Why Yang-Mills theories? Paper presented to the 2006 Annual Conference of the British Society for the Philosophy of Science in Southampton.

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July 10, 2006

Abstract

The elucidation of the gauge principle "is the most pressing problem in current philosophy of physics" (Redhead 2003, 138). This paper argues two points that contribute to this elucidation in the context of Yang-Mills theories. 1) Yang-Mills theories, including quantum electrodynamics, form a class. They should be interpreted together. To focus on electrodynamics is a mistake. 2) The essential role of gauge and BRST surplus is to provide a local theory that can be quantized and would be equivalent to the quantization of the non-local reduced theory.

Contents

1 Introduction

Three of the four best theories that model fundamental interactions are of the Yang-Mills type (YM): quantum electrodynamics, quantum electroweak theory and quantum chromodynamics. At the core of each of these theory lies a local gauge symmetry of the same kind. Gauge symmetries in YM theories are peculiar. They do not have direct empirical significance (Brading and Brown 2004). They imply indeterminism at a classical level (Earman 2003). They are postulated as a principle and are parts of a priori gauge arguments. Nevertheless these are at best heuristic and/or incomplete (Martin 2002). In fact gauge symmetry, which is necessary to keep track of adding unphysical variables to a description, seems to be a useful trick, nothing more. So is there something philosophically valuable to learn from such formal symmetry? Is gauge arbitrariness really "a deep and far reaching principle" (Itzykson and Zuber 1980, 10)?

In this paper our objectives are modest. Only two points will be argued. The first one is methodological. Since all YM theories share the same structure, they should be studied together as a class. Concentrating only on quantum electrodynamics is a mistake. In the next section we will show what is exactly the common gauge structure possessed by all YM theories and what quantum electrodynamics does not share with the others. The second point is more substantial. If the gauge structure of YM theories plays any significant role it must be in the quantization process. We show in this paper that we have good reasons to believe that gauge and BRST surplus are essentially tools to get us a local theory that can be quantized and would be equivalent to the quantization of the reduced theory. So the gauge principle is important, but not because it "dictates" interactions (Ryder 1985, 81). It is a facilitator of quantization. If this is correct, the gauge principle plays a different role in YM theories and in general relativity. Note that one originality of this paper is that all discussions about quantization are made using Feynman functional formalism. Thus this paper is a nice complement to (Earman 2003) who advocates for the constrained Hamiltonian formalism.

2 Common aspects of the gauge structure of YM theories

In their seminal paper, Yang and Mills, while discussing the possibility of a $SU(2)$ YM theory, assert one of the fundamental aspect of YM theories:

> [W]e wish to explore the possibility of requiring all interactions to be gauge invariant under *independent* rotations of the isotopic spin at all space-time points, so that the relative orientation of the isotopic spin at two space-time points becomes a physically meaningless quantity. (Yang and Mills 1954, 192, original italics)

According to Yang and Mills, local "phase" (in this case isospin) is not a meaningful physical quantity. This passage is often interpreted in a passive way but the active version follows. Furthermore, in YM theories a local property, when understood as the local value of a field, is not physical. This can be deduced from a paper from Wu and Yang. When discussing $U(1)$ YM theory (quantum electrodynamics) they write: "[Quantum] electromagnetism is thus the gauge-invariant manifestation of nonintegrable phase factor" (Wu and Yang 1975, 3846). This is the signature of a YM interaction and can be generalized to any YM theories. However nonintegrable phase factors, when calculated using the gauge potential as a connection, are not in general gauge invariant. In fact, in general only phase factors associated with the electromagnetic interaction and measured for a closed loop are gauge invariant. This is equivalent to noticing that only the field tensor of electrodynamics is gauge invariant. So electrodynamics is not the norm but the exception, even if its interaction is clearly in the YM class. The non-local aspect of gauge invariant phase factors is incompatible with a strict local ontology, where only properties of points have a physical meaning. If this is correct, we have to say that classical electromagnetism (a local theory) and the classical $U(1)$ YM theory, called also electrodynamics, are in fact very different theories. This point is already present in (Belot 1998). We will return to this in the next section.

Let us characterize the gauge structure of YM theories. Following (DeWitt 2003), in this paper we will use a condensed notation. To simplify further the exposition we will consider only theories where matter is represented by bosonic fields. Components of gauge potentials $A^a_\mu(x)$ and matter fields are represented generically by φ^i. Each index is implicitly understood to include a space-time point. The action S is a functional $S : \Phi \to \mathbb{R}$, where Φ is the space of all field histories over space-time. The full description of Φ generally requires an atlas, a collection of charts of which the φ^i are the coordinates.

The gauge invariance implies the existence, on Φ, of a set of vector fields $\mathbf{Q}_\alpha$ that leave the action invariant: $\mathbf{Q}_\alpha S \equiv 0$. In coordinates $S_{,i} Q^i_\alpha \equiv 0$, where $,i$ represents functional differentiation. The index α has also both a discrete and continuous (space-time) part. For YM theories Lie brackets of the $\mathbf{Q}$'s depend linearly on the $\mathbf{Q}$'s themselves: $[\mathbf{Q}_\alpha, \mathbf{Q}_\beta] \equiv -\mathbf{Q}_\gamma c^\gamma_{\alpha\beta}$. Using primes to distinguish the points associated with various indices: $c^{\gamma''}_{\alpha\beta'} = f^\gamma_{\alpha\beta} \delta(x'', x) \delta(x'', x')$, where $f^\gamma_{\alpha\beta}$ are the structure constants of the typical fibre of the YM principal bundle. As we can see the gauge constraint on what can be a YM theory is strong. This again suggests that YM theories form a "natural" class.

From these definitions we can deduce that the action remains invariant under infinitesimal changes of φ^i of the form $\delta\varphi^i = Q^i_\alpha \delta\xi^\alpha$, where $\delta\xi^\alpha$ are arbitrary φ independent coefficients. The group G of transformations generated by the infinitesimal transformations is called the gauge group or more precisely proper gauge group. In this paper we will put aside the notion of full gauge group which is obtained by appending to the proper gauge group all other φ independent transformations of Φ onto itself that leaves S invariant and does not arise from global symmetries.

For the discussion that follows it is convenient to make, at least conceptually, the transformation $\varphi \to I^A, K^\alpha$, where I's label points in the space of gauge orbits Φ/G (considered as fibres) and are gauge invariant $\mathbf{Q}_\alpha I^A \equiv 0$. The K's label points within each fibre (gauge orbit). I^A, K^α constitute a fibre adapted system of coordinates of Φ. As usual for a fibre bundle construction there is no canonical way of associating points on one fibre with those on another.

One often chooses a reference for K's. Usually this consists in singling out a base point φ_* in Φ and choosing the K's to be local functionals of the φ's such as the matrix $\hat{\mathcal{F}}^\alpha_\beta := \mathbf{Q}_\beta K^\alpha = K^\alpha_{,i} Q^i_\beta$ is a non singular differential operator at and in the neighborhood of φ_*. In the region where the operator $\hat{\mathcal{F}}$ is not singular it can be shown that $\frac{\delta}{\delta K^\alpha} = -\hat{\mathcal{G}}^\beta_\alpha \mathbf{Q}_\beta$, where $\hat{\mathcal{G}}$ is a Green's function of $\hat{\mathcal{F}}$. As expected vertical fields are generated by gauge transformations. In principle we can also make a specific choice for I's but in practice theses choices depend non-locally on φ^i. We will return to this in the next section.

YM theories are not the only theories where the associated Φ is structure in a fibre way. But the manner in which the fibre bundles are structured through the action action of specific $\mathbf{Q}_\alpha$ is the signature of YM theories. Quantum electodynamics, quantum electroweak theory, and quantum chromodynamics admit respectively $U(1)$, $SU(2) \times U(1)$ and $SU(3)$ as proper gauge groups. Only electrodynamics has an abelian group. Its specificity comes from this fact. By focusing on the tree of electrodynamics, however interesting it would be to explain gauge puzzling phenomenon like the Aharonov-Bohm effect, we are missing the forest.

3 Quantization of Yang-Mills theories

It is useful to compare and contrast the gauge structure between classical YM theories and other classical gauge theories. But to limit analyses to this kind of work is missing an essential point: only quantized versions of YM theories have been applied in experimental contexts. In a strict sense, electromagnetism is not the classical version of quantum electrodynamics. Electromagnetism is a theory of localized charges interacting through the electromagnetic field. The gauge structure comes only from the fact that the field-strengh tensor can be expressed using an infinite class of distinct gauge potentials. In quantum electrodynamics, understood as a YM theory, matter is described by a field. The gauge structure is the result of the particular coupling between matter and gauge potential. Contrary to electromagnetism both fields are changed in a gauge transformation. Why then, are these two different theories are considered as describing the same interaction? First, their gauge groups are isomorphic. And second, there is an intermediate theory between them: a quantized particle interacting with an unquantized gauge potential. In the case of non-abelian YM theories no such classical version has any experimental application. Here again electrodynamics is the exception.

This said, a difficulty arises. Many quantization procedures are available and we do not have a proof that they are all equivalent. In this paper the Feynman sum over histories method will be privileged. This global method of quantization, which is not well known in philosophy, has been chosen for three main reasons: 1) It is more attuned with the lessons of special relativity than canonical techniques since it does not depend on (3+1) dimensional baggage of conjugate moments and constraints. 2) This global approach provides an elegant transition between classical and quantum systems. 3) More importantly, it is the quantized versions of YM theories obtained by using this method that have been tested by experiments. At least in the context of YM theories, we can be confident in this method of quantization.

3.1 Non-relativistic quantum mechanics

Most philosophical discussions about quantized gauge theory are in the framework of nonrelativistic quantum mechanics and are about the simplest case: quantized particles interacting with an unquantized electromagnetic potential. A local gauge transformation of the potential and of the wave function (represented in Feynman formalism by the propagator) is a symmetry of the system (Aitchison and Hey 1989, 50). In this case the Feynman propagator takes the form of a sum over the possible trajectories from x to y, $K(y,x) = \int D(\vec{q}(t))e^{iS[\vec{q}(t)]}$, where S is the classical action associated with a path. The action of the electromagnetic interaction is to multiply the contribution of each path q by a nonintegrable phase factor $U(y,x) = e^{-ie\int_q A_\mu dx^\mu}$ (Wilson line). Recall that this is, according to Wu and Yang, characteristic of a YM type of interaction. Note that a Wilson line is not in general gauge independent but the relative change of phase between paths, caused only by electromagnetic interaction, is gauge invariant. In other words, Wilson loops $U(x,x)$ are gauge independent. Since in the Feynman formalism the phenomena is the result of the interference between contributions of different histories, it is tempting to attribute physical significance to Wilson loops.[1] In this case the gauge structure is the result of the freedom in producing local descriptions compatible with nonlocal entities: Wilson loops. Could this explanation be generalized to relativistic cases? We will see in the next section that it is not that simple.

3.2 Relativistic quantization: the general problem

There is a limit to what can be deduced from non-relativistic mechanics. If the focus of your study is theories modelling fundamental interactions, to do without special relativity is not an option. However, even after having chosen Feynman quantization, more than one path can bring us from a classical YM theory to its quantum version. Among them: 1) first reduce the gauge surplus then quantize in order to obtain a reduced Hilbert space, presumably the "physical" Hilbert space. 2) Get the quantum theory by quantizing an extended theory built by imposing BRST symmetry. At least in the perturbative regime the second strategy has been empirically very successful. Nonetheless, even if it has not been fully carried

[1] Note that, in the non-abelian case, the Wilson line is defined as $U(y,x) = P\left\{e^{ig\int_0^1 ds \frac{dx^\mu}{ds} A^a_\mu(x(s))t^a}\right\}$ where P is a prescription of path-ordering. The associated gauge independent Wilson loop is the <u>trace</u> of $U(x,x)$.

out, the first strategy seems more philosophically satisfying since the physical variables are identified. Both strategies will be discussed in this paper. But first a few words on gauge fixing.

A third way to quantize a YM theory would be to choose a gauge and than quantize. Fixing the gauge can be achieve by adding a covariant gauge breaking term to the Lagrangian, for example $\frac{1}{2\xi}(\partial^\mu A_\mu^a)^2$. It is well known that in the case of a non-abelian YM theory the obtained quantum theory is not unitary. It is believed the gauge condition is failing to force the system to be on an hypersurface in Φ that meets each of the gauge orbits exactly once. Equivalently, it is possible to rigorously prove that it is impossible in these cases to define coordinates K^α serving as global coordinates. This means that $\hat{\mathcal{F}}$ cannot be non-singular globally.This result is called the Gribov ambiguity (DeWitt 2005, 23).[2] This seems to exclude the possibility of defining a true global gauge. At best we can define arbitrary local gauges (local coordinates systems) which is an approach aligned with the original (Yang and Mills 1954). Since simple gauge fixing is not available let us explore the other possibilities.

3.2.1 Reduction of phase space method

The functional integral associated to any transition amplitude between "in" and "out" states takes the general form

$$\langle out|in\rangle = \int e^{i\dot{S}[I]}\dot{\mu}_I[I][dI], \quad [dI] := \prod_A dI^A \tag{1}$$

where $\dot{S}$ is the classical action S plus all counter terms that will be needed to render the amplitude finite and $\dot{\mu}_I$ is the functional measure necessary for the normalization of the integral.[3] The integral runs over distinct field histories, thus over I. In this form the integral is too abstract to be very useful. A particular set of variable I's has to be identified in order to proceed further. Since YM theories have no classical application, no classical experiments can help us to choose particular I's. However we can notice that all readily available I's depend non locally on the

[2]For a simpler model than full YM theories, the Gribov ambiguity has been understood to signal the existence of inequivalent reductions to a physical quantum theory (McMullan 1994). However it is not clear that this result is independent of the quantization procedure adopted, in this case Dirac constrained quantization.

[3]Note that to render the equation more transparent we omitted the sum over homotopy equivalence classes and sums over parameters compatible with boundary conditions. These are not essential for the rest of the discussion.

φ^i. This observation is important because it suggests that the gauge independent structure of YM theories is incompatible with a straightforward interpretation of the Humean supervenience defended by David Lewis. "[T]he doctrine that all there is to the world is a vast mosaic of local matters of particular fact" (Lewis 1986, ix). YM theories seem incompatible with a strict point ontology, which is surprising for a field theory.

Among the possible choices of specific I's, Wilson loops is a possibility. But this is not the unique choice. For example, if the asymptotic boundary conditions are empty Minkowskian, then one can introduce gauge invariant line integrals coming from infinity. For other conditions other choices can be proposed. How can we choose among these gauge invariant variables? Do we have to choose? A enlightening lesson can be drawn from a simpler case. In classical electromagnetism the interaction field can be described locally by the electromagnetic field $F_{\mu\nu}$ or non-locally by Wilson loops $e^{-ie\oint A_\mu dx^\mu}$. How do we choose the local description over the non-local one even if they are equivalent? Based on at least two reasons: 1) experimental applications are more easily explained by a local bearer of force. 2) If energy is conserved locally, $F_{\mu\nu}$ seems better suited as bearer of energy. This example illustrate that in order to choose among equivalent descriptions we need exterior constraints, experimental or theoretical. For YM theories we lack these resources. No classical applications are known and it is not clear what ontological commitment is suggested by relativistic quantum experiments. Since all available reductions are non-local, a locality commitment cannot be invoked. All we can say is that the non-relativistic case discussed above pushes us toward Wilson loops. But this is not much.

In a recent paper (Healey 2001) argues efficiently against any kind of connection substantialism and indirectly against the notion of true gauge. It is our opinion that these notions are for a big part illusions caused by a misunderstanding of the principal fibre bundle formalism. If the gauge groupoid formalism had been used from the beginning such theses would probably not have appeared. For more information see (MacKenzie 1987). Nevertheless we are puzzled by Healey's insistence on defending the concept of holonomy as physical. Of course holonomies are a better construction than the connection to represent interaction in YM theories. At least in this context the non-locality is obvious. But elements of an holonomy group are not gauge invariant for non-abelian YM theories. Thus non-physical variables are still present in the description. Choosing gauge potential or holonomy is a matter of convenience not of ontological commitment.

After choosing particular I's, another serious difficulty is the evaluation of the measure functional $\dot{\mu}_I[I][dI]$. In a system without gauge surplus, $\mu[\varphi][d\varphi]$

plays the role of a volume density in the space of field histories. To give an explicit expression for $\mu[\varphi]$ usual technics rely heavily on locality conditions. For example, $\frac{1}{\mu[\varphi]}\frac{\delta\mu[\varphi]}{\delta\varphi^i}$ should depend only on the properties of φ in the immediate vicinity of the spacetime point associated with i. Because of this when we deal with non-local variables, quantization is much more complicated, even at the level of approximation. Thus we have a compelling reason to adopt a local description, even if it implies adding unphysical variables. But note, compelling reasons are not necessities. Nothing in equation 1 forbids future developments of the method.

3.2.2 BRST symmetry method

As we said the BRST method has been empirically very successful. This success alone should have been sufficient to incite philosophers to study this method. Why this is not the case we are not sure.

In two words, BRST method consists in imposing a new *global* symmetry on Yang-Mills theories and then quantizing. In practice it consists of adding a gauge breaking term in the action and other dynamical terms that involve new unphysical fields: ghost, antighosts, and an auxiliary field. For reasons of concision, in the rest of the paper all these unphysical fields will generically be called ghosts. If this new theory is quantized the quantum theory obtained is unitary and renormalizable (Becchi, Rouet, and Stora 1976). Of course what is really puzzling is that to get rid of a gauge surplus we *add* another surplus that is apparently even more bizarre than the first one since the parameter of the BRST symmetry is non-commutative. Thus BRST symmetry has no simple classical interpretation, such as the one based on local/non-local description that we propose earlier for gauge symmetry.[4] Presented this way, the BRST method looks mysterious. It apparently gives a "Platonist-Pythagorean role for purely mathematical considerations in theoretical physics" (Redhead 2003, 138). This would be plausible if you could prove that the BRST method is incompatible with the reduced space method. What we will show is that this conclusion is not justified. The origin of the BRST surplus arises entirely from the fibre bundle structure of Φ. To get the ghosts it is not necessary to integrate over the gauge group as it is done in the usual presentation.

The use of unphysical variables to keep under control the gauge surplus can be surprising when we think in a classical physics framework, but it is much less so in quantum physics. Since BRST is a global symmetry it implies, by Noether's

[4] A nontrivial interpretation is that the BRST construction provides a cohomological description of the Poisson algebra of the reduced system (Tuynman 1992).

first theorem, the existence of a conserved charge. The operator associated with this fictitious charge is nilpotent. Its action divides nicely the extended Hilbert space into physical and unphysical subspaces of states (Kugo and Ojima 1978). Adding more structure can be a good strategy after all.

Note that the following discussion relies greatly on (DeWitt 2005). At least formally we know that the right way to quantize is equation 1. Unfortunately working directly with non-local I's is difficult. To bring the local variables φ^i into the integral one must first introduce the remaining variables K^α of the fibre adapted coordinates of Φ. Only then one will be able to transform coordinates I, K's to φ's. Let $\Omega[I,K]$ be a real scalar functional on Φ such that the integral

$$\Delta[I] := \int e^{i\Omega[I,K]}\mu_K[I,K][dK], \quad [dK] := \prod_\alpha dK_\alpha \tag{2}$$

exists and is non-vanishing for all I. For $\Delta[I]$ to be invariant under changes (generally I dependent) of the fibre adapted coordinates K^α, the measure must transform like $\mu_{K'}[I',K'] = \mu_K[I,K]\frac{\delta(K)}{\delta(K')}$. Using Δ we can write

$$\langle out|in\rangle = \int [dI] \int [dK] \dot{\mu}_{I,K}[I,K]\Delta[I]^{-1} e^{i(\dot{S}[I]+\Omega[I,K])}, \tag{3}$$

where $\dot{\mu}_{I,K}[I,K] := \dot{\mu}_I[I]\mu_K[I,K]$. To pass from I, K to the local φ one must include the Jacobian $J[\varphi] = \frac{\delta[I,K]}{\delta[\varphi]}$. So equation 3 becomes

$$\langle out|in\rangle = \int [d\varphi]\dot{\mu}_{I,K}[\varphi]\Delta[\varphi]^{-1}J[\varphi]e^{i(\dot{S}[\varphi]+\Omega[\varphi])}, \quad [d\varphi] := \prod_i d\varphi^i, \tag{4}$$

In this form the measure of the integral is not well defined. But if we study how $J[\varphi]$ behaves under coordinates changes of K^α, we can observe that $J det\hat{\mathcal{G}}$ only depends on I's and hence is gauge invariant. Moreover this product transforms as a scalar density of unit weight under transformations of the coordinates φ^i (DeWitt 2003, chapter 10). This product is thus an essential element for building the functional measure.

Now if we place ourselves in the context of loop expansion (the context in which ghost fields were used), we can pretend that the K^α can be global coordinates. In other words the K^α are coordinates of the tangent space. In this case an interesting choice for Ω is $\Omega := \frac{1}{2}\kappa_{\alpha\beta}K^\alpha K^\beta$, where $\kappa_{\alpha\beta}$ is a symmetric ultralocal invertible continuous matrix.[5] Since we are staying in a single chart we can

[5] A continuous matrix is ultralocal if it is of the form $\gamma_{\alpha\beta}\delta(x,x')$ and does not contain a differentiated δ function.

choose $\mu_{I,K}[K] = 1$ then $\Delta = const \times (det\kappa)^{-1/2}$. Equation 4 takes the form

$$\langle out|in\rangle = \int [d\varphi]\dot{\mu}[\varphi](det\hat{\mathcal{G}})^{-1} e^{i(\dot{S}[\varphi]+\frac{1}{2}\kappa_{\alpha\beta}K^{\alpha}K^{\beta})}, \quad (5)$$

where $\dot{\mu}[\varphi] = const \times \dot{\mu}_I[\varphi](det\kappa)^{1/2} J[\varphi] det\hat{\mathcal{G}}$. This new measure is to be used when the integration is carried out over the whole space of histories Φ rather than just over the base space Φ/G.

Two remarks about equation 5. 1) There is now a gauge breaking term in the exponent of the integrand. It appeared naturally to guarantee the good behaviour of the integral when summing over K's. 2) A factor $(det\hat{\mathcal{G}})^{-1}$ appears in the integrand. It is this factor that gives rise to all ghosts loops in loop expansion (DeWitt 2003, chapter 24). This derivation shows that the surplus of structure of BRST construction was expected. It is the result of the fibre structure of Φ when defined with local fields and from the Jacobian of the transformation from I, K's to φ's.

To quantize YM theories is a tricky business. It is not sufficient to produce a local version of the the theory. This local description must be built in such way that its quantization is faithful to what would be a quantized version of the theory expressed in physical non-local variables. Does this mean that we should have included ghosts in the definition of Φ? We do not think so. Equation 5 suggests that the two kinds of unphysical variables, gauge and ghost, play a different role in the quantization process. Ghosts are the result of the use of gauge variables. The reverse does not seem true. Gauge variables are doing the localizing job. Ghosts are assuring equivalence between quantum theories.

4 Conclusion

We have shown that YM theories should be discussed together. It is possible to conceptually define quantization of a YM theory expressed only in the language of its physical nonlocal variables, but for practical reasons a gauge structure is preferable. The role of the added unphysical variables is precisely to give a local version of the theory. Still, to achieve commutation between quantization paths more unphysical variables (ghosts) are needed.

In conclusion gauge symmetries in YM theories are not as mysterious as it is usually thought. The gauge principle is an important step of the quantization process. The real mystery lies behind the gauge structure. How should we characterize physical non-local variables, the ones that have been forgotten all along?

Already some interesting discussions can be found in the literature, for example (Belot 1998) or (Healey 2001), but all of them build on electrodynamics, which is problematic since this theory is the exception in YM theories.

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