Losing energy in classical, relativistic and quantum mechanics

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ABSTRACT

A Zenonian supertask involving an infinite number of colliding balls is considered, under the restriction that the total mass of all the balls is finite. Classical mechanics leads to the conclusion that momentum, but not necessarily energy, must be conserved. Relativistic mechanics, on the other hand, implies that energy and momentum conservation are always violated. Quantum mechanics, however, seems to rule out the Zeno configuration as an inconsistent system.

1 Introduction

As every schoolboy knows, the energy and the momentum of an isolated mechanical system do not change with time: they are conserved. This is deemed to be so, no matter whether the system is considered as a whole, or as the sum of its parts. Thus one routinely calculates the energy and the momentum of an isolated system of balls as the scalar, respectively vector, sum of the energy and momentum of the individual balls. If the collisions are elastic, the total energy and momentum remain unaffected, although their distribution among the various balls may change as time goes on.

However, it has been claimed that this textbook knowledge is not as robust as it seems. Pérez Laraudogoitia (1996) has pointed out that, if an infinite number of equally massive balls collide, one after another, the total energy and momentum can completey disappear after a finite time. The momentum is passed undiminished from one ball to another, but at the end of a finite time every ball has lost the momentum that it had acquired, and the sum of the momenta of all the balls has fallen to zero, there being no carrier for the momentum after the infinite number of collisions has taken place. The straightforward conclusion is that the equations of classical mechanics do not imply the conservation of momentum and energy, inasmuch as exceptions to the general rule apparently exist.

It would be deeply troubling if it were physically possible for energy to disappear without a trace. The question arises as to whether some feature of Laraudogoitia's model is out of bounds, relegating his result to the arcana of a mathematical curiosity shorn of physical relevance. In this connection it is important to make a conceptual distinction between the following two questions:

(1) Do the laws of mechanics entail conservation of the total momentum and energy of a spatially finite, isolated system that interacts elastically during a finite time?

(2) Do the laws of mechanics entail conservation of the total momentum and energy of such a system, when other relevant laws of physics are also taken into account?

Laraudogoitia has shown by his model that the answer to (1) is 'no'. Are there reasons for disqualifying his model as an attempt to answer (2)? Indeed there are, for the law of gravitation makes nonsense of a system containing an infinite number of balls of equal mass. In the Zeno configuration of Laraudogoitia, the balls are placed at the positions $x_n = 2^{-n}$, $n = 0, 1, 2, ...$, on a straight line, and the introduction of the gravitational interaction invalidates the model, for the infinite mass accumulating at the origin implies infinite attractive forces. However, to say that the model collapses on the adjunction of gravity is not to say that the answer to (2) is 'yes'. After all, the possibility remains open that nonconservation might occur for a different set of balls, one that is not so fragile as to collapse as soon as gravity is turned on.

To focus on such possibilities, one could tighten (1) to

(1′) Do the laws of mechanics entail conservation of the total momentum and energy of a spatially finite, isolated system *of finite mass* that interacts elastically during a finite time?

With finite total mass, a suitably constructed Zeno system could easily be arranged in such a way that gravitational effects constitute a small correction, so that their inclusion would not affect the main conclusions.

Can momentum and energy disappear if one allows the balls to have different masses, under the restriction of finite total mass? It will be proved in this paper that in *classical mechanics* the total momentum is necessarily conserved, but energy can be lost (depending on the specific model for the masses). However, in *relativistic mechanics* neither momentum nor energy is conserved whenever there is an infinite number of balls with monotonically decreasing masses, the total mass of all the balls being finite (Atkinson, 2006). So the answer to $(1')$ is still 'no'! The demise of the conservation laws cannot thus be traced to the infinite mass of Laraudogoitia's model, but has its origin elsewhere.

2 Nonrelativistic Zeno balls

Suppose that there is a ball of mass m_n at the Zeno point x_n, and that all the balls are at rest except the one at $x_0 = 1$, which at time $t = 0$ has a negative velocity, $u_0 < 0$, so it will collide after a finite time with the ball at x_1. After this collision, we suppose its velocity to be v_0.

For $n = 1, 2, 3, ...$, let u_n be the velocity of the nth Zeno ball, after it has been struck by the $(n-1)$th Zeno ball, from its right, and let v_n be its velocity after it has struck the $(n+1)$th Zeno ball, to its left. Conservation of momentum and energy at the nth collision lead to

$$\begin{aligned} u_{n+1} &= \frac{2m_n}{m_n + m_{n+1}} u_n \\ v_n &= \frac{m_n - m_{n+1}}{m_n + m_{n+1}} u_n = u_{n+1} - u_n \,. \end{aligned} \tag{1}$$

In the case that all the masses are equal, the final velocity of each Zeno ball vanishes, $v_n = 0$, and so globally neither energy nor momentum is conserved, indeed they both have vanished after a finite time, when all the collisions have taken place.

In this paper, we shall mainly be concerned with Zeno processes in which the total mass is finite:

$$M \equiv \sum_{n=0}^{\infty} m_n < \infty \,. \tag{2}$$

It can be easily checked from Eq.(1) that, if $m_{n+1} < m_n$ for all n, then $u_{n+1} > u_n$, and so $v_n > 0$, for all n. For simplicity of analysis, we shall limit ourselves to models satisfying this constraint, for if it were to break down for some n, v_n would be negative, and so the nth ball would collide for a second time with the $(n-1)$st ball, after having struck the $(n+1)$st ball. With this constraint,

$$\mu_n \equiv \frac{m_{n+1}}{m_n} < 1 \,, \tag{3}$$

and one can check that each ball, except the first, will suffer precisely two collisions. The recurrence relations (1) can be rewritten

$$\begin{aligned} u_{n+1} &= \frac{2}{1+\mu_n}\, u_n \\ v_n &= u_{n+1} - u_n \,. \end{aligned} \tag{4}$$

2.1 Geometric sequence

Let Δ be a constant, and let

$$m_n = \Delta^n \qquad \text{so} \qquad \mu_n = \Delta\,; \tag{5}$$

thus the total mass is finite if $\Delta < 1$, but infinite if $\Delta \geq 1$. From the first of the relations (4),

$$u_{n+1} = \frac{2}{1+\Delta}\, u_n\,,$$

with solution

$$u_n = \left(\frac{2}{1+\Delta}\right)^n u_0\,,$$

and from the second of the relations (4),

$$v_n = u_{n+1} - u_n = \frac{1-\Delta}{1+\Delta}\left(\frac{2}{1+\Delta}\right)^n u_0\,. \tag{6}$$

If $\Delta < 1$, both momentum and kinetic energy are conserved, for

$$\begin{aligned} P &\equiv \sum_{n=0}^{\infty} m_n v_n = \frac{1-\Delta}{1+\Delta}\left[1 - \frac{2\Delta}{1+\Delta}\right]^{-1} u_0 = m_0 u_0 \\ 2T &\equiv \sum_{n=0}^{\infty} m_n v_n^2 = \left[\frac{1-\Delta}{1+\Delta}\right]^2 \left[1 - \frac{4\Delta}{(1+\Delta)^2}\right]^{-1} u_0^2 = m_0 u_0^2\,. \end{aligned} \tag{7}$$

However, if $\Delta > 1$, momentum conservation is maximally violated, for in this case $P = -\infty$. On the other hand, perhaps surprisingly, the total kinetic energy is conserved.

The matter is interestingly different when $\Delta = 1$, for then

$$\frac{2\Delta}{1+\Delta} \qquad \text{and} \qquad \frac{4\Delta}{(1+\Delta)^2}$$

are both equal to 1, and so both series in Eq.(7) are divergent. They are however multiplied respectively by factors

$$\frac{1-\Delta}{1+\Delta} \qquad \text{and} \qquad \left[\frac{1-\Delta}{1+\Delta}\right]^2 ,$$

both of which are zero. So for $\Delta = 1$ the expressions for the total momentum and the total energy have the indeterminate form $0 \times \infty$.

2.2 Momentum conservation

It will now be proved that, if $M < \infty$ and $\mu_n < 1$, the total momentum is conserved. At a time after the first, but before the second collision of the nth ball, the balls labelled $p = 0, 1, 2, ..., n-1$ have velocity v_p, the nth ball has velocity u_n, and all the others are still at rest. The total momentum and the total kinetic energy of all the balls at such a time are given by

$$\begin{aligned} P &\equiv \sum_{p=0}^{n-1} m_p v_p + m_n u_n = m_0 u_0 \\ 2T &\equiv \sum_{p=0}^{n-1} m_p v_p^2 + m_n u_n^2 = m_0 u_0^2 , \end{aligned} \qquad (8)$$

where conservation of momentum and energy during a finite number of collisions has been used. As time goes on, more and more balls partake in the motion, and, after a finite time, they have all collided, and then

$$\begin{aligned} P &= \sum_{p=0}^{\infty} m_p v_p + \lim_{n\to\infty} m_n u_n \\ 2T &= \sum_{p=0}^{\infty} m_p v_p^2 + \lim_{n\to\infty} m_n u_n^2 . \end{aligned} \qquad (9)$$

The total momentum is conserved if $m_n u_n$ vanishes in the limit $n \to \infty$.

Suppose, *per impossibile,* that $m_n u_n$ does not tend to zero as n tends to infinity. Since the sum of all the m_n is finite, it follows that m_n must tend to zero, and therefore that u_n must tend to infinity (the m_n and u_n all being positive). However, from the second of Eqs.(8), one has $m_0 u_0^2 > m_n u_n^2$. This is impossible, since by supposition $m_n u_n$ does not tend to zero, so $m_n u_n^2$ must tend to infinity. This contradiction implies that $m_n u_n$ does tend to zero, and thus that the total momentum is conserved.

From the above proof one cannot conclude that the total kinetic energy is conserved. From the inequality $m_0 u_0^2 > m_n u_n^2$ it does follow that the only option is that of losing energy, i.e. the infinite series in the second of Eqs.(9) can only be *less* than $2T$. In the next subsection a model is given in which one half of the kinetic energy 'leaks' away at infinity, showing that the phenomenon is not a shortcoming of the method of analysis, but is actually allowed by Newton's mechanics.

2.3 Energy nonconservation

Consider the model specified by

$$m_n = \frac{2}{(n+1)(n+2)} = \frac{2}{n+1} - \frac{2}{n+2}\,, \tag{10}$$

for which the total mass is

$$M = \sum_{n=0}^{\infty} m_n = \sum_{n=0}^{\infty} \left[\frac{2}{n+1} - \frac{2}{n+2}\right] = 2\,.$$

Since

$$\mu_n = \frac{m_{n+1}}{m_n} = \frac{n+1}{n+3}\,, \tag{11}$$

it follows from the first of the relations (4) that

$$u_{n+1} = \frac{2}{1+\mu_n}\, u_n = \frac{n+3}{n+2}\, u_n\,.$$

The solution of this recurrence relation is

$$u_n = \tfrac{1}{2}(n+2)u_0\,,$$

and from the second of the relations (4),

$$v_n = u_{n+1} - u_n = \tfrac{1}{2}u_0\,,$$

which is an interesting result: each particle acquires the same velocity, after its final collision, namely one half of the initial velocity of the zeroth Zeno ball. The total momentum and kinetic energy are given by

$$\begin{aligned} P_\infty &\equiv \sum_{n=0}^{\infty} m_n v_n = \tfrac{1}{2} M u_0 = u_0 = m_0 u_0 \\ 2T_\infty &\equiv \sum_{n=0}^{\infty} m_n v_n^2 = \tfrac{1}{4} M u_0^2 = \tfrac{1}{2} u_0^2 = \tfrac{1}{2} m_0 u_0^2\,. \end{aligned}$$

The total momentum is conserved, being equal to what it was at the beginning, but the total energy is only one half what it was at the beginning. One half of the energy has been lost into the sink of the Zeno process!

3 Relativistic Zeno process

The relativistic equations for the conservation of energy and momentum just before and just after the nth collision are

$$\begin{aligned} m_n\gamma(u_n) + m_{n+1} &= m_n\gamma(v_n) + m_{n+1}\gamma(u_{n+1}) \\ m_n u_n\gamma(u_n) &= m_n v_n\gamma(v_n) + m_{n+1}u_{n+1}\gamma(u_{n+1})\,, \end{aligned} \tag{12}$$

where the Lorentz factor is

$$\gamma(u) = (1-u^2)^{-\frac{1}{2}}\,, \tag{13}$$

the speed of light having been set equal to unity. By addition and subtraction of these two equations, one obtains

$$m_n[1\pm u_n]\gamma(u_n) + m_{n+1} = m_n[1\pm v_n]\gamma(v_n) + m_{n+1}[1\pm u_{n+1}]\gamma(u_{n+1})\,,$$

and thence

$$\begin{aligned} \varepsilon^{-1}(v_n) &= \varepsilon^{-1}(u_n) + \mu_n\,[\,1-\varepsilon^{-1}(u_{n+1})] \\ \varepsilon(v_n) &= \varepsilon(u_n) + \mu_n\,[\,1-\varepsilon(u_{n+1})]\,, \end{aligned} \tag{14}$$

where the definition (3) has been used, and where

$$\varepsilon(u) = (1-u)\gamma(u) = \sqrt{\frac{1-u}{1+u}}\,.$$

On multiplying the first of the equations (14) by the second, one eliminates v_n to obtain the nonlinear recurrence relation

$$\varepsilon(u_{n+1}) = \frac{\mu_n + \varepsilon(u_n)}{1+\mu_n\varepsilon(u_n)}\,\varepsilon(u_n)\,, \tag{15}$$

on condition that $\varepsilon(u_{n+1}) \neq 1$. Since $u_0 \neq 0$, i.e. the intial velocity of the zeroth ball is not zero, it follows that $\varepsilon(u_0) \neq 1$. It will be proved below that this implies $\varepsilon(u_n) \neq 1$ for all n.

By eliminating u_{n+1} from Eqs.(14), one finds

$$\varepsilon(v_n) = \frac{\mu_n + \varepsilon(u_n)}{1 + \mu_n \varepsilon(u_n)}\,, \tag{16}$$

and by inspection of Eqs.(15)-(16) one sees that

$$\varepsilon(v_n) = \frac{\varepsilon(u_{n+1})}{\varepsilon(u_n)}\,. \tag{17}$$

For small velocities, Eqs.(15)-(17) reduce to the nonrelativistic relations (4).

3.1 Monotonicity of $\varepsilon(u_n)$

The proof that, if $\mu_n < 1$ for all finite n, and $u_0 \neq 0$, then $\varepsilon(u_n) < 1$ for all n, proceeds by induction:

1. Since u_0 does not vanish, $\varepsilon(u_0) < 1$.
2. If $\varepsilon(u_n) < 1$, then

$$\varepsilon(u_{n+1}) = \left\{1 - \frac{[1-\mu_n][1-\varepsilon(u_n)]}{1+\mu_n\,\varepsilon(u_n)}\right\} \varepsilon(u_n) < \varepsilon(u_n) < 1\,.$$

The induction is complete, and we see from the proof that the sequence $\varepsilon(u_n)$ is in fact monotonically decreasing, and this implies that it always has a limit. The strict monotonicity is a consequence of the strict inequality $\mu_n < 1$.

In fact, if $a > 0$ exists such that $\mu_n \le 1 - \frac{a}{n+2}$ for all finite n, then

$$\begin{aligned}
\varepsilon(u_{n+1}) &\le \frac{n+2-a+(n+2)\varepsilon(u_0)}{n+2+(n+2-a)\varepsilon(u_0)}\,\varepsilon(u_n) \\
&= \frac{n+\frac{2-a+2\varepsilon(u_0)}{1+\varepsilon(u_0)}}{n+\frac{2-a\varepsilon(u_0)+2\varepsilon(u_0)}{1+\varepsilon(u_0)}}\,\varepsilon(u_n)\,.
\end{aligned}$$

Iteration of this inequality leads to

$$\varepsilon(u_n) \le \frac{\Gamma\left[n+\frac{2-a+2\varepsilon(u_0)}{1+\varepsilon(u_0)}\right]}{\Gamma\left[n+\frac{2-a\varepsilon(u_0)+2\varepsilon(u_0)}{1+\varepsilon(u_0)}\right]}\,\frac{\Gamma\left[\frac{2-a\varepsilon(u_0)+2\varepsilon(u_0)}{1+\varepsilon(u_0)}\right]}{\Gamma\left[\frac{2-a+2\varepsilon(u_0)}{1+\varepsilon(u_0)}\right]}\,\varepsilon(u_0)\,. \tag{18}$$

Since $\Gamma[n+c]/\Gamma[n+d] \sim n^{c-d}$ as $n \to \infty$, it follows that the right-hand side of inequality (18) behaves asymptotically like $n^{-a\delta}$, where

$$\delta = \frac{1-\varepsilon(u_0)}{1+\varepsilon(u_0)} > 0\,.$$

Thus $\varepsilon(u_n)$ tends to zero as $n \to \infty$.

In other words, if $m_{n+1}/m_n = \mu_n \leq 1 - a/(n+2)$, where $a > 0$, then the velocity of the nth ball, after its first collision, tends to light velocity in the limit $n \to \infty$. In fact $a > 1$ is a condition of convergence of the series $M = \sum_n m_n$, according to Raabe's test. The above proof is applicable to this case, but also when $0 < a \leq 1$, which extends the conclusion to series of masses that are slowly divergent.

3.2 Nonconservation of energy-momentum

After the first, but before the second collision of the nth ball, the total energy of the balls is

$$E \equiv \sum_{p=0}^{n-1} m_p \gamma(v_p) + m_n \gamma(u_n) + \sum_{p=n+1}^{\infty} m_p = m_0[\gamma(u_0) - 1] + M\,, \tag{19}$$

and the total momentum is

$$P \equiv \sum_{p=0}^{n-1} m_p v_p \gamma(v_p) + m_n u_n \gamma(u_n) = m_0 u_0 \gamma(u_0)\,. \tag{20}$$

In the limit $n \to \infty$, Eqs.(19)-(20) become

$$\begin{aligned} E &= \sum_{p=0}^{\infty} m_p \gamma(v_p) + \lim_{n\to\infty} m_n \gamma(u_n) \\ P &= \sum_{p=0}^{\infty} m_p v_p \gamma(v_p) + \lim_{n\to\infty} m_n u_n \gamma(u_n)\,. \end{aligned} \tag{21}$$

Conservation of energy would be guaranteed if the limit term on the right of the first of these equations were to vanish. However, this never happens if the total mass is finite, for then $\varepsilon(u_n) \to 0$, as has just been shown. Accordingly, for large n the recurrence relation Eq.(15) becomes asymptotically linear,

$$\varepsilon(u_{n+1}) \sim \mu_n\, \varepsilon(u_n)\,,$$

or equivalently, since $\mu_n = m_{n+1}/m_n$,

$$\frac{\varepsilon(u_{n+1})}{m_{n+1}} \sim \frac{\varepsilon(u_n)}{m_n}\,. \tag{22}$$

In words, $\varepsilon(u_n)$ becomes asymptotically proportional to m_n. Introducing the asymptotic constant κ by

$$\frac{m_n}{2\varepsilon(u_n)} \to \kappa\,, \tag{23}$$

one obtains

$$m_n\gamma(u_n) = \tfrac{1}{2}m_n\left[\varepsilon(u_n) + \varepsilon^{-1}(u_n)\right] \to \kappa\,, \tag{24}$$

in the limit $n \to \infty$. This is the measure of the violation of energy conservation, and it is not zero. Since $u_n \to 1$ (light speed), momentum conservation is violated by the same amount.

m_n	μ_n	κ/E $u_0 = 0.1$	κ/E $u_0 = 0.5$	κ/E $u_0 = 0.9$
$(\frac{1}{2})^n$	$\frac{1}{2}$	0.0014	0.075	0.455
$(\frac{9}{10})^n$	$\frac{9}{10}$	0.0045	0.125	0.550
$\frac{2}{(n+1)(n+2)}$	$\frac{n+1}{n+3}$	0.0026	0.081	0.434

Table 1. Fraction of energy lost at infinity

The relative energy loss, κ/E, is given in Table 1 for three models. The first two rows correspond to the geometric sequence of Subsect. 2.1, with $\Delta = 0.5$ and $\Delta = 0.9$, while the third row corresponds to the model of Subsect. 2.3, but with relativistic instead of classical mechanics. The last three columns give the results for three different initial velocities, as fractions of light velocity, of the zeroth ball.

It should be noted that the relative energy loss increases as the initial velocity is increased. For very small, nonrelativistic initial velocities, the loss is insignificant, but for ultrarelativistic velocities it is large. There is incidentally no contradiction between the fact that one half of the energy is lost in the model of Subsect. 2.3, according to classical mechanics, but much less according to relativistic mechanics (see the third row of Table 1).

4 Conclusions

It has been known for some time that the conservation laws of energy and momentum can break down when one considers *infinite times,* or envisages *unbounded spatial separations* (Earman 1986). Laraudogoitia's model showed that momentum can also disappear in a spatially finite system during a finite time interval (Pérez Laraudogoitia, 1995, and Earman and Norton, 1998). In the present article it has been shown that nonconservation can occur even if the *mass* of the system is *finite.* What is essential to the mechanism of loss is that the number of subsystems be infinite.[1] Energy and momentum pass from subsystem to subsystem, and, so long as only a finite number of them have participated in the motion, the last subsystem carries the balance of energy and momentum, and both are globally conserved. After all the subsystems have been involved, which occurs for Zenonian systems in a finite time, energy and momentum can be lost, precisely because there is no last system to carry the balance away.

One could simply stop there. The answer to the question ($1'$) in Sect. 1 is 'no': momentum and energy are not in general conserved, even if the total mass is finite. Granting the truth of this assertion, one can still ask question (2) and look for other laws of physics which might rule out the Zenonian configuration and reinstate the old conservation laws. Since the successive balls would have to be smaller and smaller in order to fit into the finite length of the Zeno set, the configuration cannot be practically realized. After all, implementation would require making balls smaller than the smallest atoms, smaller even than their component particles. Yet according to our present state of knowledge, there is no law that explains the existence and properties of electrons, protons and neutrons. The Standard Model of elementary particle physics takes the existence and properties of the electron, and of the quarks and gluons that make up the protons and neutrons, to be given, not as the consequence of a law.

[1]Pérez Laraudogoitia (1999) specifies as a further requirement the convergence of the sum of the *intervals* between the balls, claiming that this does not require the sum of the *diameters* of the balls to converge. However, Zeno systems that are infinitely extended in space go beyond the class of models considered in the present paper, and in any case they would be excluded automatically in relativistic mechanics. For no rigid bodies can exist relativistically, and elastic waves can be transmitted through the balls at no more than light velocity, so the basic requirement of a finite time for completion of the process implies that the spatial extent of the system must also be finite.

However, the fact that the Zeno configuration cannot be implemented in practice is distinct from the question as to whether there is a general principle or law that *forbids* its existence. The fact that something is not possible in practice does not imply that it is impossible in principle. Quantum mechanics may well be a principled way of ruling Zeno out of court. It has been remarked that the losses in energy and momentum that were calculated are equal to one another, and indeed this is intuitively clear. For if the number of balls were large but finite, energy and momentum would be conserved, the balance being carried off by the last ball, which would have a very small rest mass. In the limit in which the number of balls is infinite, the rest mass relating energy to momentum is zero, but there is no last ball to carry the energy away. In quantum theory there is however a way in which the balance of energy and momentum could be propagated in the absence of a material carrier. It could appear as one or more quanta of light, i.e. as photons.

This way of rescuing the threatened conservation laws is strengthened by the consideration that, for a given initial velocity, u_0, the missing momentum has a well-defined value, as illustrated in Table 1. To this momentum there corresponds a de Broglie wavelength, namely the inverse of the momentum, in units in which the Planck constant is unity. A large initial velocity leads to a large but finite momentum loss from the Zeno configuration. Such a large momentum, however, corresponds to a small, but finite de Broglie wavelength, and smaller separations between successive balls than this are not well defined, according to quantum mechanics. The uncertainty principle shows that the infinite Zeno configuration does not make sense, for it is effectively truncated when the separation between the balls is about equal to the de Broglie wavelength associated with the escaping momentum. It makes little difference whether one says that the Zeno sequence of balls is terminated by the quantum mechanical lack of precision in the definition of position, or whether one claims that the missing energy and momentum are cashed out as photons, rather than in the quantally smeared-out end of the would-be Zeno sequence of balls. In either case the conservation of energy and momentum might after all be assured.

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