologically $\Sigma\times\mathbf{R}$. But this reformulation turns out to be much more convenient for our purposes.[68]

It works as follows. Given an instant in a solution of the standard equations, we can ask for a description of the instantaneous disposition of the geometric and material degrees of freedom of the system from the point of view of the dust motes. To any geometric or matter tensor defined on an instant of spacetime, there corresponds one defined on S describing the same situation.

There is a small set, Φ, of such tensors defined on S—comprising the metric structure of the instant, the 'rate of change' of this metric structure, and the matter density along the instant—which form a dynamically closed set, capturing the full content of the theory.[69]

We form a space of states for our theory, where each state is of the form $(\Phi,\boldsymbol{T})$, specifying the values of our chosen tensors on S and the identity of the instant at which they assume these values. As in our formulation of classical mechanics in the extended phase space, the differential equations of our theory serve to give our space of states a geometrical structure which encodes the dynamics of the theory.

In the former case, this structure determined one dimensional families of states, corresponding to possible histories of the system: writing $(q',p';t')\rightarrow(q'',p'';t'')$ if the two states lie in the same one dimensional family of states, which occurs iff the dynamics evolves one into the other. And unless dynamical singularities develop, each family of dynamically related states includes exactly one state for each instant of time.

In the present case, a possible history for our system (that is, dust coupled to gravity) again involves an assignment of values to our dynamical variables, Φ, for each possible instant—where now the possible instants, being parameterized by smooth real-valued functions on S, form an *infinite* dimensional family. The geometric structure on our space of states singles out infinite dimensional families of states—where such a family includes all and only those states which belong to a single dynamically possible history of our system.[70] Let us call such a family a *dynamical history*, and write $(\Phi,\boldsymbol{T})\rightarrow(\Phi',\boldsymbol{T}+\boldsymbol{\tau})$ when $(\Phi,\boldsymbol{T})$ and $(\Phi',\boldsymbol{T}+\boldsymbol{\tau})$ belong to the same dynamical history. And as in the classical mechanical case, we have a well posed initial value problem: specifying the state of the dynamical variables at a given instant determines their values at each other instant (unless singularities develop, and the state is not defined at some instants).[71]

In this form, general relativity has the sort of temporal symmetry that we seek. Suppose that we have chosen a fiducial instant, and that relative to this choice we consider another instant labeled by a function $\boldsymbol{\Theta}(z)$ on S. We now consider what happens if we take this new instant as our fiducial instant. The result is that an instant labeled by $\boldsymbol{T}(z)$ relative to our original choice of fiducial instant is now labeled $\boldsymbol{T}'(z)=\boldsymbol{T}(z)-\boldsymbol{\Theta}(z)$; this induces a transformation on the space of instantaneous dust states such that $(\Phi,\boldsymbol{T})$ gets mapped to $(\Phi,\boldsymbol{T}-\boldsymbol{\Theta})$; this mapping preserves all of the structure of our space which plays a role in determining the dynamics—so the group of smooth real-valued functions on the space of dust motes is a symmetry group for our theory.[72] And as in the classical mechanical case, we find that the dynamics is indifferent to this transformation: if $(\Phi,\boldsymbol{T})\rightarrow(\Phi',\boldsymbol{T}+\boldsymbol{\tau})$, then $(\Phi,\boldsymbol{T}-\boldsymbol{\Theta})\rightarrow(\Phi',\boldsymbol{T}-\boldsymbol{\Theta}+\boldsymbol{\tau})$ for any smooth functions $\boldsymbol{T}$, $\boldsymbol{\tau}$, and $\boldsymbol{\Theta}$ defined on S—each dynamical history is an invariant set of these mappings. Note that whereas in the classical mechanical case the group of time translations was one dimensional (corresponding to the freedom to reset the origin of a single clock), in the present case it is infinite dimensional (corresponding the freedom of each dust mote to *independently* reset the origin of its clock).

4.4 Is this Second Argument Successful?

So dust general relativity admits of an attractive formulation invariant under a symmetry group that permutes the identity of instants. The analogous result in the classical mechanical case secured a sense in which the laws of that theory fail to select a preferred origin for time. The present result appears to carry us quite a bit further. Each possible fiducial instant is associated in a natural way with a time—that time whose instants correspond to the level sets of proper time of the dust motes, under the supposition that they agree to set their clocks to zero along the given instant. Given any choice of fiducial instant and its associated time, we can always find another possible fiducial instant whose corresponding time, considered as a set of instants, is wholly disjoint from that of the fiducial instant that we started with. The fact that the laws of the theory are indifferent to the identity of instants shows that it is impossible that they could ever favor the first time over the second. This seems to give us an excellent start towards showing that in this formulation the laws of general relativity do not favor any time over any other.

But can this really be so? Let us keep working in this formalism, but restrict attention to those states which lie in the rotation-free solutions. Instant-relabeling remains a symmetry of this restricted space of states (thought of as equipped with the geometrical structure it inherits from the larger space).[73] But in this smaller class of states it is quite clear that there *is* a preferred absolute time—namely, that given by instants orthogonal to the dust worldlines! Why isn't this fact registered in the symmetry properties of our formulation of the theory?

The problem is that in restricting attention to instantaneous states that give rise to non-rotating solutions, we have done nothing to rule out the choice of an *arbitrary* instant within such a solution as our fiducial instant.[74] In order to get a preferred time, we must impose a further constraint, restricting our attention to instants which are orthogonal to the dust worldlines.[75] This means restricting attention to a small subset of possible instantaneous states for each non-rotating solution. This subset is no longer invariant under any instant re-labelings which carry us out of the class of orthogonal fiducial instants. And we at last have a means, invariant under the (relatively restricted) symmetries of our latest space of states, of recognizing within that space the privileged nature of the absolute time composed of hypersurfaces of orthogonality in nonrotating solutions.

One lesson to be learned is that here, as elsewhere, symmetry arguments can be got round—one can *always* argue that the structure studied ought to be replaced by another with a different set of symmetries. Against Plato's argument: the Earth is not a sphere, so couldn't *up* be above *here*? Against the argument of the previous subsection: the universe doesn't appear to rotate—so can't we employ the orthogonal foliation present in standard cosmological models? In both of these cases, the symmetry argument is wrecked as we include more information about the actual world. In the first case, the structure under consideration is a representation of our world; so taking more information into account means augmenting our structure, yielding a more detailed representation of the world. In the second case, our structure is a representation of salient possibilities; taking more information into account means eliminating some of the original possibilities as irrelevant, resulting in a thinning-out of our structure.

We are in an uncomfortable, if unsurprising, situation.

Every student of special relativity should be exposed to arguments like those of §3.1 above, which delimit what can be expected from a notion of simultaneity in Minkowski spacetime. But, of course, these arguments are polemically inert in the dispute between the orthodox, who hold that the metric structure of Minkowski spacetime encodes the physical kernel of Einstein's theory, and those reactionaries and heretics who supplement this structure by the choice of a preferred inertial frame. For what is at stake in that dispute is whether or not Minkowski spacetime provides a perspicuous starting point for physical reasoning—and so symmetry arguments taking that structure as their point of departure are liable to be dismissed as question-begging.

Just so in the present case—the choice of a conception of instant, on the one hand, and the adoption of a view about the nature of time in general relativity, on the other, form a very tight circle indeed. In this situation we cannot hope that a symmetry argument would convince a party to a debate about the nature of time, because the debate would in part concern the proper point of departure for any such argument.

We have run up against one aspect of a very general problem.[76] Physical theories typically admit a variety of formulations. Often the difference between variant formulations is founded upon a difference in the set of possibilities countenanced. Sometimes, interpretative questions will hang upon such differences—and judgments as to the correct formulation will be influenced in turn by interpretative judgments.[77] Then we can be faced with an intractable problem—for it is not always easy to see what grounds we could adduce in arguing for a preferred formulation.

5 Conclusion

Neither of the arguments considered achieve their desired target—namely, showing that there is no more place for absolute time in general relativistic cosmology than in special relativity. Where does this leave us regarding the status of absolute time in general relativity?

If we are asking about the structure of individual solutions, then the answer is relatively clear. On the one hand, solutions like the Big Bang models admit a unique time invariant under the symmetries of the solution. On the other, Gödel constructed a solution in which no candidate for a notion of time is invariant under the relevant symmetries. What do these examples tell us about our world? Not so much. Gödel's example is very far from being empirically adequate. The Big Bang models are much closer—but their high degree of symmetry represents an idealization that will sooner or later have to be dropped. More realistic models will lack any symmetry—and here the problem lies not constructing a time invariant under symmetries, but in singling out a single such time from among the huge multitude available.

If we focus on the structure of the laws rather than the structure of individual solutions, we are faced with this same problem: the symmetries of the laws do not seem to single out any preferred notion of time. But then such features depend on the formulation of the laws chosen. And here there is the potential for a clash of intuitions—in the case of dust general relativity, for instance, some will be happy to restrict attention to non-rotating solutions and to formulate the laws in such a way that there will be a preferred time. Others will think that, even if the restriction to non-rotating solutions is made, a formulation singling out such a time is anathema.

At best, the arguments of this paper represent one way into the issues that would have to be resolved before we can fully understand the nature of time in general relativity.

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Department of Philosophy
University of Pittsburgh
Pittsburgh PA 15260
USA

gbelot@pitt.edu

References

Abraham, R. and J. Marsden [1985]: *Foundations of Mechanics*, Cambridge, MA: Perseus.

Adams, R. [1979]: 'Primitive Thisness and Primitive Identity', *Journal of Philosophy*, **LXXVI**, pp. 5–26.

Belot, G. [2000]: 'Geometry and Motion', *The British Journal for the Philosophy of Science*, **51**, pp. 561–95.

Brown, J. and K. Kuchař [1995]: 'Dust as a Standard of Space and Time in Canonical Quantum Gravity', *Physical Review* D, **51**, pp. 5600–29.

Budic, R., J. Isenberg, L. Lindblom, and P. Yasskin [1978]: 'On the Determination of Cauchy Surfaces from Intrinsic Properties', *Communications in Mathematical Physics*, **61**, pp. 87–95.

Cornford, F. [1997]: *Plato's Cosmology*, Indianapolis: Hackett.

Crnkovic, C. and E. Witten [1987]: 'Covariant Description of Canonical Formalism in Geometrical Theories', in S. Hawking and W. Israel (*eds*.), 1987, *Three Hundred Years of Gravitation,* Cambridge: Cambridge University Press, pp. 676–84.

Dorato, M. [2002]: 'On Becoming, Cosmic Time, and Rotating Universes', in C. Callender (*ed*.), 2002, *Time, Reality, and Existence,* Cambridge: Cambridge University Press, pp. 253–76.

Earman, J. [1995]: *Bangs, Crunches, Whimpers, and Shrieks*, Oxford: Oxford University Press.

Eddington, A. [1920]: *Space, Time, and Gravitation*, Cambridge: Cambridge University Press.

Eddington, A. [1923]: *The Mathematical Theory of Relativity*, Cambridge: Cambridge University Press.

Eichhorn, E. [1992]: 'Felix Hausdorff–Paul Mongré: Some Aspects of his Life and the Meaning of his Death', in W. Gähler, H. Herrlich, and G. Preuss (*eds*.), 1992, *Recent Developments of General Topology and its Applications,* Berlin: Akademie-Verlag, pp. 85–117.

Ellis, G. [1967]: 'Dynamics of Pressure-Free Matter in General Relativity', *Journal of Mathematical Physics*, **8**, pp. 1171–91.

Ellis, G. [1996]: ‘Contributions of K. Gödel to Relativity and Cosmology’, in P. Hájek (*ed*.), 1996, *Gödel ’96,* Berlin: Springer-Verlag, pp. 34–49.

Ellis, G. [2000]: ‘Editor’s Note’, *General Relativity and Gravitation*, **32**, pp. 1399–407.

Fischer, A. and V. Moncrief [1996]: ‘A Method of Reduction of Einstein’s Equations of Evolution and a Natural Symplectic Structure on the Space of Gravitational Degrees of Freedom’, *General Relativity and Gravitation*, **28**, pp. 207–19.

Giulini, D. [2001]: ‘Uniqueness of Simultaneity’, *The British Journal for the Philosophy of Science*, **52**, pp. 651–70.

Gödel, K. [1946/9-*B2*]: ‘Some Observations about the Relationship between Theory of Relativity and Kantian Philosophy’ version *B2* in S. Feferman, J. Dawson, W. Goldfarb, C. Parsons, and R. Solovay (*eds*.), 1995, *Kurt Gödel. Collected Works. Volume III: Unpublished Essays and Lectures,* Oxford: Oxford University Press, pp. 230–46. Citations in the text follow marginal page numbers (keyed to Gödel’s manuscript).

Gödel, K. [*1946/9-*C1*]: ‘Some Observations about the Relationship between Theory of Relativity and Kantian Philosophy’ version *C1*, in S. Feferman, J. Dawson, W. Goldfarb, C. Parsons, and R. Solovay (*eds*.), 1995, *Kurt Gödel. Collected Works. Volume III: Unpublished Essays and Lectures,* Oxford: Oxford University Press, pp. 247–59. Citations in the text follow marginal page numbers (keyed to Gödel’s manuscript).

Gödel, K. [1949]: ‘An Example of a New Type of Cosmological Solutions of Einstein’s Field Equations’, in S. Feferman, J. Dawson, S. Kleene, G. Moore, R. Solovay, and J. Van Heijenoort (*eds*.), 1990, *Kurt Gödel. Collected Works. Volume II: Publications 1938–1974,* Oxford: Oxford University Press, pp. 190–98. Citations in the text follow marginal page numbers (keyed to the first published version).

Gödel, K. [1949*a*]: ‘A Remark about the Relationship between Relativity Theory and Idealistic Philosophy’, in S. Feferman, J. Dawson, S. Kleene, G. Moore, R. Solovay, and J. Van Heijenoort (*eds*.), 1990, *Kurt Gödel. Collected Works. Volume II: Publications 1938–1974,* Oxford: Oxford University Press, pp. 202–7. Citations in the text follow marginal page numbers (keyed to the first published version).

Gödel, K. [*1949*b*]: ‘Lecture on Rotating Universes’, in S. Feferman, J. Dawson, W. Goldfarb, C. Parsons, and R. Solovay (*eds*.), 1995, *Kurt Gödel. Collected Works. Volume III: Unpublished Essays and Lectures,* Oxford: Oxford University Press, pp. 269–87. Citations in the text follow marginal page numbers (keyed to Gödel’s manuscript).

Gödel, K. [1952]: ‘Rotating Universes in General Relativity Theory’, in S. Feferman, J. Dawson, S. Kleene, G. Moore, R. Solovay, and J. Van Heijenoort (*eds*.), 1990, *Kurt*

Gödel. Collected Works. Volume II: Publications 1938–1974, Oxford: Oxford University Press, pp. 208–16. Citations in the text follow marginal page numbers (keyed to the first published version).

Hacking, I. [1975]: 'The Identity of Indiscernibles', *The Journal of Philosophy*, **LXXII**, pp. 249–56.

Hawking, S. and G. Ellis [1973]: *The Large-Scale Structure of Space-Time*, Cambridge: Cambridge University Press.

Henkel, O. [2002]: 'Global Prescribed Mean Curvature Foliations in Cosmological Spacetimes with Matter', *Journal of Mathematical Physics,* **42**, pp. 2439–65.

Hewitt, C. [1991]: 'An Exact Tilted Bianchi II Cosmology', *Classical and Quantum Gravity*, **8**, pp. L109–14.

Isenberg, J. and J. Marsden [1982]: 'A Slice Theorem for the Space of Solutions of Einstein's Equations', *Physics Reports*, **89**, pp. 179–222.

Isenberg, J. and A. Rendall [1998]: 'Cosmological Spacetimes not Covered by a Constant Mean Curvature Slicing', *Classical and Quantum Gravity*, **15**, pp. 3679–88.

Ismael, J. and B. van Fraassen [2003]: 'Symmetry as a Guide to Superfluous Theoretical Structure', in K. Brading and E. Castellani (*eds*.), 2003, *Symmetries in Physics: Philosophical Reflections*, Cambridge: Cambridge University Press, pp. 371–92.

Jeans, J. [1936]: 'Man and the Universe', in J. Jeans, W. Bragg, E. Appleton, E. Mellenby, J. Haldane, and J. Huxley, 1936, *Scientific Progress,* London: The MacMillan Company, pp. 11–38.

Jeans, J. [1942]: *Physics and Philosophy*, Cambridge: Cambridge University Press.

Kant, I. [1768]: 'Concerning the Ultimate Grounds of the Differentiation of Directions in Space', in Walford, D. and R. Meerbote (*eds*.), 1992, *Immanuel Kant: Theoretical Philosophy* 1755–1770, Cambridge: Cambridge University Press, pp. 361–72.

King, A. and G. Ellis [1973]: 'Tilted Homogeneous Cosmological Models', *Communications in Mathematical Physics*, **31**, pp. 209–42.

Kragh, H. [1996]: *Cosmology and Controversy*, Princeton, NJ: Princeton University Press.

Kramer, D., H. Stephani, E. Herlt, and M. MacCallum [1980]: *Exact Solutions of Einstein's Field Equations*, Cambridge: Cambridge University Press.

Kuchař, K. [1986]: 'Canonical Geometrodynamics and General Covariance', *Foundations of Physics*, **16**, pp. 193–208.

Lanczos, C. [1997]: 'On a Stationary Cosmology in the Sense of Einstein's Theory of Gravitation', *General Relativity and Gravitation*, **29**, pp. 363–99.

Mach, E. [1960]: *The Science of Mechanics*, La Salle, IL: Open Court.

Malament, D. [1977]: 'Causal Theories of Time and the Conventionality of Simultaneity', *Noûs*, **11**, pp. 293–300.

Malament, D. [1995]: 'Introductory Note to *1949*b*', in S. Feferman, J. Dawson, W. Goldfarb, C. Parsons, and R. Solovay (*eds*.), 1995, *Kurt Gödel. Collected Works. Volume III: Unpublished Essays and Lectures,* Oxford: Oxford University Press, pp. 261–9.

Malament, D. [2002]: 'A No-Go Theorem about Rotation in Relativity Theory', in D. Malament (*ed*.), 2002, *Reading Natural Philosophy,* La Salle, IL: Open Court, pp. 267–93.

Martin, D. [2001]: 'Charles Johnson, 76, Proponent of Flat Earth', *New York Times*, March 25.

Minkowski, H. [1953]: 'Space and Time', in H. Lorentz, A. Einstein, H. Minkowski, and H. Weyl, 1953, *The Principle of Relativity,* New York: Dover, pp. 73–91.

Olver, P. [1993]: *Applications of Lie Groups to Differential Equations*, Berlin: Springer-Verlag.

Ozsváth, I. and E. Schücking [1969]: 'The Finite Rotating Universe', *Annals of Physics*, **55**, pp. 166–204.

Poincaré, H. [1963]: *Mathematics and Science*, New York: Dover.

Sachs, R. and H. Wu [1977]: *General Relativity for Mathematicians*, Berlin: Springer-Verlag.

Savitt, S. [1994]: 'The Replacement of Time', *Australasian Journal of Philosophy*, **72**, pp. 463–74.

Scherfner, M. [1998]: 'History of Cosmic Rotation and New Developments', in H.-J. Schmidt and M. Rainer (*eds*.), 1998, *Current Topics in Mathematical Cosmology,* Singapore: World Scientific, pp. 419–24.

Smolin, L. [2001]: 'The Present Moment in Quantum Cosmology: Challenges to the Arguments for the Elimination of Time', PITT-PHIL-SCI pre-print 00000153.

Souriau, J.-M. [1997]: *Structure of Dynamical Systems*, Basel: Birkhäuser.

Stein, H. [1970]: 'On the Paradoxical Time-Structures of Gödel', *Philosophy of Science*, **37**, pp. 589–601.

Stein, H. [1991]: 'On Relativity and Openness of the Future', *Philosophy of Science*, **58**, pp. 147–67.

Stein, H. [1995]: 'Introductory Note to *1946/9', in S. Feferman, J. Dawson, W. Goldfarb, C. Parsons, and R. Solovay (*eds*.), 1995, *Kurt Gödel. Collected Works. Volume III: Unpublished Essays and Lectures,* Oxford: Oxford University Press, pp. 202–29.

van Elst, H. and G. Ellis [1996]: 'The Covariant Approach to LRS Perfect Fluid Spacetime Geometries', *Classical and Quantum Gravity*, **13**, pp. 1099–127.

van Fraassen, B. [2004]: 'Structure: Its Shadow and Substance', to appear in R. Jones and P. Ehrlich (*eds*.), 2004, *Reverberations of the Shaky Game*, Oxford: Oxford University Press.

Wald, R. [1984]: *General Relativity*, Chicago: Chicago University Press.

Weiner, P. (*ed*.) [1951]: *Leibniz: Selections*, New York: Charles Scribner's Sons.

Woodhouse, N. [1980]: *Geometric Quantization*, Oxford: Clarendon Press.

Yourgrau, P. [1991]: *The Disappearance of Time*, Cambridge: Cambridge University Press.

Yourgrau, P. [1995]: *Gödel Meets Einstein*, La Salle, IL: Open Court.

[1] In the sense, e.g., of Adams ([1979], §1). See Ismael and van Fraassen ([2003]) for a quite different sense and a discussion of another aspect of symmetry arguments.
[2] A property, P, is fixed by a permutation, Φ, of the individuals if: $P(a)$ iff $P(\Phi(a))$ for each individual, a. A dyadic relation, R, is fixed by such a Φ if: $R(a,b)$ iff $R(\Phi(a),\Phi(b))$, for all individuals a and b. And so on.
[3] Our symmetries are functions, so composition of functions gives us an associative binary operation on the set of symmetries; the identity mapping is an identity for this operation; the existence of an inverse for each symmetry follows from the fact that symmetries are bijections.
[4] This is very clear under an alternative approach to characterizing symmetries. We consider descriptions of the structure in a language which includes a predicate symbol for each property and relation, and enough constants to serve as names for the individuals. Relative to an assignment of names to individuals, we can form the set of all the atomic sentences in this language stating truths about our structure. A symmetry can be thought of as a permutation of the *names* of the individuals which leaves invariant this set of atomic truths. The group of such permutations is isomorphic to the automorphism group of the structure.
[5] I.e., we can find individuals, $x_1,\ldots,x_n$, a symmetry, Φ, of the original structure, and a new relation, R, such that $R(x_1,\ldots,x_n)$ but not $R((\Phi(x_1), \ldots,\Phi(x_k))$.
[6] If, on the other hand, the proposed extension respects the symmetries of the original structure, then the solution that it serves has passed a test. If it can be shown that it is the unique invariant extension of the sort under consideration, then we have excellent reason to accept the proposed solution—so long as we have good reason to restrict attention to the approach at hand.
[7] Of course, one man's *modus ponens* is another's *modus tollens*: Charles Johnson, late president of the Flat Earth Research Society, is said to have remarked that 'If earth were a ball spinning in space, there would be no up or down' (see Martin [2001]).
[8] Gödel's cosmological investigations resulted in several papers, whose nature and inter-relations it may be helpful to recall here. Gödel himself published three papers on our topic: Gödel ([1949*a*]), with which we are principally concerned, is a non-technical discussion of the philosophical significance of Gödel's stationary rotating solutions; Gödel ([1949]) is a technical paper containing proofs of a number of remarkable results concerning these solutions; Gödel ([1952]) is a technical exposition of some of the properties of the *expanding* rotating solutions later discovered by Gödel. Three relevant papers unpublished by Gödel are now also available: Gödel ([*1946/9-*B2*]) and ([*1946/9-*C1*]) are ancestors of Gödel ([1949*a*]); Gödel ([*1949*b*]) is the manuscript for a talk in which Gödel presented his results to his colleagues at the Institute for Advanced Studies. (I follow the conventions of the editors of Gödel's collected works in citing these papers.)
[9] For discussion of (iv), see Stein ([1970], §3), Yourgrau ([1991], pp. 53–5; [1999], pp. 47 f. and 84–103), Savitt ([1994], §1), Earman ([1995], Appendix to Chapter 6), and Dorato ([2002], §4). Each of these emphasizes considerations deriving from (iii*) rather than (iii).
[10] Gödel appears to be sympathetic to the worry that the notion of successively realized instants is, ultimately, incoherent—as he mentions with approval arguments to this effect due to McTaggart and Mongré/Hausdorff ([1949*a*], fnn. 1 and 4; [*1946/9-*B2*], fn. 14).

In an early draft of the article, Gödel enigmatically refers to the divisibility of spacetime into instants as concerning the (order) *structure* of our idea of time, while the successive actualization of such instants concerns the *content* of that idea ([*1946/9-*B2*], p. 6). Felix Hausdorff, writing as Paul Mongré, appears to have drawn a related distinction in a work cited by Gödel. For discussion of Mongré's view, see Eichhorn ([1992], pp. 87 f.).
[11] ([1949*a*], p. 558). See also ([1949*a*], fn. 5; [*1946/9-*C1*], fn. 28). In Gödel's use, 'objective' is opposed to 'subjective,' while 'absolute' is opposed to 'relative.' The paradigm of a subjective notion for Gödel is, perhaps, time or space as conceived of by Kant; the paradigm of a relative notion is simultaneity relative to an observer in Minkowski spacetime.
[12] This is somewhat more restrictive than the usual definition—see, e.g., Abraham and Marsden ([1985], p. 95).
[13] I take the structure of Minkowski spacetime to be given by a flat metric of Lorentz signature on $\mathbf{R}^4$. The symmetries of this structure are the (inhomogeneous) Lorentz transformations. So here invariance means that the image of an equivalence class of this relation under a Lorentz symmetry is again an equivalence class of the relation. In fact, the only invariant equivalence relations on Minkowski spacetime are the trivial

ones: that under which each point is equivalent only to itself, and that under which each point is equivalent to every other. See, e.g., Giulini ([2001], Theorem 4).

[14] There are a number of other invariant equivalence relations in this case (e.g., count two events as simultaneous if light signals sent by the observer coincidentally and reflected by the events in question return coincidentally to the observer, thus partitioning each Einstein simultaneity class by concentric spheres). But the Einstein equivalence relation is the only one whose equivalence classes intersect exactly once the worldline of each inertial observer comoving with the privileged observer. If, however, one adds a sense to the privileged worldline (eliminating temporal inversion as a symmetry), one *does* get further such invariant equivalence relations; see Giulini ([2001], §6). See Malament ([1977]) and Stein ([1991]) for discussion of related results departing from the causal structure of Minkowski spacetime, rather than from its metric structure.

[15] More precisely, a dust cosmology is an ordered triple, consisting of: a spacetime manifold carrying a metric tensor of Lorentz signature; a positive spacetime scalar, ρ; and a timelike vector field u_a, normalized so that $u^a u_a=-1$. The metric satisfies the Einstein field equations for a stress-energy tensor of the form $T_{ab}=\rho u_a u_b$. The dust worldlines are the integral curves of u_a.

Gödel investigates the equations for dust cosmology with a cosmological constant term. When, as in the case of his stationary rotating universes, this constant is assigned a negative value, such solutions can be reinterpreted as involving a perfect fluid with positive pressure and a vanishing cosmological constant (in which case the dust motes no longer follow geodesics). See, e.g., Kramer *et al*. ([1980], §5.2) for the recipe. Adopting this reinterpretation would make no difference to the considerations of the present paper.

[16] Note that instants, in this sense, are *not* required to be spacelike. This represents a liberalization of our ordinary use—one which proves helpful for present purposes (first in discussing Gödel's argument, since Gödel's stationary rotating cosmologies admit slicings into instants (the level sets of the coordinate x_0 in Gödel [1949]) but do not contain *any* spacelike instants (property (7) in Gödel [1949]); then again in the reformulation of general relativity in dust variables, discussed in §4.3 below).

[17] As noted above, in the dust case, we are able to reconstruct the matter variables from knowledge of the geometry alone. So everything goes well, in the sense that we do not need to distinguish between the study of the structure 'spacetime geometry + matter fields' and the study of the structure 'spacetime geometry.' Things do not always go well however: there are solutions of the Einstein-Maxwell equations where the matter configuration and the geometry have different symmetry groups (see Kramer *et al*. [1980], p. 114)).

[18] It is convenient, and harmless, to ignore a further condition that Gödel explicitly builds into his notion of absolute time: namely, that the instants arise as the level sets of a time function that increases for all possible future-moving observers (for this see ([1949], p. 447), ([1949*a*], fn. 13), and ([1952], p. 179); but see also ([*1949*b*], p. 11)). Note that stable causality is a necessary and sufficient condition for the existence of such a time function (see Hawking and Ellis [1973], p. 198 and Sachs and Wu [1977], §8.3.5).

[19] Asymmetric models form a dense and open subset of the space of solutions; see Isenberg and Marsden ([1982]).

[20] I.e., typically we expect our cosmological spacetime to be a trivial **R**- or S^1-principal bundle over the space of dust motes—in this case, any section will provide an instant, and a time can be constructed by setting all of the dust motes' proper time clocks to zero at this instant, then taking the level surfaces of proper time as the instants.

[21] The most widely studied candidate is the (generically unique) foliation by surfaces of constant mean curvature. But there exist dust cosmologies which do not admit any such foliation; see Isenberg and Rendall ([1998]). See Henkel ([2002]) for an alternative approach—which also has trouble with dust.

[22] De Sitter spacetime was discovered at around the same time, and enjoyed a period of popularity as a cosmological model in the early days of relativistic cosmology, despite being a vacuum solution. Like Minkowski spacetime, de Sitter spacetime is a homogeneous spacetime of constant curvature, and does not admit an absolute time unless supplemented by a freely falling congruence.

[23] The spacetime can be written as $S^3\times\mathbf{R}$, with the dust worldlines assuming the form $\{x\}\times\mathbf{R}$ for $x\in S^3$ and the hypersurfaces of orthogonality assuming the form $S^3\times\{t\}$ for $t\in\mathbf{R}$. The temporal translations map (x,t) to $(x,t+\alpha)$ for some $\alpha\in\mathbf{R}$. The spatial isometries arise by identifying S^3 with the unit quaternions and allowing it to act on itself by left or right multiplication. See Ozsváth and Schücking ([1969], §3).

[24] Eddington ([1920], p. 163; see 12 f. and 28 f. for the earlier discussions alluded to). Eddington seems to moderate his enthusiasm for absolute time in his ([1923], §§1 and 71).

[25] Here and below, I use 'Big Bang model' to denote FRW solutions with vanishing cosmological constant.
[26] Rather, this is true if the solution expands forever. In solutions that expand and then contract, there will be two instants corresponding to each (non-minimal) instantiated value of the mass density; these will be related by a discrete spacetime symmetry (a temporal reflection).
[27] Jeans ([1936], pp. 23 and 24). Jeans appears to be somewhat more cautious in his ([1942], pp. 63–9).
[28] Gödel ([1949*a*], p. 560). The claim is stated more precisely in the technical companion paper: 'If Σ is any system of mutually exclusive three-spaces, each of which intersects each world line of matter in one point, then there exists a transformation which carries [the solution] S … into itself, but does *not* carry Σ into itself; i.e., an *absolute* time does not exist, even if it is not required to agree in direction with the times of all possible observers (where "absolute" means: definable without reference to individual objects, such as, e.g., a particular galactic system)' ([1949], p. 447).
[29] A dust solution is *stationary* if it is possible to foliate it by instants with isomorphic geometries and matter distributions. Gödel later discovered a family of rotating solutions that are expanding (and therefore non-stationary); see Gödel ([1952]).
[30] That this is so is especially clear in an earlier version of the argument: 'a definition in terms of physical magnitudes of an absolute world time is demonstrably impossible. If, however, such a world time were to be introduced in these worlds as a new entity, independent of all observable magnitudes, it would violate the principle of sufficient reason, insofar as one would have to make an arbitrary choice between infinitely many physically completely indistinguishable possibilities, and introduce a perfectly unfounded asymmetry' ([*1946/9-*B2*], p. 10).
[31] As Gödel puts it, 'the angular velocity we are interested in is the angular velocity relative to the directions of space defined by axes of gyroscopes moving along with matter. For this is the angular velocity which the astronomer who, with his measuring apparatus, moves along with matter will observe' ([*1949*b*], p. 6). See Malament ([2002]) for details concerning this notion of rotation, and a delimitation of what one can hope for from a standard of rotation in general relativity.
[32] For helpful discussion, see Malament ([1995]).
[33] What happens if the dust motes apply the Einstein simultaneity convention? In the Einstein static universe, any dust mote applying this convention will arrive at the instants of orthogonality as its surfaces of simultaneity. But the existence of particle horizons in Big Bang models means that in such solutions the Einstein simultaneity convention leads to pathological results—for any dust mote, there will be events which are not simultaneous with any event on its worldline (see Wald [1984], §5.3b). Event horizons will cause similar problems in other spacetimes. Some of these difficulties can be circumvented by considering the result of allowing each of a set of privileged communicating observers to apply the Einstein convention. When this procedure works for dust motes, it leads to the surfaces of orthogonality (see Sachs and Wu [1977], §5.3).
[34] See Gödel ([1952]). The matter density time consists of well-behaved spacelike instants in those solutions with low rates of rotation. But those with high rates of rotation feature closed timelike curves and *timelike* instants of constant matter density.
[35] See King and Ellis ([1973], p. 221) or Hewitt ([1991]). In such cosmologies, the foliation by hypersurfaces of constant mass density is, while the foliation by instants of orthogonality is not, fixed leaf-wise by symmetries (or rather, by those in the connected component of the identity in the symmetry group).
[36] See Lanczos ([1997]). According to Ellis ([2000], pp. 1401 f.), this solution was not widely known or well-understood at the time; Kragh ([1996], pp. 109 f.) is interesting in this connection. In any case, Lanczos's solution had only a very brief career of cosmological plausibility: Lanczos conceived of his dust cylinder as representing our galaxy, which he took to exhaust the universe, and he suggested that our sun lay near the axis of symmetry ([1997], p. 378); this picture no longer looked reasonable once it was conclusively established (in the mid-1920s) that the universe is much larger than our galaxy.

Gödel seems to have been unaware of the existence of this rotating solution: 'All cosmological solutions with non-vanishing density of matter known at present have the common property that, in a certain sense, they contain an "absolute" time coordinate, owing to the fact that there exists a one-parametric system of three-spaces everywhere orthogonal to the worldlines of matter. It is easily seen that the non-existence of such a system of three spaces is equivalent with a rotation of matter relative to the compass of inertia' ([1949], p. 447).
[37] In Gödel's solution matter everywhere rotates at the same rate. But the solution is spacetime

homogeneous, so there is no distinguished axis of rotation and the dust worldlines do not diverge. As Gödel remarks ([*1949*b*], pp. 4 f.): 'The world may be perfectly homogeneous and still rotate locally in every place…. Nevertheless, the world may be said to rotate as a whole (like a rigid body) because the mutual distances of any two material particles (measured by the orthogonal distance of their world lines) remains constant during all times. Of course, it is also possible and even more suggestive to think of this world as a rigid body at rest and of the compass of inertia as rotating everywhere relative to this body. Evidently this state of affairs shows that the inertial field is to a large extent independent of the state of motion of matter. This contradicts Mach's principle but it does not contradict relativity theory.' This is perhaps misleading in one respect: *finite* rigid bodies do after all have distinguished axes of rotation (as does Lanczos's rotating cylinder); but see Gödel ([*1949*b*], pp. 2 f.) for an infinite Newtonian rigid body which rotates without having a distinguished axis of rotation. See Ozsváth and Schücking ([1969]) for a spatially closed anti-Machian rotating universe.

[38] This argument can be found at Gödel ([1949], 449). Gödel's solution is not the unique solution with the requisite properties. But amongst such solutions, it has maximal symmetry, and is of the greatest physical interest. For other locally rotationally symmetric everywhere rotating spacetimes and their shortcomings, see Ellis ([1967], *Case Ib*) and van Elst and Ellis ([1996], §§4.2 and 4.5). On the interest and influence of Gödel's solution, see Ellis ([1996]; [2000]).

[39] Yourgrau ([1991], pp. 53–5; [1999], pp. 47 f.) suggests that Gödel has in mind a third sort of argument, which is supposed to turn upon the local indistinguishability of the experience of time in his solutions from that in solutions admitting an absolute time. The argument (but not its attribution to Gödel) is taken up by Savitt ([1994], §1); it is criticized by Earman ([1995], Appendix to Chapter 6) and Dorato ([2002], §4); for a reply, see Yourgrau ([1999], pp. 93–8).

I set this argument aside here because I do not think see any textual basis for attributing it to Gödel: the crucial sentence, 'But, if the experience of the lapse of time can exist without an objective lapse of time, no reason can be given why an objective lapse of time should be assumed at all' ([1949*a*], p. 561), appears in the penultimate paragraph of the article, a paragraph explicitly devoted to the significance of closed timelike curves in Gödel's stationary rotating solutions for the existence of objective lapse of time in *those* worlds; the topic of the significance of the lack of an objective lapse of time in Gödel's solutions for the nature of time in *other* worlds is not announced and addressed until the following paragraph. (Further, I doubt that the determination that the experience of time would be the same in the two sorts of universe can be arrived at in a non-question begging fashion.)

[40] See Gödel ([1952], §3). The surfaces of constant matter density form an absolute time, in the present weak sense, in all of these solutions. The further condition that Gödel requires in order for a time to be absolute (see fn. 18 above) is satisfied in solutions with low rates of rotation, but violated in those that rotate more rapidly (see fn. 34 above).

[41] ([1949*a*], p. 562). For variant formulations, see ([*1946/9-*B2*], p. 10) and ([*1946/9-*C1*], p. 17).

[42] For the objection below, see Earman ([1995], p. 198); see Savitt ([1994]) and Dorato ([2002]) for further discussion.

[43] And, as Kant observed, there *do* exist such processes and object types: 'In the case of human beings, the hair on the crown of the head grows in a spiral from the left to the right. All hops wind around their poles from left to right, whereas beans wind in the opposite direction. Almost all snails, with the exception of perhaps, only three species, have shells which, when viewed from above, that is to say, when their curvature is traced from the apex to the embouchure, coil from left to right'; and, furthermore, 'all the peoples of the world are right-handed (apart from a few exceptions which, like that of squinting, do not upset the universality of the regular natural order' ([1768], pp. 368 and 369).

[44] The two construals lead to distinct (though not entirely unrelated) questions: while one does not expect the solutions of a given set of equations to have the same symmetry properties as the equations themselves (see fn. 53 below), certain empirical asymmetries of a fundamental nature would lead one to suspect an asymmetry in the laws.

[45] Here we see a disanalogy between the question of parity and the question of the direction of time. In the latter case, the success of thermodynamics is founded upon a tremendously broad array of time-asymmetric regularities—the existence of which is arguably as deep a fact as the behavior under time reversal of any dynamical laws. But it is just this feature of thermodynamics which makes it such an unusual case—the challenge of understanding the status of what looks like a probability distribution over initial conditions but

which behaves in many respects like a law of nature is the source of many of the conceptual problems surrounding statistical thermodynamics.

[46] ([*1946/9-*C1*], p. 1). See also very similar ([*1946/9-*B2*], p. 1).

[47] ([*1946/9-*B2*], fn. 1). There is no corresponding passage in ([1946/9-*C1*]).

[48] ([*1946/9-*B2*], p. 13). See also ([*1946/9-*C1*], fnn. 6 and 16).

[49] ([*1946/9-*B2*], fn. 24). See also very similar ([*1946/9-*C1*], fn. 27).

[50] ([*1946/9-*B2*], p. 18). See also very similar ([*1946/9-*C1*], p. 27).

[51] ([*1946/9-*B2*], p. 21; [*1946/9-*C1*], p. 30). The principle in question appears to be also in play at ([*1946/9-*B2*], p. 10 and fn. 18).

[52] But, of course, this falls short of establishing Gödel's point. For note: if we ask 'What does this theory tell us about future physics?' then it does indeed seem that we ought to focus on examining the structural features of the theory for ones which we believe will survive (in one form or another) in future physics, perhaps almost to the exclusion of paying attention to the physics of particular solutions; but if we ask instead 'What does this theory tell us about the world?' then considerations about the empirical patterns of prediction of one or another solution suddenly begin to seem much more salient. See van Fraassen ([2004]) for a similar point, raised in the course of criticism of the neo-Kantianism of Reichenbach and Friedman.

[53] See Olver ([1993], Chapter 3). Thus, while the equations of Newtonian mechanics are invariant under temporal and spatial translation, very few of their solutions are boring enough to be likewise invariant.

[54] The space which parameterizes the physical possibilities of the theory *without redundancy* is singular at the points corresponding to symmetric possibilities—although the singularities have a relatively tame and tractable structure; see Isenberg and Marsden ([1982]). On the conceptual side, those who share the intuitions of Hacking's ([1975]) Leibniz—see, e.g., Smolin ([2001])—will hesitate to include symmetric models in the space of possibilities.

[55] Note that general relativity had not yet been cast into a form in which questions about the symmetries of its laws could be precisely posed when Gödel was working on cosmological topics.

[56] Let Q be the manifold of possible positions of the particles; then the phase space is the cotangent bundle, T^*Q. This latter manifold carries a canonical symplectic structure (i.e., a closed non-degenerate two -form).

[57] See, e.g., Abraham and Marsden ([1978], §§3.1–3.3).

[58] See Abraham and Marsden ([1978], Theorem 5.1.13). The Hamiltonian can now be allowed to depend on t as well as on q and p.

[59] From §6 of Leibniz's third letter to Clarke. See Weiner ([1951], p. 224).

[60] The shift from the phase space of the theory to its extended phase space has further conceptual and technical advantages; see Souriau ([1997], pp. xviii f.) and Woodhouse ([1980], pp. 33–5).

[61] That is, a symmetry is a diffeomorphism on the extended phase space which preserves the contact form that determines the dynamics.

[62] Note that our choice to employ real numbers as names for instants did not foreclose the question whether there is a preferred instant which deserves to be called *The Origin*, because all of the dynamically relevant structure is invariant under systems of renaming which change which instant is labeled by zero. Compare with Plato's case: even if we chose to name a certain point on the celestial sphere 'Polaris' or '(0,0,1)' this would not help to pick out *up*, for all of the relevant cosmological structure is invariant under transformations which permute such names. See fn. 4 above.

[63] For present purposes, let us say that Cauchy surfaces count as instants in non-dust solutions.

[64] Why? Restrict attention to globally hyperbolic *vacuum* solutions on $\mathbf{R}\times\Sigma$ for a given spatial topology, Σ. If we omit symmetric solutions (or choose Σ so that they do not arise—see Fischer and Moncrief [1996]) then the space of solutions is a principal fiber bundle over the space of solutions modulo spacetime diffeomorphisms. A generally applicable technique for constructing foliations by Cauchy surfaces amounts, roughly speaking, to choosing from each equivalence class of solutions one in which the sets of the form $\{t\times\Sigma\}$, $t\in\mathbf{R}$, are all Cauchy surfaces (the preferred timeslices). (Actually, this is a bit too strong—what we want is this, modulo the choice of a preferred parameterization of the family of slices.) Such a recipe amounts to selecting a section of the principal bundle satisfying a certain open condition. Now, since the bundle is principal, if it admits any sections, it admits an infinite dimensional family of sections (for some spatial topologies the bundle is trivial and so admits sections; Fischer and Moncrief ([1996], p. 215)). And since the condition is open, if any section satisfies it, then so do sufficiently nearby sections (i.e., those

differing from the given one by the action of an automorphism of the bundle sufficiently close to the identity).

[65] See Crnkovic and Witten ([1987]) for this approach. A diffeomorphism is just a smooth permutation of the spacetime manifold. Intuitively, the action is given by lifting up the geometrical and matter fields, stretching and contracting the underlying manifold however you like, then laying the fields back down. The geometry of the new solution is identical to that of the old—all that changes is which spacetime point plays which geometrical role.

[66] See, e.g., Wald ([1984], Appendix E.2).

[67] That this is possible is established by Fischer and Moncrief ([1996]).

[68] The difference is in some ways analogous to the difference between the formulation of fluid mechanics in Lagrangian variables and in Eulerian variables. In the present case the formulation in terms of dust variables has a large number of advantages—in addition to those noted below, it leads to a Hamiltonian constraint linear in the momenta.

[69] The formulation that Brown and Kuchař produce includes a further set of tensors which give a mapping from Σ to S; these play a role in reconstructing spacetime solutions from dust space solutions. But the corresponding degrees of freedom are pure gauge and, presumably, can be factored out to yield the formulation described in the text.

[70] These are the gauge orbits obtained by integrating the null distribution of the presymplectic form defined on our space of states.

[71] The ordinary gauge freedom of general relativity reappears if we attempt to reconstruct a spacetime solution from a dust space solution: each dust space solution corresponds to a diffeomorphism-equivalence class of spacetime solutions. It is in this sense that the dust formulation captures the full (diffeomorphism-invariant) content of the theory. (It is also true that *many* dust space dynamical histories correspond to any equivalence class of diffeomorphic spacetime solutions.)

[72] As in the case of classical mechanics, a change of origin for our set of instants merely re-poses the same initial data at another instant. This is not manifest if one works in the usual variables; see Brown and Kuchař ([1995], equation 2.27).

[73] $(\Phi,\boldsymbol{T})$ and $(\Phi,\boldsymbol{T}-\boldsymbol{\Theta})$ typically lie in *distinct* dynamical histories—our symmetry group permutes dynamical histories which correspond to the same diffeomorphism-class of spacetime solutions

[74] Brown and Kuchař ([1995], equation 7.10).

[75] Brown and Kuchař ([1995], equation 7.19).

[76] It is, I think, part of what Mach ([1960], p. 284) and Poincaré ([1963], p. 19) meant to advert to in speaking of the world as having been given only once. See also Souriau's discussion of the 'paradox of the physicist' ([1997], pp. xxiv ff.).

[77] In Newtonian mechanics one is faced with the choice between the standard formulation of the theory and that which results when one first restricts attention to nonrotating systems, then identifies points in the standard phase space related by the action of the group of symmetries of Euclidean space. Given that the cosmos appears to have vanishing rotation, it seems that both are viable options. Each enjoys an advantage over the other: the former is simpler and more familiar; the latter admits a pretty interpretation under which space is relational and motion is relative. See Belot ([2000]) for discussion and references.