Implications for a spatially discrete transition amplitude in the twin-slit experiment

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A discrete path integral formalism is used to obtain the transition amplitude between 'sources' (slits and detector) in the twin-slit experiment of quantum mechanics. This method explicates the normally tacit construct of dynamic entities with temporal duration. The resulting amplitude is compared to that of standard wave mechanics in order to relate 'source' dynamics and spatial separation. The implied metric embodies non-separability, in stark contrast to the metric of general relativity. Thus, this approach may have implications for quantum gravity.

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I. INTRODUCTION

According to Feynman[1], the twin-slit experiment "has in it the heart of quantum mechanics. In reality, it contains the *only* mystery." Herein we address this "mystery" by taking to heart Pauli's admonition that[2] "in providing a systematic foundation for quantum mechanics, one should start more from the composition and separation of systems than has until now (with Dirac, e.g.) been the case." Our result resonates strongly with Smolin's belief[3] that what "we are all missing" in the search for quantum gravity "involves two things: the foundations of quantum mechanics and the nature of time."

We start in section 2 by introducing a discrete path integral formalism whence quantum mechanics (QM) follows in the temporally continuous and spatially discrete limits while quantum field theory (QFT) follows in the temporally and spatially continuous limits. Per this formalism we are able to explicate the manner in which relations (as opposed to the wavefunction) may be viewed as fundamental to relata (such as particles) as suggested by our previous work on the *Relational Blockworld* interpretation[4] of QM. In section 3, we argue that the fully spatiotemporally discrete starting point is, in a sense, more fundamental than its QM and QFT limits. Then, by relating our particular discrete Lagrangian to its QM counterpart, we expose the notion of trans-temporal identity[5] employed tacitly in the construct of an action (classical or quantum). This may shed light on the "nature of time" as necessary, per Smolin, for quantum gravity. In section 4, we obtain the transition amplitude and interaction energy for an exchange of momentum between two QM 'sources', i.e., a source and detector in the parlance of QM. We use this result in section 5 to obtain the QM amplitude for the twin-slit experiment.

When we compare this amplitude to that of standard wave mechanics, we find a relationship between spatial distance and 'source' dynamics quite unlike that of Einstein's equations of general relativity (GR). In particular, the implied metric isn't an "extreme embodiment of the separability principle[6]." To wit, there are no waves, particles or fields propagating from source to detector through otherwise empty space during the exchange of momentum. Indeed, this notion of spatial distance is not defined between points of empty space, but only between 'sources'. Thus, our rendition of the twin-slit experiment necessarily circumvents "a fundamental incompatibility between general relativity and quantum mechanics[7]," i.e., QM embodies non-separability via quantum entanglement while the GR metric and its underlying differentiable manifold embody pervasive spatiotemporal separability. In this sense, QM's "only mystery" may indeed be a foundational issue relevant to quantum gravity per Smolin's suggestion.

II. DISCRETE PATH INTEGRAL FORMALISM

In QFT for a scalar field without scattering we have for the transition amplitude[8]

$$Z = \int D\varphi \exp\left[i\int d^4x\left[\frac{1}{2}(d\varphi)^2 - V(\varphi)\right]\right]. \qquad (1)$$

According to Zee, the QM counterpart then obtains in (0+1) dimensions. In the derivation of Eq. (1) *from* QM, the field φ is obtained in the continuum limit of a discrete set of oscillators q_a distributed in a spatial lattice. Any *one* of these q_i is supposed to replace φ in Eq. (1) in order that it reduce to QM. However, each q_i is fixed in space so the notion that we're integrating over all possible paths *in space* (standard treatment) from a source to a detector when we compute Z is not ontologically consistent with the fact that we integrate over all values of q but *not* over all values of the index 'i' in q_i. We rather suggest that the method for obtaining QM is to associate sources $J(x)$ with elements in the experimental set up (all of which may be deemed "sources" *and* "detectors" in a relational reality) while maintaining a discrete collection q_i(t). Thus, we want to obtain a QM situation from the spatially discrete counterpart to

$$Z = \int D\varphi \exp\left[i\int d^4x\left[\frac{1}{2}(d\varphi)^2 - V(\varphi) + J(x)\varphi(x)\right]\right]. \qquad (2)$$

More generally, we assume that the truly fundamental starting point is both spatially *and* temporally discrete so we start with[9]

$$Z = \int \ldots \int dQ_1 \ldots dQ_N \exp\left[\frac{i}{2} Q \cdot A \cdot Q + iJ \cdot Q\right] \qquad (3)$$

where A_{ij} is the discrete matrix counterpart to the differential operator of Eq. (2) while J_m and Q_n are the discrete vector versions of *J(x,t)* and q_i(t). [More on this in section 3.] The solution to Eq. (3) is

$$\left(\frac{(2\pi i)^N}{\det(A)}\right)^{1/2} \exp\left[-\frac{i}{2} J \cdot A^{-1} \cdot J\right]. \tag{4}$$

Since A_{ij} has an inverse, it has a non-zero determinant so it's composed of N linearly independent vectors in its N-dimensional, representational vector space. Thus, any vector in this space may be expanded in the set of vectors comprising A_{ij}. Specifically, the vector J_m, which will be used to represent elements in the experimental set up, can be expanded in the vectors of A_{ij}. In this sense it is clear how relations, represented by A_{ij}, can be fundamental to relata, represented by J_m.

Again, by construction, QM is the spatially discrete but temporally continuous limits of Eq. (4) in order that our action models a collection of denumerable particles q_i(t). [QFT obtains in the temporally *and* spatially continuous limits, i.e., φ(x,t).] For two coupled quantum oscillators (particles) q_1(t) and q_2(t) each of mass m with potential energy given by

$$V(q_1, q_2) = \sum_{a,b} \frac{1}{2} k_{ab} q_a q_b = \frac{1}{2} k q_1^2 + \frac{1}{2} k q_2^2 + k_{12} q_1 q_2 \tag{5}$$

where $k_{11} = k_{22} = k$ and $k_{12} = k_{21}$, our Lagrangian is

$$L = \frac{1}{2} m \dot{q}_1^2 + \frac{1}{2} m \dot{q}_2^2 - \frac{1}{2} k q_1^2 - \frac{1}{2} k q_2^2 - k_{12} q_1 q_2 . \tag{6}$$

When computing the action, integration by parts yields $\dot{q}_i^2 \rightarrow -q_i \ddot{q}_i$, so the spatially and temporally discrete version of A_{ij} in Eq. (3) would be

$$A = -\begin{pmatrix}
\frac{m}{\Delta t^2} + k & \frac{-2m}{\Delta t^2} & \frac{m}{\Delta t^2} & 0 & & \dots & 0 & k_{12} & 0 & & \\
0 & \frac{m}{\Delta t^2} + k & \frac{-2m}{\Delta t^2} & \frac{m}{\Delta t^2} & & 0 & \dots & 0 & k_{12} & & \\
 & & & \ddots & & & & & & & \\
k_{12} & 0 & \dots & \frac{m}{\Delta t^2} + k & & \frac{-2m}{\Delta t^2} & \frac{m}{\Delta t^2} & 0 & \dots & & \\
0 & k_{12} & 0 & \dots & & \frac{m}{\Delta t^2} + k & \frac{-2m}{\Delta t^2} & \frac{m}{\Delta t^2} & 0 & & \\
 & & & & & \ddots & & & & &
\end{pmatrix}. \tag{7}$$

where again Q_n denotes a single vector which must ultimately be divided into q_1(t) [first half of entries] and q_2(t) [second half of entries] in the temporally continuous limit to recover QM for Eq. (6). The process of temporal identification $Q_n \rightarrow q_i(t)$ may be encoded in the blocks along the diagonal of A_{ij} whereby the spatial division between the q_i(t) would then be encoded in the relevant off-diagonal (interaction) blocks:

$$A = -\begin{pmatrix} \boxed{\begin{matrix}\ddots & & \\ & q_1(t) & \\ & & \ddots\end{matrix}} & q_1(t) \Leftrightarrow q_2(t) & \cdots \\ q_1(t) \Leftrightarrow q_2(t) & \boxed{\begin{matrix}\ddots & & \\ & q_2(t) & \\ & & \ddots\end{matrix}} & \cdots \\ \cdots & \cdots & \boxed{\begin{matrix}\ddots & & \\ & \ddots & \\ & & \ddots\end{matrix}} \end{pmatrix}. \tag{8}$$

Thus,

$$-\left(A \cdot q\right)_1 |_{t=0} \approx m\frac{Q_1 - 2Q_2 + Q_3}{\Delta t^2} + kQ_1 + k_{12}Q_{N/2+1} \tag{9}$$

where

$$\ddot{q}_1(t=0) \approx \frac{Q_1 - 2Q_2 + Q_3}{\Delta t^2}. \tag{10}$$

In the temporally continuous limit when $Q_n \rightarrow q_i(t)$ for two particles, Eq. (7) becomes

$$A = -\begin{pmatrix} m\frac{d^2}{dt^2} + k & k_{12} \\ k_{12} & m\frac{d^2}{dt^2} + k \end{pmatrix} \tag{11}$$

so that

$$-\frac{1}{2}Q \cdot A \cdot Q \rightarrow \int \left(\frac{m}{2}q_1\ddot{q}_1 + \frac{1}{2}kq_1^2 + \frac{m}{2}q_2\ddot{q}_2 + \frac{1}{2}kq_2^2 + k_{12}q_1q_2\right)dt \tag{12}$$

which instantiates the relevant spacetime symmetries of our particular QM situation per Bohr *et al.*[10].

III. THE NATURE OF TIME: COMPOSITION

We believe the discrete view, as in Eq. (3), is fundamental to that of QM's continuous temporal and discrete spatial distribution because the form of Eq. (3) represents a more general view than a "sum over paths," which is possible when the action contains "dynamical bodies" (quantum particles) and takes a form as in Eq. (12). That since, without *a priori* dynamic constraint, it's possible to construct A_{ij} and J_m such that the phase $P(Q_n)$ cannot be interpreted as an action with distinct dynamical objects. For example, a diagonal A_{ij} has an inverse but does not represent entities with temporal duration, i.e., trans-temporal objects; dynamically, it would represent N entities with zero spacetime dimension and, therefore, no temporal extent. And, A_{ij} in a form such as that of Eq. (7) cannot be diagonalized via mere rotation, since the rows do not represent an orthogonal set. Indeed, non-orthogonality (non-zero projection between adjacent rows) is precisely what allows for a discrete formulation of acceleration. Thus, the possible stationary $P(Q_n)$ resulting from its symmetries in Q_n is a set which subsumes and exceeds stationary $P(q_i(t))$ obtained via extrema of the action, whence the classical equations of motion.

More importantly, it is clear from the discrete formulation that the QM description tacitly assumes an *a priori* process of trans-temporal identification, $Q_n \rightarrow q_i(t)$, as well as an implicit specification of spatial distribution via a restriction on coefficients in $P(Q_n)$. Indeed, there is no principle which dictates the construct of trans-temporal objects fundamental to the formalism of dynamics in general – these objects are "put in by hand." Thus, the approach herein suggests the need for a fundamental principle which would explicate the trans-temporal identity employed tacitly in QM, QFT and all dynamical theories. Since this principle restricts the form of both A_{ij} and J_m, it is likely a self-consistency relationship between what is meant by objects/substances and the spatiotemporal measurements pertaining thereto. Again, this resonates strongly with Smolin's quote in section 1 and we will see this further intimated in the analysis of the twin-slit experiment below (section 5).

IV. SIMPLE TWO-SOURCE RESULT

To obtain the QM amplitude between a single pair of 'sources' we need the spatially discrete and temporally continuous counterpart to Eq. (4). Therefore, we must find $D_{im}(t-t')$ in

$$Z(J) \propto \exp\left[\frac{-i}{2}\iint dtdt' J_i(t) D_{im}(t-t') J_m(t')\right] \tag{13}$$

where we've implied sums over repeated indices. $D_{im}(t-t')$ is given by

$$-\begin{pmatrix} m\frac{d^2}{dt^2}+k & k_{12} \\ k_{12} & m\frac{d^2}{dt^2}+k \end{pmatrix} D_{im}(t-t') = \begin{pmatrix} \delta(t-t') & 0 \\ 0 & \delta(t-t') \end{pmatrix}. \tag{14}$$

Using

$$\delta(t-t') = \int \frac{d\omega}{2\pi} e^{i\omega(t-t')} \tag{15}$$

and assuming

$$D_{im}(t-t') = -\begin{pmatrix} \int \frac{d\omega}{2\pi} A(\omega) e^{i\omega(t-t')} & \int \frac{d\omega}{2\pi} B(\omega) e^{i\omega(t-t')} \\ \int \frac{d\omega}{2\pi} B(\omega) e^{i\omega(t-t')} & \int \frac{d\omega}{2\pi} A(\omega) e^{i\omega(t-t')} \end{pmatrix} \tag{16}$$

we find

$$A = \frac{\omega^2 m - k}{k_{12}^2 - \left(\omega^2 m - k\right)^2} \tag{17}$$

and

$$B = \frac{k_{12}}{k_{12}^2 - \left(\omega^2 m - k\right)^2} \tag{18}$$

so the QM amplitude in this simple case is given by

$$Z(J) \propto \exp\left[-\frac{i}{\hbar}\iint dtdt' J_1(t) D_{12} J_2(t')\right] = \exp\left[\frac{i}{\hbar}\iiint \frac{dt'dtd\omega}{2\pi} \frac{J_1(t) k_{12} e^{i\omega(t-t')} J_2(t')}{k_{12}^2 - (\omega^2 m - k)^2}\right] \tag{19}$$

having restored $\hbar$, used $D_{12} = D_{21}$ and ignored the "self-interaction" terms $J_1 D_{11} J_1$ and $J_2 D_{22} J_2$. We can simplify the expression via the Fourier transform

$$j_2(\omega) \equiv \int J_2(t') e^{-i\omega t'} dt' \tag{20}$$

so that

$$Z(J) \propto \exp\left[\frac{i}{\hbar}\iint \frac{dtd\omega}{2\pi}\frac{J_1(t)k_{12}e^{i\omega t}j_2(\omega)}{k_{12}^2-(\omega^2 m-k)^2}\right]. \tag{21}$$

With

$$J_1(t) = \int j_1(\omega')e^{i\omega' t}\frac{d\omega'}{2\pi} \tag{22}$$

we have

$$Z(j) \propto \exp\left[\frac{i}{\hbar}\iiint \frac{d\omega' dtd\omega}{(2\pi)^2}\frac{j_1(\omega')k_{12}e^{it(\omega+\omega')}j_2(\omega)}{k_{12}^2-(\omega^2 m-k)^2}\right]. \tag{23}$$

Using Eq. (15) we find

$$Z(j) \propto \exp\left[\frac{i}{\hbar}\int \frac{d\omega}{2\pi}\frac{j_1(-\omega)k_{12}j_2(\omega)}{\left(k_{12}^2-(\omega^2 m-k)^2\right)}\right] \tag{24}$$

or

$$Z(j) \propto \exp\left[\frac{i}{\hbar}\int \frac{d\omega}{2\pi}\frac{j_1(\omega)^* k_{12}j_2(\omega)}{\left(k_{12}^2-(\omega^2 m-k)^2\right)}\right] \tag{25}$$

with J_1(t) real. The interaction energy E is then given by

$$E = \frac{-1}{T}\int \frac{d\omega}{2\pi}\frac{j_1(\omega)^* k_{12}j_2(\omega)}{\left(k_{12}^2-(\omega^2 m-k)^2\right)} \tag{26}$$

where T is the interaction time.

5. TWIN-SLIT EXPERIMENT AND SEPARABILITY

We now use the amplitude of section 4 to analyze the twin-slit experiment. There are four J's which must be taken into account when computing the amplitude (figure 1), so we will use the solution obtained in section 4 to link J_1 with each of J_2 and J_4, and J_3 with each of J_2 and J_4, i.e., $J_1 \leftrightarrow J_2 \leftrightarrow J_3$ and $J_1 \leftrightarrow J_4 \leftrightarrow J_3$. In doing so, we ignore the contributions from other pairings, i.e., the exact solution would contain one integrand with $Q_n \rightarrow q_i(t)$, i = 1,2,3,4 reflecting a discrete 'field theoretic' correction to QM. Also, we're finding interference effects while ignoring diffraction effects, i.e., the exact solution would employ two J's for each slit – one J for each edge of each slit.

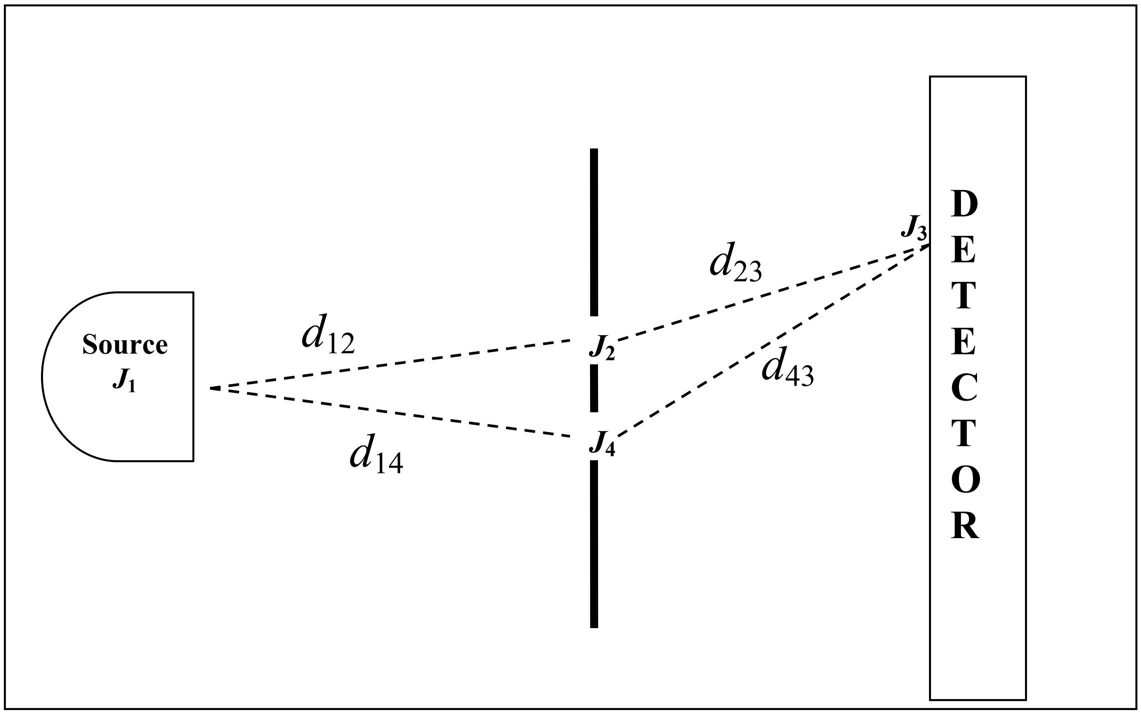

FIG. 1. Twin-slit Experiment

Finally, we assume a monochromatic source of the form $j_1(\omega)^* = \Gamma_1\delta(\omega - \omega_o)$ with Γ_1 a constant, so the amplitude between J_1 and J_2 is

$$Z(j) \propto \exp\left[\frac{i}{2\pi\hbar}\frac{\Gamma_1 k_{12} j_2(\omega_o)}{\left(k_{12}^2 - (\omega_o^2 m - k)^2\right)}\right] \tag{27}$$

whence we have for the amplitude between J_1 and J_3 via J_2 and J_4

$$\psi \propto \exp\left[\frac{i}{2\pi\hbar}(\Gamma_1 d_{12} j_2 + \Gamma_2 d_{23} j_3)\right] + \exp\left[\frac{i}{2\pi\hbar}(\Gamma_1 d_{14} j_4 + \Gamma_4 d_{43} j_3)\right] \tag{28}$$

where

$$d_{im} = \frac{k_{im}}{\left(k_{im}^2 - (\omega_o^2 m - k)^2\right)} \tag{29}$$

with ψ the QM amplitude. With the source equidistance from either slit we expect the phase $\Gamma_1\, d_{12} j_2$ equals the phase $\Gamma_1\, d_{14} j_4$ so we have the familiar form

$$\psi \propto \exp\left[\frac{i}{2\pi\hbar}(\Gamma_2 d_{23} j_3)\right] + \exp\left[\frac{i}{2\pi\hbar}(\Gamma_4 d_{43} j_3)\right]. \tag{30}$$

The interaction energy between slit J_i and detector region J_3 is then

$$E = \frac{1}{2\pi T}\frac{\Gamma_i k_{i3} j_3}{\left((\omega_o^2 m - k)^2 - k_{i3}^2\right)}. \tag{31}$$

Per standard wave mechanics the phases in the exponents of Eq. (30) differ according to the interference given by $\cos\left(\frac{p(x_{23} - x_{43})}{\hbar}\right)$ in $\psi^*\psi$ where x_{i3} is the distance from slit J_i to detector region J_3 and p is the momentum exchanged for each detector event. Thus, the phases in Eq. (30) must relate to spatial separation via

$$\frac{p}{\hbar}\left(x_{23} - x_{43}\right) = \frac{j_3}{2\pi\hbar}\left(\Gamma_2 d_{23} - \Gamma_4 d_{43}\right). \tag{32}$$

Assuming the impulse j_3 is proportional to the momentum transfer p, we have

$$g_{im} \propto \frac{\Gamma_i k_{im}}{\left(k_{im}^2 - (\omega_o^2 m - k)^2\right)} \tag{33}$$

relating the spatial separation g_{im} of the trans-temporal objects J_i and J_m to their intrinsic (m, k, ω_o) and relational (k_{im}) dynamical characteristics.

While Eq. (33) suggests a relationship between the spacetime metric and dynamics *a la* GR, g_{im} is distinct from $\overset{\Rightarrow}{g}(\vec{e}_i, \vec{e}_m)$, where $\{\vec{e}_j\}$ spans the tangent space T of the spacetime manifold and $\overset{\Rightarrow}{g} \in T^* \otimes T^*$ is the spacetime metric with T* dual to T. The metric implied by Eq. (33) is defined only between trans-temporal objects, in stark contrast to the field $\overset{\Rightarrow}{g}(\vec{e}_i, \vec{e}_m)$ which takes on values for all points of the differentiable spacetime manifold, even in regions where the stress-energy tensor is zero. Indeed, as is clear from our presentation, there is 'no thing' being displaced spatially by J_i(t) and there is no particle or wave (of momentum p or otherwise) moving 'through space' from the source to the detector, even though there is a transfer of momentum. Thus, our simple analysis of Feynman's "mystery," in accord with Pauli's dictum concerning the articulation of composition and separability, resonates strongly with Smolin's sentiment that the nature of time and QM's foundational issues may be highly relevant to quantum gravity.

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