Imaginary-Constitutives: The Ontology of Scientific Models[†] [‡]

Abstract

Hitherto, there have been two approaches towards models in the philosophy of science. One could be called "formalistic" and the other one "methodological". The first coincides with the so-called semantic view, which employs the notion of semantic models, as used in mathematical logic. The other approach, which became popular through *Models as Mediators* (Morgan and Morrison 1999) is very much based on case studies and inquires into the methodological role models hold in scientific practice. Where the formalistic approach offers a well-defined notion of models but struggles to justify its use for the empirical sciences, the proponents of the methodological approach present us with detailed case studies but fall short of providing a theory of models; their claims reduce to the hardly falsifiable and almost vacant claim that models are "autonomous agents" or "instruments", which "mediate" between theory and the world. Neither the formalistic nor the methodological approach has managed to tackle the ontology of scientific models, which I attempt to provide in this paper in terms of *Imaginary-Constitutives*.

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0. Introduction

In the current philosophy of science two approaches towards models can be tracked. One approach tries to gain an understanding about models formal-theoretically, the other approach tries to do so empirically. The theoretical approach coincides with the so-called semantic view. The semantic view construes scientific models as semantic models, which are extralinguistic realizations or interpretations of an abstract theoretical structure. Suppes (1989; 1967) was the first to apply this notion of a model – invented in the domain of mathematical logic by Tarski (1933; 1956) – to the empirical sciences. Distinct from the formal approach – though not necessarily irreconcilable with it – is the methodological approach. Instead of merely subsuming scientific models under the umbrella of mathematics, the methodologists attempt to discern the methodological function of models in relation to the theory and to the real system under investigation. In contrast to the unificatory scope of the formal approach, the methodological approach, due to it being based on case studies, is very local, exhibits a minimum of theorizing ("we do not see ourselves in providing a 'theory' of models"; Morgan and Morrison 1999, 12) and reduces to hardly falsifiable claims like "models mediate between theory and data" (*ibid*.). The questions being discussed most frequently within the methodological framework are whether models are truthful to the real system they are to represent and whether their construction is motivated by theory (top-down) or by the real system (bottom-up). Here are two contrary examples for this debate:

I suggest that [one should take] more into account the role of *background theories and theoretical frameworks* in suggesting, choosing and evaluating models. (Psillos 1995; my emphasis)

Instead of a theory-driven view of models, we suggest a *phenomenologically-driven* one. (Cartwright 1995; my emphasis)

The "models as mediators view" is located somewhere between these two extremes. Morgan, Morrison and others claim that models are "autonomous agents", independent from theory and data (cf. Morgan and Morrison 1999). In general, the top-down approach is often associated with the semantic approach and is utterly rejected by the methodologists[1]. Neither the formalistic nor the methodological approach has managed to tackle the ontology of scientific models[2], which I shall attempt to provide in the following sections.

1. The Standard View and Paradigmatic Models

I take it that the philosopher of science, regardless of whether he/she adheres to the formalistic or the methodological approach, would agree with the following claim: *Models are idealizing assumptions and are simplifications of the real system.* Less popular nowadays, but once very central in the discussion about models is the claim that models are arrived at by analogical reasoning (cf. Hesse 1966; cf. Psillos 1995 for an attempted re-animation of the "analogical approach"). The standard view will be our starting point in analyzing the nature of models in the succeeding sections.

Another observation the philosopher of science can make by reviewing the relevant literature is that a handful of models re-appear in the discussion. These are:

- the billiard ball model of a gas,
- the Bohr model of the atom,
- the harmonic oscillator,
- Maxwell‘s ether model and
- the model of the DNA.

These models are paradigmatic models, i.e. those models the philosopher of science regards as representing the typical characteristics of models in general[3]. Therefore, the conclusions I shall reach by investigating some of these paradigmatic models should warrant a broad validity[4].

2. The Constitutive Character of Imaginary Entities

There is probably not one philosopher of science, who doesn't think that models don't involve idealizing assumptions[5]. Compare, for instance the billiard ball model of a gas and the Bohr model of the hydrogen atom:

The Bohr Model of the Atom:

1. The electron moves in a circular orbit about the proton under the influence of the electric force of attraction.
2. Only certain electron orbits are stable; only on these do we find electrons. As long as the electrons stay in these orbits, the total energy of the atom remains constant.
3. Electromagnetic radiation is emitted from the atom when an electron „jumps" from one orbit to another.
4. The size of the orbits depends on the electron‘s orbital angular momentum.

Billiard ball model of a gas:

1. A gas consists of identical perfectly elastic spheres in motion;
2. The size of these particles is negligible;
3. Particles move along a straight line with constant speed until they hit the wall of the container;
4. The particles are in random motion;
5. The forces of attraction between the particles are negligible.

Is there anything we can say about these two models apart from the fact that they involve idealizing assumptions? If we compare the assumptions of each model with each other, it should be possible to see that both models contain one assumption, which all the other assumptions are based on. If we don't assume that an electron moves about the proton on a circular (or elliptical) orbit we cannot assume that only certain electron orbits are stable (assumption 2). Neither can we state that the electron jumps from one orbit to the other (assumption 3), nor that the size of the orbits depends on the electron's orbital momentum (assumption 4). Likewise, if we do not assume that a gas consists of (elastic) particles in motion, we cannot assume that the size of the particles and the forces acting between them is negligible (assumption 2 and 5). If we do not make assumption 1, we cannot assume that the particles move along a straight line with constant speed (assumption 3) or that their motion is random (assumption 4). I shall call the assumption, which all other assumptions of the model are based on, the *fundamental assumption.* The other assumptions of the respective models only seem to specify the fundamental assumption. Notice however that the referent of the fundamental assumption is not observable. We cannot observe an electron circulating about a proton. We are not able to observe molecules hitting the wall of their container. So how do we know that these events actually take place? The answer is that we postulate them. To put it in more provocative terms, we *imagine* these events. Fig. 1 depicts these events[6].

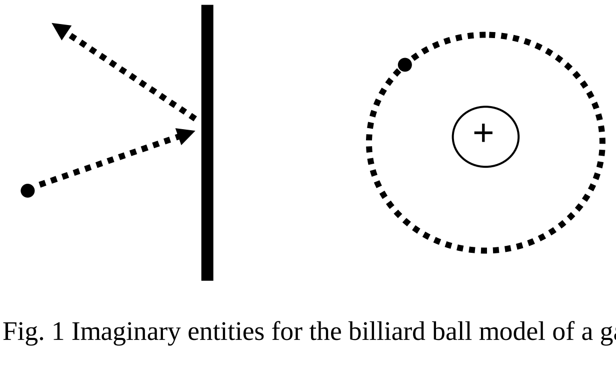

Fig. 1 Imaginary entities for the billiard ball model of a gas (left) and the Bohr model of the atom (right).

In virtue of being the referent of the fundamental assumption of the model, which all other assumptions of the model are modifications of, imaginary entities are *constitutive* of models. I thus baptise the entities, which underlie scientific models, *Imaginary-Constitutives* or simply ICs. In order to rule out misunderstandings I wish to emphasize three points: first, I don't want the constitutive character of ICs to be interpreted temporally. I don't claim that the scientist first imagines an IC and then goes on to build his model on it. I hold the temporal order of "discovery" to be a contingent. What does matter is that the model is, in a way "justified" by the IC. The centripetal acceleration $a = v^2/r$ is an integral part of deriving the Rydberg equation. In order to apply it, one needs to *imagine* that the electron revolves about the proton in an orbit. If one doesn't presume that pressure is equivalent to the *imaginary event* of molecules hitting the wall of the container elastically, one wouldn't be able to write down the change of momentum for the molecule being *2mv* after collision with the wall. Second, ICs are not what is known as "unobservables" in philosophical discussions. Unobservables are usually assumed to be a part of the world. However, my ICs are imagined, i.e. products of our thought. Also, unobservables are usually taken to be electrons, atoms, quarks etc. Yet, it is not the components that matter here. Rather, what matters is what could insufficiently be described as a structure (Bohr model) or behaviour (Billiard ball model)[7] or more formally "X revolves about Y" and "X hits Y elastically". Again, these entities cannot be observed but are *imagined*. Although the realist still might want to argue that the constituents of the models (electrons, protons, molecules) are real, he cannot claim that the ICs of "X revolves about Y" and "Xs hit Y elastically" are real. We know well that, even though it is capable of predicting[8] the Rydberg equation, the Bohr model of the atom and therefore its IC of "the electron revolves about the proton" is strictly speaking "false". Likewise, the IC of "Xs hit Y elastically" of the billiard ball model lacks any considerations of intermolecular forces, for instance, and is thus not "the real thing" either. Nevertheless, both ICs are constitutive of their respective models. The third point I want to stress is that ICs, although they are visual entities, are not visualizations. ICs are not merely a useful means of visualizing theory and its statements, as suggested by the logical positivists:

"[T]he postulates of the theory appear to be statements at least part of whose content can be *visually imagined*. Such a presentation is adopted, among other reasons, *because it can be understood with greater ease* than can an inevitably longer and more complicated purely formal exposition." (Nagel 1979, 135)

Rather than taking imaginary entities as merely helping us to understand formal statements more easily, I claim that formal statements describe ICs. Thus ICs and therefore also models are non-linguistic entities. This explains why Giere (1988) and the proponents of the semantic view characterize models as abstract and non-linguistic entities. That is also why models are neither true nor false, as probably Hutten (1954) first pointed out. The models' statements refer to an imagined entity and specify it. All this takes place in the realm of imagination. "Are imaginary entities true or false?" does not seem to be meaningful question to ask.

In the following section I shall first point to shortcomings in Hesse's classic account of analogies and then offer an alternative in terms of ICs.

3. Models and Analogies

In the 1960s and 1970s in particular, but still nowadays (cf. Psillos (1995), Magnani (1999) et al. and Hallyn (2000)), models are said to depend on or are the result of analogies. In the classic *Models and Analogies* Hesse introduced a terminology, which is used until this day[9]. Hesse distinguished between three kinds of analogies holding between a source and a target system: positive, negative, and neutral analogies[10]. *Positive analogies* stand for those properties which are common to both the source and the target system. *Negative analogies* denote those properties which are absent in the one and present in the other system and *neutral analogies* are either positive or negative analogies, whose status has not been settled yet. The model is the set of positive and neutral analogies, where neutral analogies are what Hesse calls the "growing points" of the model, i.e. those properties, which encourage further research (9p.). Neutral analogies are crucial for Hesse's notion of a model:

"A model, for me, is any system, whether buildable, picturable, imaginable, or none of these, *which has the characteristic of making a theory predictive* […]" (1966, 19). "A model is not simply a system of formal rules, for it carries with it suggestions for its own extension and generalization." (1953, 212)

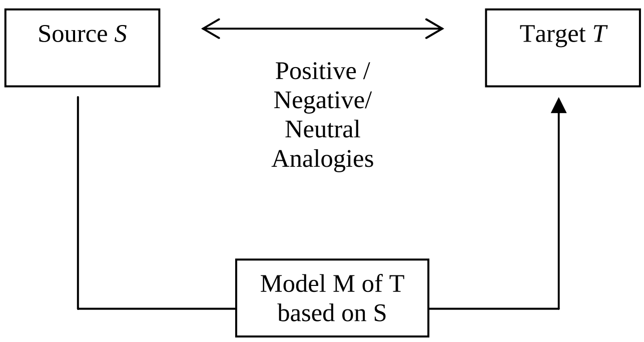

Fig. 2: The relationships of source and target system including positive, negative, and neutral analogies. Graphic adopted from Psillos (1995, 114).

The model Hesse uses to exemplify this terminology is the billiard ball model of a gas. Billiard balls are coloured, hard and shiny which molecules are not (negative analogy). However, "motion and impact" are indeed properties which we want to assign to molecules of a perfect gas (positive analogy). Given that neutral analogies are of a central importance to her account, Hesse is surprisingly vague about them. In contrast to the positive and negative analogies neutral analogies don't seem to consist of a relation between the *properties* of the source and target system. Instead Hesse says about neutral analogies that "*from our knowledge of the mechanics* of billiard balls we may be able to make new predictions about the expected behaviour of gases" (9; my emphasis). The relations between billiard balls and molecules of a gas in the "Hessean" terminology are summarized in Tab. 1.

Sort of analogy	***Properties of billiard balls***	***Properties of the molecules of gas***
Negative	colour	-
Positive	elasticity	perfect elasticity
Neutral	?	?

Tab. 1: Hessian analogies between billiard balls and molecules of a gas.

There is one property of the two systems, which Hesse does not mention but which could serve as the basis for the neutral analogy: the size and the rotation of the considered objects. The "real world size" and rotation of the billiard balls could be the incentive for allowing a size for the particles. This seems to be rather absurd. In no textbook or historical account one will find that van der Waals came up with his modified equation of state because billiard balls have a size and the particles of the ideal gas. However this is not what Hesse seems to imply. Rather, she seems to say that the neutral analogy consists of the applicability of Newtonian mechanics, which is assumed to hold in the case of molecules in the same fashion as in the case of billiard balls (see her quote above). In other words, the neutral analogy (=the applicability of Newtonian mechanics) would be based on the positive analogy (=the assumption that molecules of a gas behave like billiard balls in regard to their elasticity). Thus, the *only* property billiard balls and molecules share is elasticity. However, the positive analogy between the two systems is not even a perfect similarity: billiard balls are elastic but not perfectly elastic. There is a further, maybe even more severe, difficulty. On which basis does one draw an analogy between an observable and an unobservable system? On which basis, for instance, does one judge that the behaviour of unobservable gas molecules is analogous to the behaviour of billiard balls? One can not observe whether gas molecules bounce back from the wall of their container as billiard balls would do. Let me call this unsolved and so far unaddressed problem the *riddle of analogies*[11]:

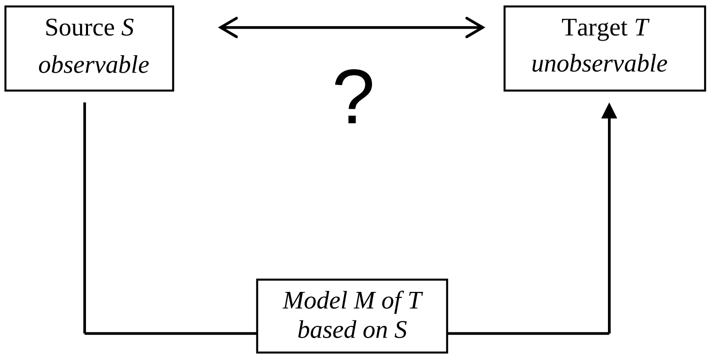

Fig. 3 The riddle of analogies. How can one draw a relation of similarity between two systems of which one is unobservable?

3.1 Analogy is IC-identity

In the last section it should have become clear that Hesse's terminology is inaccurate. Is there anything we can offer in its stead? If we consider again the physical treatment of billiard balls and molecules of an ideal gas it turns out that there is a deeper relationship between the two systems than can be captured by Hesse's terminology. It is not the case that source and target are somehow similar, i.e. that they exert some positive and some negative analogies. The physical description of the source and target system rather is based on the *identical* set of IC-entities and properties (Tab. 3).

Source System	Target System	Shared IC-system
Billiard balls	Molecules of a gas	Particles with perfect elasticity
Solar system	Bohr model of the atom	Particle revolving around particle

Tab. 2: Source and target systems and their shared IC system.

That is, the relation between the source and the target system is *not* to be sought in the real physical systems but rather in their underlying IC-systems. Billiard balls and molecules of a gas in a container are both physically and mathematically described by *imagining* particles with perfect elasticity. The solar system and the Bohr model of the atom are both physically and mathematically described by *imagining* a particle revolving around another particle. That is, the analogy between the two systems holds true because the two systems share the *identical* IC system. This is the resolution of our *riddle of analogies*: the analogy between source and target is an analogy between two imaginary systems.

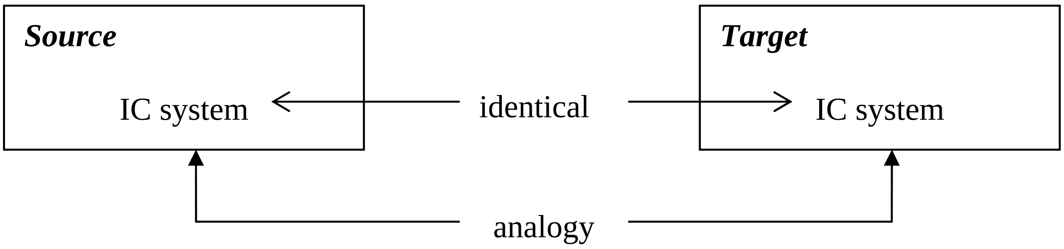

Fig. 4 The IC systems of the source and the target are identical.

It then comes as no surprise that the same theory can be used in both systems. Billiard balls and molecules can thus be treated with Newtonian mechanics and Bohr's model of the atom is to some degree classical because it consists of the identical IC system as our solar system. Thus, an electron revolving about the nucleus obeys the same laws (namely Kepler's laws) as a planet revolving about the sun[12]. Of course, the Bohr model of the atom is not identical with the, if you wish, "model" of the solar system. This is due to the fact that the particles of the Bohr model possess some *additional* imaginary properties, namely the property of having a charge, which warrants the application of electromagnetic theory (which then necessitates Bohr's quantum mechanical postulates). If we compare the descriptions of the physical systems mentioned in this section we find that the particle figures in all of them. The difference lies in the properties the particle is endowed with (Tab. 3).

Physical System	**Property /-ies of the particle(s)**
Billiard balls / molecules	Perfect elasticity
Solar system	Structural property of one particle revolving about the other
Bohr model of the atom	One particle revolving about the other / electrical charge

Tab. 3: Physical Systems and the properties of their respective IC systems.

4. Deriving ICs

4.1 The Particle

At the end of the previous section we saw that the particle served as a kind of "building block" of the IC-models. Both the billiard ball model of a gas and the Bohr model of the atom consisted of particles. Let us take a closer look at particles by considering a quote from a physics textbook written by Jones and Childers (1993):

> "By a "particle" we mean an object of negligible size and constant mass. We use the idea of a particle as a *way of creating a simplified model* of a real physical situation (132). By a particle we mean an *idealized object* so small that we can *imagine* it as a single point in space with no size and no internal structure." (footnote on page 162; my italics)

Notice that the particle is described as "a way of creating a simplified model". This description very well matches our observation in the last section: the particle is used as a kind of building block for different models. The particle itself is an *imagined* entity with a location in space but no size or internal structure, i.e. the particle is also an *idealized* entity. Size and structure are disregarded in favour of motion:

> "The two primary conditions for using the particle model are as follows:
> - the size of the actual object is of no consequence in the analysis of its motion
> - any internal processes occurring in the object are of no consequence in the analysis of its motion."
>
> (Serway and Jewitt 2002, 25)

In other words, the particle could be defined as "something moving". This gives support to my claim in Section 1 that it is not the components of the models that matter. It is rather an electron *moving* or a molecule *moving* in a particular fashion specified by the respective IC. In the first case the particle revolves about a centre, in the other a particle collides with an obstacle elastically.

4.2 IC_{AbId} and IC_{post} Systems

In the Section 3 we saw that an analogy can be drawn between the source and the target system because both systems are based on the same IC-systems. Isn't there any difference between the source and the target system? The answer is "Yes". The difference is that the IC system of the source is an abstraction and idealization from a real system, (IC_{AbId}) whereas the IC system of the target is postulated (IC_{post}). Cf. Fig. 5.

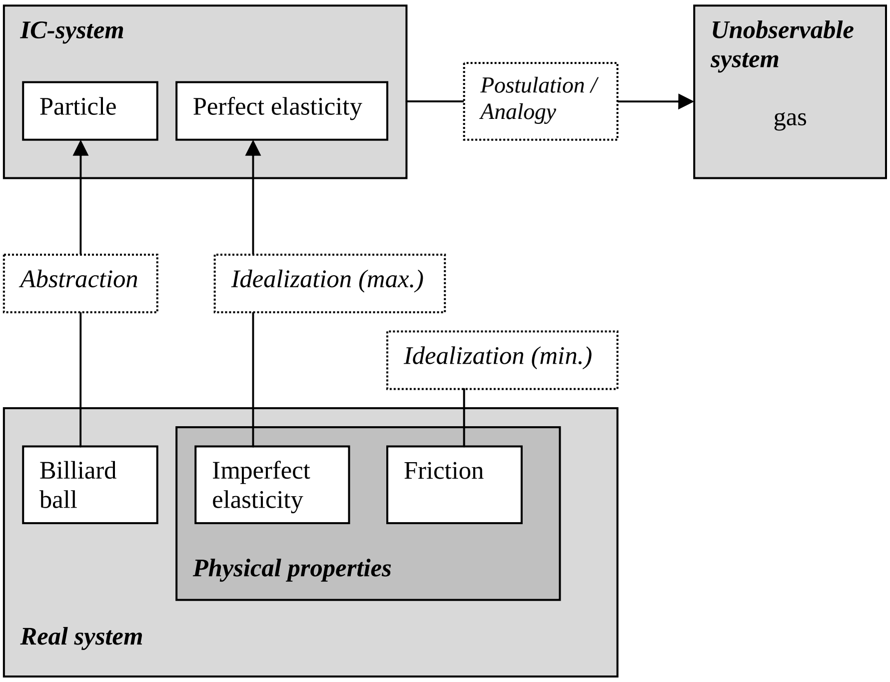

Fig. 5: Real system, IC-system, and unobservable system with the relations of abstraction, idealization, and postulation and *post hoc* analogy.

The real system consisting of the real object of the billiard ball and its real property of being imperfectly elastic is abstracted and idealized into the IC-system. Abstraction I take to be a *subtraction* in the sense of stripping away all its *irrelevant* features. In the case of the billiard ball features like colour, shape, size are abstracted away in favour for the entity's position and its mass. Idealization I take to be the *minimizing* or *maximizing* of *relevant* factors[13]. Air resistance and friction are set to zero and imperfect elasticity is maximized to perfect elasticity. Then, the gained IC system is postulated for an unobservable system. In order to reveal the basic relationship between source and target (analogous to Psillos's diagram of Hesse's account on page 6, Fig. 2) we can simplify Fig. 5 into Fig. 6, which is not only valid for the billiard ball model of a gas but *mutatis mutandis* also of the Bohr model of the atom.

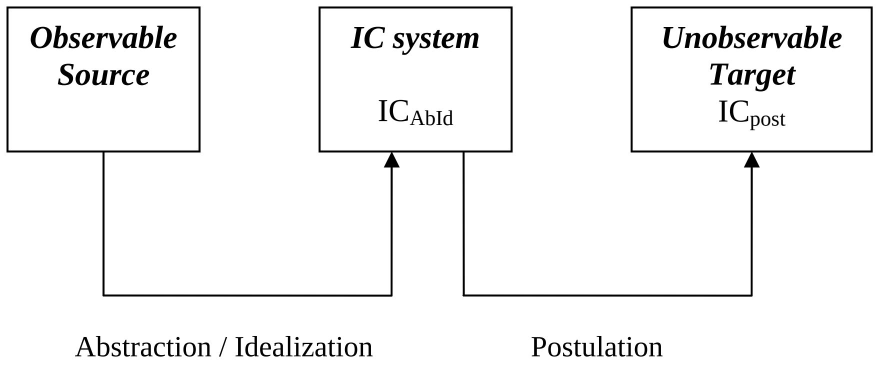

Fig. 6 Simplification of Fig. 5. IC_{AbId} is abstracted and idealized from the observable real system. The idealized and abstracted system IC_{AbId} is then postulated for the unobservable target.

That is, the ICs of the Bohr model and the Billiard ball model are both postulated but arrived at by abstracting and idealizing from an observable system.

5. Conclusion

At the beginning of this paper I promised to provide an ontology of scientific models. I showed that the idealizing assumptions of scientific models are all based on one *fundamental assumption*. This fundamental assumption refers to what I call *Imaginary-Constitutives (ICs).* The IC of the Bohr model states that "a particle revolves about a centre" and the IC of the Billiard ball model of a gas says that "a particle collides with an obstacle elastically". Although ICs cannot be found in the world but are products of our thought they are constitutive of the model. The rest of the model's assumptions function as specifications of the fundamental assumption, such as "only particular orbits are stable" or "the particles move along a straight line before they hit an obstacle".

The notion of ICs allowed us to derive a satisfactory revision of Hesse's account of analogies, discarding the latter's shortcomings. The relation between the source and the target system is not a similarity between two physical systems, as Hesse suggested, but rather an identity between two ICs, of which one is idealized from the real system (source) and the other is postulated (target).

ICs provide a useful cognitive scheme that allows us to analyse models ontologically and offers us a genuine alternative to the "formalistic" of the proponents of the semantic view and the "methodological" approach of Morgan & Morrison, Cartwright and others.

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[1] That the semantic approach needn't be construed top-down is argued by da Costa and French (2000) and (2003).

[2] The only truly systematic attempt of an ontological characterization I am aware of is Achinstein (1964). Achinstein's tripartite classification is discussed by Schindler (2005).

[3] Even in accounts on quantum mechanical models – seemingly far removed from our discussion – authors refer to the classical models (cf. instance Hartmann 1995).

[4] Due to spatial limitations, I cannot discuss Maxwell's ether model, the harmonic oscillator and the model of the DNA. Maxwell's model is a special case and rather particular to the 19th century physics and the harmonic oscillator can be easily derived from the basics, which will be discussed here. The model of the DNA is often mentioned as a paradigm case for a "scale" model but I hold that the latter is just a concretization of an IC and as such subject to the same analysis as other ICs discussed in this paper (see Schindler, forthcoming).

[5] Achinstein (1964 and 1968) put much emphasis on this observation in his characterization of "theoretical models".

[6] The images of Fig. 1 can be found in every modern textbook. It goes without saying that these line-drawings only illustrate or represent imaginary entities. Imaginary entities per se are a product of our thought.

[7] Achinstein put this thus: 'A theoretical model describes a type of object or system by attributing to it what might be called an inner structure, composition, or mechanism.' (1968, 213)

[8] "Prediction" is used in a logical sense here, not in a chronological one.

[9] Leatherdale (1974) gives the most detailed and the most complete overview of the discussion of analogies, models and metaphors in the philosophy of science. Not very much has been added to the discussion of the relationship of models, metaphors and analogies ever since. Hesse's account is still the authority in the field (cf. Psillos 1995, for instance).

[10] Instead of the nowadays common terms 'source' and 'target' Hesse used 'model$_2$' and 'model$_1$' respectively. The idea is that the similarity between source and target is a symmetrical relationship.

[11] It seems as though Hesse does not even realizes that the analogy is drawn between an observable and an unobservable system. She speaks of "observable similarities" between the

behaviour of bouncing billiard balls and "the effects of pressure on a surface due to a hail of particles […]" (Hesse 1966, 69p.; my emphasis). Only the pressure of the gas on the wall of the container is measurable. The "hail of particles" is not.

[12] For both systems the formula for centripetal acceleration $F = - mv^2/r$ applies (cf. Achinstein 1964, 341).

[13] In regard to idealization and abstraction, accounts worth mentioning are McMullin (1985), Nowak (1979), Shapere (1969), and Cartwright (1989, chapter 5).