The present moment in quantum cosmology: Challenges to the arguments for the elimination of time

Lee Smolin*

Center for Gravitational Physics and Geometry
Department of Physics
The Pennsylvania State University
University Park, PA, USA 16802
and
The Blackett Laboratory
Imperial College of Science, Technology and Medicine
South Kensington, London SW7 2BZ, UK

August 30, 2000

ABSTRACT

Barbour, Hawking, Misner and others have argued that time cannot play an essential role in the formulation of a quantum theory of cosmology. Here we present three challenges to their arguments, taken from works and remarks by Kauffman, Markopoulou and Newman. These can be seen to be based on two principles: that every observable in a theory of cosmology should be measurable by some observer inside the universe, and all mathematical constructions necessary to the formulation of the theory should be realizable in a finite time by a computer that fits inside the universe. We also briefly discuss how a cosmological theory could be formulated so it is in agreement with these principles.

This is a slightly revised version of an essay published in Time and the Instant, Robin Durie (ed.) Manchester: Clinamen Press, 2000

*smolin@phys.psu.edu

Contents

1 Introduction

There is a general agreement that the notion of time is problematic in cosmological theories. During the last years discussions about time in cosmology have tended to focus on the question of whether time can be eliminated from the fundamental statement of the laws and principles of the theory. The argument that time can be eliminated has been put forward in its strongest form to date by Julian Barbour[1, 2]. I am convinced by Barbour's argument. If a quantum cosmological theory can be formulated along the lines contemplated by many people in the field it follows, as he has argued, that time can be eliminated from the theory. One of my first tasks here will be to outline the logic of the argument and show how that it does indeed follow from premises often assumed when talking about quantum cosmological theories.

However, before accepting the conclusion we should ask whether the premises are correct or not. The main part of my argument will be that there are strong reasons to doubt the correctness of several of the key premises. These come from the unjustified reliance of the proposed quantum theories of cosmology on mathematical structures that have no relevance for the representation of observations that can be made by real observers inside the universe. Once these are eliminated it is no longer possible to make Barbour's argument. However it also becomes necessary to find a new way to formulate a fundamental theory of cosmology that does not allow the introduction of formal quantities which are observables in principle but unobservable in practice.

Thus, the aim of my essay is to argue that the problem of time is genuine, but that its resolution requires, not the elimination of time as a fundamental concept, but instead a reformulation of the basic mathematical framework we use for classical and quantum theories of the universe.

This reformulation is in any case necessary in order to bring our theories into line with what working cosmologists actually do when they compare theory and observation. Real practice requires a theory that satisfies the following two principles.

- *Every quantity in a cosmological theory that is formally an observable should in fact be measurable by some observer inside the universe[3, 4, 5, 6, 7, 8].*
- *Every formal mathematical object in a cosmological theory should be constructible in a finite amount of time by a computer which is a subsystem of the actual universe[9, 10, 11, 12].*

These principles have been enunciated before, in the cited references and elsewhere, but their implications have perhaps not been sufficiently explored. What I aim to show here is that they are in contradiction with the argument for the elimination of time. Further, they imply that time will play an even more central role in the formulation of fundamental physical and cosmological theories.

My argument will have four parts. In the first, I review the standard constructions used to formulate classical and quantum cosmological theories and show that, in combination with the two principles just quoted, they imply five postulates. In the second I sketch the basic argument why time is not a fundamental concept in theories which satisfy the five postulates. I believe that this account strengthens Barbour's claim that time is not a fundamental concept in theories that satisfy these postulates. In the third part I will give three objections to the argument, which attack one or more of the postulates. In the final part I mention some features that a dynamical theory may have if it is to be consistent with these two principles.

In order to make the structure of the argument clear what follows is only a quick sketch of the argument. There is much more that could be said on the subject[1]. However my view is that this question is not one that can be settled by philosophical argument alone. Were that possible the problem would already have been resolved. The point of my argument is that the problem of time both requires and points to a chance in the structure of our physical theories. What needs to be done next is to see if theories of this kind can be constructed and if they may also help to resolve other issues in physics such as quantum gravity and the foundations of quantum theory. I believe that the answer is likely yes, but this is something that cannot be argued for, it must be tried to see if it succeeds or fails.

Very little of my argument is original[2]. Julian Barbour has greatly strengthened the argument for the elimination of time in cosmological theories, and the argument here was invented mainly in reaction to his recent papers[1] and book[2]. I disagree with his conclusions but see no escape short of the kind contemplated here.

The arguments against the elimination of time I present here are due to Stuart Kauffman[12, 10], Fotini Markopoulou[4, 5, 6] and Ted Newman[18]. Discussions with a number of other people, especially Jeremy Butterfield, Saint Clair Cemin, Louis Crane, Fay Dowker, Chris Isham, Louis Kauffman, Jaron Lanier, Seth Major, Carlo Rovelli, Simon Saunders and Rafael Sorkin have been crucial for my understanding of these issues.

Finally I must stress that both the positive and negative steps of my argument do not depend very much on the details of the cosmological theory under consideration. They apply as strongly in string theory as in quantum general relativity and they appear in all versions of these theories so far known.

1.1 Some questions

I would like to preface my argument with several basic questions about cosmology.

- What is an observable in a cosmological theory, based on general relativity? What is the actual physical content of the theory and how can we separate it out from its mathematical presentation?
- What will be an observable in the ultimate theory of quantum cosmology?

[1]Other implications of these issues are explored in [15, 16].

[2]The title is taken from a debate with Jaron Lanier, held in Brooklyn, New York, April 3, 1999, under the auspices of Universitat Universalis.

- Is novelty possible? Does general relativity allow the existence of entities that do not exist at some time, but exist at some other time (where a moment of time is identified with a spacelike surface)?
- Will the ultimate quantum theory of cosmology allow the possibility of novelty?

The importance of the first question is that presently we can give a formal characterization of observables in general relativity, but we are actually not able to explicitly construct many examples of quantities that satisfy it. This stems from the fact that general relativity does not, as is often said, identify the history of a physical universe with a manifold on which are defined a metric and perhaps other fields. The correct statement is that the history of a universe is defined by an *equivalence class of manifolds and metrics under arbitrary diffeomorphisms*[3].

This is a key point, the significance of it is still often overlooked, in spite of the fact that it is far from new[4] One major consequence of it is that there are no points in a physical spacetime. A point is not a diffeomorphism invariant entity, for diffeomorphisms move the points around. There are hence no observables of the form of the value of some field at a given point of a manifold, x.

If observables do not refer to fields measured at points what in the world do they refer to? We have to begin with only the characterization that an observable must be a functional of the fields on a manifold, including the metric, which is invariant under the action of arbitrary diffeomorphisms.

This is easy enough to state. It is harder, but still possible, to describe a few observables in words. For example the spacetime volume is an observable for compact universes. So is the average over the spacetime, of any scalar function of the physical fields[5]. If the theory contains enough matter fields one can attempt to use the values of some of the matter fields as coordinates to locate points in generic solutions. Once points are labeled by fields, the argument goes, they have a physical meaning and one can then ask for the values of other fields at those points.

There are however several unsolved problems that make it doubtful that this is a satisfactory way to describe the observables of the theory. Past the first few simple ones such as spacetime volume we do not know whether the others are actually well defined on the whole space of solutions to Einstein's theory. For example all attempts made so far to use the values of some physical fields as coordinates on the space of solutions fail because of the unruly behavior of the fields in generic solutions. As a result, we have control over only a

[3]A diffeomorphism is in this context a map of a manifold to itself that preserves the notion of infinitely differentiable functions. Thus, it moves the points around, but in a way that takes differentiable functions to differentiable functions. It thus preserves relationships between functions that can be described by coincidences of values at points

[4]The original argument for the identification of the physical spacetime with a diffeomorphism equivalence class of metrics is due to Einstein and is called the *hole argument.* It was strengthened by Dirac, Higgs, Bergman, DeWitt and others. It is discussed in many places including [13, 14, 15, 16].

[5]Where the average is taken using the volume element defined by the spacetime metric.

handful of observables. But we need an infinite number of observables if we are to use them to distinguish and label the infinite number of solutions of the theory.

Finally, we must ask if any of the observables are actually measurable by observers who live inside the universe. If they are not then we cannot use the the theory to actually explain or predict any feature of our universe that we may observe. If we cannot formulate a cosmological theory in terms that allow us to confront the theory with things we observe we are not doing science, we are just playing a kind of theological game and pretending that it is science. And the worrying fact is that none of the quantities which we have control over as formal observables are in fact measurable by us. We certainly have no way to measure the total spacetime volume of the universe or the spacetime average of some field.

The reader may ask what relativists have been doing all these years, if we have no actual observables. The answer is that most of what we know about general relativity comes from studying special solutions which have large symmetry groups. In these cases coordinates and observables can be defined using special tricks that depend on the symmetry. These methods are not applicable to generic spacetimes, furthermore there is good reason to believe that many observables which are defined for generic spacetimes will break down at symmetric solutions.

Thus, relativists have sidestepped the problem of defining observables for general relativity and solved instead a much simpler problem, which is defining observables for fields moving on certain fixed backgrounds of high symmetry. It is fair to say that the result is that we do not really understand the physical content of general relativity, what we understand is instead the physical content of a set of related theories in which fields and particles move on fixed backgrounds which are themselves very special solutions of the Einstein's equations. There is nothing wrong with this, so far as it goes, the problem is that this approach does not give us much information about the observables of the full theory, in which generic initial data evolves into a generic spacetime.

The questions about novelty get their relevance from the fact that we observers in the real universe do genuinely observe novelty, in the sense that we observe things to happen that could not have been predicted on the basis of all the information that was, even in principle, available to us. One source of novelty is that each year new stars and galaxies come into view that we could not, on the standard models of cosmology, have received light signals from before, due to the universe's finite age and the finiteness of the speed of light. One may try to evade this argument in the context of an inflationary model, but this requires that we be able to predict the precise details of the light received from these distant galaxies on the basis only of the physics of their past during the inflationary era. But this is impossible in principle as the patterns of inhomogeneities that, according to the models of inflation, become the seeds for galaxy formation, are themselves seeded by quantum fluctuations in the vacuum state of a field during the inflationary era. Thus it is impossible in principle to predict the light that we will see next year arriving from a star presently too far to see, even assuming that inflation is correct.

A second source of novelty has to do with the fact that we live in a complex universe, so that we are constantly confronted with novel biological, sociological and cultural phenomena.

It may seem that this has nothing to do with the problem of time in physics, but it is not completely obvious, given that a cosmological theory is supposed to allow us to make sense of all observed phenomena. We will see below that this kind of worry might under certain circumstances indeed affect how we formulate cosmological theories.

In any case the first source of novelty is genuine and this is worrying enough. How are we to reconcile the fact that there is a necessarily unpredictable component to what we observe with the claim that time can be eliminated from our fundamental cosmological theories? There may be an answer to this question, but this is an issue we will have to consider carefully before judging the claim that time can be eliminated from physics.

2 The arguments for the elimination of time in cosmological theories

Before giving the arguments for the elimination of time I must emphasize a crucial point, which is that they concern cosmological theories. There is no problem of time in theories of isolated systems, embedded inside the universe or theories of systems with boundaries, at either finite or infinite distances. The reason is that if the system modeled is understood to include only part of the universe one has the possibility of referring to a clock in the part of the universe outside the system which is modeled by the theory. This is generally what the t in the equations of classical and quantum mechanics refers to[6]. The problem of time arises only in cosmological theories in which the whole universe is included in the degrees of freedom modeled in the theory, so that any clock, and any measuring instruments referred to in the interpretation of the theory must be part of the dynamical system which is modeled.

For the purpose of this discussion a universe is a closed system, which contains all that any part of it may interact with, including any observers and observing instruments and any clocks used to measure time. We will call any description of the physics of such a system a cosmological theory whether or not it is believed to be the actual universe, as its description faces the formal problems we are concerned with here.

There are two closely related arguments for the elimination of time in cosmological theories, the first classical, the second its consequence for quantum theories of cosmology.

2.1 The standard framework for classical cosmological theories

The argument for the elimination of time in classical physics begins with the definition of a *configuration* of a universe. A configuration is a possible state or situation that the universe can have at a given moment of time. The arena in which the argument takes place is the

[6]For example in asymptotically flat or Anti-DeSitter spacetime one can define time evolution with respect to a time coordinate at infinity. There is a problem with how to continue this into the interior of the spacetime, but this is not the same as the problem which occurs in cosmological theory. It can be resolved by an appropriate choice of gauge, which means that, while it is a serious technical problem it is not a deep problem.

configuration space of the universe denoted $\mathcal{C}$. This is defined to be the space of all possible *configurations* that the universe can be in at a given moment of time.

For a single particle restricted to be in a room the configuration space, $\mathcal{C}_{room}$ is defined to be the three dimensional space contained by the room; each point in the room is a place the particle might be and hence a possible configuration of the system. For a system of N particles in the room the configuration space is $\mathcal{C}_{room}^N$.

In classical physics it is assumed that universes have *histories*, which are described as curves $x^a(s) : R \to \mathcal{C}$ in the configuration space. $s \in \mathbf{R}$ is called the *time parameter*. Part of the question we are concerned with is the role of these histories in the description of the universe and the relationship between time parameters and time as measured by a clock carried by an observer inside the universe[7].

In the introduction I enunciated two principles that a cosmological theory must satisfy to be relevant to observations made by real observers inside the universe. In combination with the formalism of classical cosmological theories they lead to five postulates. We begin with the first three.

- **Postulate A: Constructibility of the configuration space.** It is possible by a finite mathematical procedure for observers inside a universe to construct its configuration space $\mathcal{C}$.

- **Postulate B: Deterministic evolution** There is a law for the evolution of universes which, given a position $x^a(0) = q^a \in \mathcal{C}$ and a velocity vector, $v^a \in T_q$ at q^a, picks a trajectory $x^a(s)$which is unique up to redefinitions of the time parameter s. The law is given by an action principle $S[x^a(s), v^a(s)]$.

The third postulate should tell us something about the observability of the configuration space. Ideally we should require something like,

- **Idealized Postulate C: Observability of the configuration space** It is possible for observers inside a universe to make measurements which are sufficient to determine which trajectory $x^a(s)$ describes the history of our universe.

However this is probably too strong, as real observers are much smaller than the universe and hence are unlikely to be able to gather and store enough information to determine its precise history. However, at the same time we want our observers to be able to measure enough observables to make some predictions as to the future values of some observables. The solution to this was understood a long time ago by the people working on the consistent histories approach to quantum theory. They proposed a notion of coarse grained histories. This will not, however, serve for us here, because that expresses the cosmological theory irreducibly in terms of histories, hence it cannot be a context in which we can run the

[7]A rather different question is the relationship between the time parameters and our experience of the flow of time. For the purposes of this paper we will be able to ignore this question, as the answers to the questions that will be considered will change considerably the context for its consideration..

argument for the elimination of time. What is required to do that is some notion of a coarse grained configuration space, $\tilde{\mathcal{C}}$ whose observables correspond to suitable averages of observables of the physical configuration space. From the evolution law specified in Postulate B we can then derive a law for the evolution of probability densities on the coarse grained configuration space. However, for the usual practice of classical cosmology we require more: it must be the case that we can derive an approximate deterministic evolution law on the coarse grained configuration space. An example is the reduction to a minisuperspace model, by eliminating the spatial dependence of the spacetime metric, which then satisfies a reduction of the classical Einstein equations with a finite number of degrees of freedom.

- **Postulate C: Observability of a coarse graining of the configuration space.** It must be possible to define, by a suitable averaging procedure from the configuration space, $\mathcal{C}$, a coarse grained configuration space, $\tilde{\mathcal{C}}$, together with an evolution law, such that an observer inside the universe can make measurements sufficient to determine the trajectory in $\tilde{\mathcal{C}}$.

We may note that each postulate is necessary for cosmology to be treated according to the usual methods of classical physics. If **A** is not satisfied than we observes inside a universe cannot describe it according to the methods of classical physics. If **B** is not satisfied than we cannot use the theory to make predictions of the future or retrodictions of the past. If **C** is not satisfied than we cannot compare those predictions and retrodictions with observations.

Given the definition we have given of a universe as a closed system, which contains all that any part of it interacts with, there is no role whatsoever in the notion of *an observer outside the universe.* Any recourse to such an observer in discussions of cosmological theories represents an implicit admission that the theory under discussion is not a proper framework for doing cosmology. This is the reason we insist on the two principles enunciated in the introduction.

All modern approaches to cosmology take on board the principle that observations of the configuration of the universe are relational in the sense that they refer to coincidences in the values of variables observable from inside the universe. This principle was first enunciate by Leibniz and the exact sense in which it is satisfied by general relativity is explained in writings of Barbour[1, 2]. We may express this as a fourth postulate

- **Postulate D: C is a relative configuration space.** This means that a point $x^a \in \mathcal{C}$ is completely determinable by measurements made by observers inside the universe.

This has the following important consequence: *Given that any two distinct points x^a and y^a of $\mathcal{C}$ must refer to configurations that can be distinguished by observers inside the universe there can be on $\mathcal{C}$ no global symmetry such as Euclidean invariance and no local gauge invariance such as diffeomorphism invariance.*

The exact form of the relative configuration space[8] depends on the content of the theory. For a system of N particles in d dimensional space, the relative configuration space may be

[8]Motion on such relative configurations spaces was first studied by Barbour and Bertotti and Barbour[17].

constructed as a quotient,

$$\mathcal{C}_{rel} = \frac{R^{dN}}{\textbf{Euclid}} \tag{1}$$

where **Euclid** is the Euclidean group of rotations and translations in R^d.

For general relativity and related theories such as supergravity

$$\mathcal{C}_{gr} = \frac{\text{metrics and fields on} \Sigma}{Diff(\Sigma)} \tag{2}$$

where $Diff(\Sigma)$ is the group of diffeomorphisms on a compact manifold Σ, which is taken to represent "a spatial slice of the universe."

An important point, to which we will return later, is that the relative configuration spaces are, at least in the examples studied so far, defined as quotients of a well defined space by the action of a group.

Nor can there be any notion of a clock which is something other than a degree of freedom measurable by observers inside the universe. Since all observables are assumed to be relational, and hence to measure coincidences in values of measurable quantities, there can be no reason why one clock can be preferred over another, so long as the values of the two of them can be related to each other uniquely. The result is our final postulate.

- **Postulate E: Reparameterization invariance** Two trajectories $x^a(s)$ and $x^a(s')$ which differ only be a redefinition of the time parameter, $s' = f(s)$ are deemed to describe the same physical history of the universe. This implies that the action $S[x^a(s), v^a(s)]$ is invariant under these redefinitions.

2.2 The classical argument for the elimination of time

The five axioms we have given define a classical cosmological theory. They rule out the existence or relevance of any clocks outside the system. Postulate **E** also rules out any absolute internal time, in that all time parameters appear, at least at first, to be on an equal footing. We must then wonder what the proper notion of time is in such a theory and, in particular, if it contains any concept of time that can be connected with our observations.

I will claim, following arguments by Barbour and others, that the theory allows no fundamental notion of time. By this we mean that the theory can be formulated in such a way that no reference is made to a time parameter. This does not exclude the introduction of parameters which have, at least in particular solutions, some of the properties of time in ordinary theories. Often these are degrees of freedom which behave in some regimes like physical systems we call clocks. We may thus call these clock variables. Typically these behave as would be expected in some regions of the space of solutions, but not in all. For these reasons, they may provide an approximate or effective notion of time in some domains in some solutions. But if the theory can be formulated without any notion of time then there is no guarantee that these approximate clock variables measure some more fundamental quantity which deserves the name of a universal time for the theory as a whole.

The basic argument for the elimination of time is the following. Given **B**, any pair $(x^a(s), v^a(s))$ determines a unique history $x^a(s)$. Conversely, given any history determined by the laws, and an arbitrary choice of initial time labeled by $s = s_0$, the whole history of the universe $x^a(s)$ is determined by its initial data $(x^a(s_o), v^a(s_0))$. But any observable $\mathcal{O}$ measurable by an observer inside the universe must be a function of the pair $(x^a(s), v^a(s))$. As a result any such function is actually determined completely by the initial data $(x^a(s_o), v^a(s_0))$.

But no physical observable can depend on the actual value of the time parameter, s because, by **E**, all observables must be invariant under rescaling of the time parameter. This means that time can only be measured in terms of coincidences between the values of the $(x^a(s), v^a(s))$ which do not depend explicitly on the specification of the actual parameter s at which that coincidence occurred. (An example of such a coincidence is to ask what the value of x^2 is when x^1 equals 17.) But such observables must be, by determinism, functions only of the trajectory. This means that any such observable must have the same value on any two points of the same trajectory.

This conclusion may seem counterintuitive when first encountered, if so it may help to go over. the argument a bit more carefully. The point is that the time parameter s is not measurable by any observer. Because it can be changed without consequence to the physical history, its only role is as a mathematical device to label the different points on a trajectory. Since that labeling is arbitrary, it cannot correspond to anything that an observer inside the universe can measure, in particular it cannot correspond to the reading on a physical clock.

Something which is observable then must be expressed as a correlation between different functions of the pairs $(x^a(s), v^a(s))$ that is independent of the parameterization s. For example it may be a correlation between the reading of one dial of an instrument and another. These correlations are independent of the parameterization and can be used to define a physical notion of time which is observable by an observer inside the system. But as it is independent of the parameterization, such a correlation must be determined by the $(x^a(s_0), v^a(s_0))$ for any arbitrarily chosen parameter value s_0. But since s_0 is arbitrary this means the observable is actually a function of the trajectory. If it is to be expressed in terms of the $(x^a(s), v^a(s))$ it must be in a way that is constant along each trajectory.

This is a key step of the argument, for the complete version I refer the reader to papers by Barbour[1, 2] and Rovelli[14].

Mathematically this has the following consequence. Because each pair $(x^a(s), v^a(s))$ determines a unique history, the evolution along each trajectory defines a one parameter group of diffeomorphisms E of $\mathcal{C}$, each member of which takes each point in $\mathcal{C}$ to another point on the same history. The argument just given says that any physical observable $\mathcal{O}$ must be invariant under the action of E. As a consequence one can take the quotient by the action of E. To do this it is convenient to first go to the phase space of the system, Γ, which is defined in simple theories[9] to be the $2N$ dimensional space of pairs $(x^a(s), v^a(s))$. The reason is that the evolution on the phase space is first order in time. This means that

[9] In all theories the phase space is defined by position-momentum pairs, only in simple theories is the momentum proportional to the velocity. This technical point is not relevant to the argument being made here.

only one trajectory can go through any point on the phase space. One can then define the quotient,

$$\mathcal{U} = \frac{\Gamma}{E}. \tag{3}$$

This defines $\mathcal{U}$, the space of physically distinct histories of the universe. Since all observables must be invariant under the action of E, it follows that any observable $\mathcal{O}$ is actually a function on $\mathcal{U}$.

The final form of the theory is then that the possible universes are described as points of $\mathcal{U}$. All observables are functions on $\mathcal{U}$. The notions of time and trajectory have disappeared from the final form of the theory.

The definition of the quotient may involve some technical work, but is assumed to be always possible. One way to do it is to defined a gauge condition, which is an equation for a surface in Γ that intersects each history exactly once. One may then identify $\mathcal{U}$ with this surface. Physically, one way to do this is to define a physical time parameter in terms of some observable quantity $T(x^a)$ which is a function on Γ which has the property that $(x^a) = 0$ defines such a surface.

The result is that time has been eliminated completely from the theory. One does physics by determining which trajectory one is on and then determining the value of the observables, expressed as correlations between quantities measurable by local observables. Some of these quantities may be interpretable as readings on devices we call clocks, in which case we can recover at some level a notion of time, defined just as the readings on a device called a clock. But there is no reason we must interpret the observables in terms of such clock variables. Time may or may not be a useful construct, good for some level of description. But it has no fundamental role in the theory.

2.3 The argument for the elimination of time in quantum cosmology

We now turn to discussion of the quantum theory. We will take the conventional view that a quantum theory of cosmology should be a quantization of a classical theory of cosmology. This is of course unlikely to be true, as the quantum theory is assumed to be the fundamental theory and the classical theory should be derived from it by some suitable approximation procedure. But we will adopt the conventional view here as it is the context in which the argument for the elimination of time in quantum cosmology follows. As before I will only sketch the argument, details can be found in [1, 2, 14].

There are several different approaches to turning a classical cosmological theory into a quantum theory. We describe only one here, the logic in the other approaches are similar. This is the approach based on the Wheeler-DeWitt, or *hamiltonian constraint equation.* This approach arises in the context of hamiltonian quantization of theories which satisfy postulate, **E**, invariance under arbitrary redefinitions of the time parameter. To make the argument as transparent as possible to those unfamiliar with it, and to avoid boring those who are, I skip the steps of the construction and exhibit only the result.

In this approach, a quantum theory of cosmology depends also on five postulates. Postulate **A** is taken over directly from the postulates of classical cosmology. The postulates **B** and **C** are replaced by

- **Postulate** B': **Existence of the wavefunctional of the universe.** A quantum state of the universe is defined to be a normalizable complex functional Ψ on $\mathcal{C}$. This means that there exists a measure $d\mu$ on $\mathcal{C}$ such that

$$1 = \int_{\mathcal{C}} d\mu |\Psi|^2. \tag{4}$$

  The normalizable states define a Hilbert space on which μ defines an inner product according to

$$< \Phi|\Psi >= \int_{\mathcal{C}} d\mu \Phi^* \Psi \tag{5}$$

  Observables must then be represented as self-adjoint operators in this inner product space. As in ordinary quantum mechanics there must be complete commuting sets of observables whose spectrum determine the quantum state uniquely. This gives rise to

- **Postulate** C': **Observability of the configuration space** It is possible for observers inside a universe to make measurements which are sufficient to determine a density matrix ρ which has sufficient information to determine, in a suitable classical limit, a trajectory in the coarse grained congifuration space $\tilde{\mathcal{C}}$.

Postulate **D** is either taken over directly, or replaced by a set of first order functional differential equations called the diffeomorphism constraints $D(v)$, which depend on an arbitrary vector field v^a on Σ, the spatial manifold. This expresses the requirement that the quantum states are valued only on relative configurations which are in this case diffeomorphism equivalence classes of metrics on Σ. In the latter case we have

- **Postulate** D': **Diffeomorphism constraints.** The wavefunctionals satisfy also

$$D(v)\Psi = 0 \tag{6}$$

  for all vector fields v^a on Σ.

Finally, the crucial postulate **E** that expresses time reparameterization invariance is replaced by

- **Postulate** E': **Solution to the Hamiltonian constraint.** There is a second order functional differential operator, H on $\mathcal{C}$ such that

$$H\Psi = 0 \tag{7}$$

Of course many technical problems arise in the realization of these conditions. An important motive for taking them seriously is that they can in fact be realized in quantum general relativity in $3+1$ dimensions, coupled to arbitrary matter fields[21][10].

There are many interesting issues concerning the construction and interpretation of such observables. We do not need to go further into this discussion than to note that as in the classical case time plays no role in the actual formulation of the theory. As many have remarked, the Hamiltonian constraint equation, (7) contains the dynamics of the theory, but there is no d/dt on its right hand side. This is because any t not contained in the wavefunctional would be a non-dynamical time, disconnected from the dynamical system described by the wavefunctional. But the basic postulates of the theory tell us there can be no such external time parameter. As in the classical case, time parameters may be introduced, but the physical observables cannot depend on the actual value of any time parameter. Hence the right hand side of the equation is 0. Rather than expressing the dependence of the state on an external time, as in the Schroedinger equation, the constraint expresses only the fact that the quantum state has no dependence on such an external time.

A full specification of the theory is given by the choice of configuration space, Hamiltonian (and possibly other) constraints. There is no place for a time parameter, time has truly been eliminated in the theory. Nor does time necessarily show up in the form of any solutions, many solutions are known [21, 22, 23] that have nothing like a time parameter.

There may be approximate notions of time which arise from properties of the solutions or observables, but no notion of time is needed to construct the theory. If a theory formulated along these lines is correct, time has disappeared from the fundamental notions needed to describe the physical universe.

3 Challenges to the argument for the elimination of time

We now turn to several challenges which have been made recently to the argument just sketched. To my knowledge these are new and in my opinion they deserve careful consideration. If they are right then not only is time still a necessary concept, we will have to find a different way to frame dynamical laws. The success of these challenges then cannot depend only on whether or not they point up a failure of the argument for the elimination of time. Their contribution must be at least equally positive as negative, they must point us towards the invention of a new dynamical framework in which time plays a different, and more essential, role than at present.

The arguments for the elimination of time as a concept necessary for the expression of the fundamental laws of physics follows from the five postulates mentioned above. If they

[10]This formulation, called loop quantum gravity, comes from using for the configuration space the diffeomorphism and gauge equivalence class of a certain connection on Σ, rather than the metric. This change makes it possible to obtain precise results, but does not affect the conceptual arguments under discussion here.

all hold then the argument goes through. At a purely formal level, they do seem to hold for quantum general relativity in $3+1$ and more dimensions, coupled to any kind of matter. The progress of the last 16 years using the Sen-Ashtekar formalism[19, 20] and loop quantum gravity[21] strengthens the argument as it leads to the explicit construction of solutions to the Hamiltonian constraints[22, 23]. Supersymmetry seems to make no difference, nor is there any reason to believe that they are not satisfied for string theory, even given that we do not know the framework of string theory at the background independent or non-perturbative level[11].

If the argument goes through then there seems little alternative to agreeing with the point of view expressed eloquently by Julian Barbour in [2]. This is the end of the concept of time in fundamental physics.

There seems only one real hope of evading the argument, which is that there is in fact no way to realize all five principles on which the argument depends consistently in a single theory. Is this possible? It is certainly the case that there are consistent and completely worked out *models* of quantum theories of cosmology which have only a few degrees of freedom. These include quantum gravity in $1+1$ and $2+1$ dimensions as well as some models of very symmetric universes. However these only satisfy postulates **B, D** and **E**. They fail to satisfy the other postulates because they are too simple to contain subsystems complex enough to be called an observer. Further, despite all the progress made on the quantization of general relativity, supergravity and related theories[21, 22, 23], the explicit construction of an infinite number of physical states has *not* been followed by the construction of an infinite number of physical observables. It is then an open question whether there are any theories which satisfy all of the postulates.

3.1 A first challenge: are there observables without time?

Postulate **C** requires the construction of a sufficient number of observables of the theory to distinguish the solutions from each other. As we are dealing with a theory with an infinite number of degrees of freedom this means we must have an infinite number of observables. In the introduction we discussed some of the issues involved with the construction of observables in general relativity. To sharpen this discussion we may distinguish two possible approaches to the construction of observables in classical and quantum theories of gravity

- **Causal observables**. These are instructions for the identification of observables that make explicit reference to the causal structure of a classical or quantum spacetime. Since the causal structure is a diffeomorphism invariant of the metric, such an observable may be diffeomorphism invariant by construction. Examples are known which are of the following form: Identify a particular localized system as a local reference system and identify one of its degrees of freedom as a clock. Define observables in terms of the values of other local degrees of freedom coincident with the clock variable taking

[11]However, it is clear that time can be eliminated in the proposals that have so far been made for a background independent form of M theory[25, 26].

on particular values. These correspond to actual observations that could be made in a spacetime, which, using the causal structure, give information about the spacetime to the past of the event when that local clock variable had a particular value.

Such an observable can be constructed explicitly in a histories formulation of the theory without solving any additional conditions. They can be in principle be directly implemented in a path integral formulation of the theory based on summing over Lorentzian histories.

- **Hamiltonian constraint observables.** These are observables which are constructed according to the rules of the hamiltonian formulation for systems with time reparameterization invariance. They must do at least one of the following things, i) have vanishing Poisson bracket with the classical hamiltonian constraint, ii) be expressed directly as a functional on the reduced phase space which is the constraint surface mod gauge transformations. iii) commute with the quantum hamiltonian constraint.

Observables of the first kind make explicit use of the causal structure and hence use time in an essential way in their construction. If we only work with these observables then we have not eliminated time from the theory. Furthermore, there are no obstacles to defining and working with such observables as no equations need to be solved. They are diffeomorphism invariant by construction.

On the other hand, if we eliminate time from the theory, as sketched above, by either reducing the classical theory to its reduced phase or configuration space or constructing a quantum theory from solutions to the Hamiltonian constraint, then we have available only the second class of observables. We then conclude that Postulate B requires that we be able to construct an infinite number of observables of the second kind. Further, all observables of the first kind must be reducible to observables of the second kind.

The problem is that Hamiltonian constraint observables are extremely difficult to construct in real field theories of gravitation. There are formal proposals for how to construct such observables, which have been implemented in toy models. But these toy models are too simple and do not have local degrees of freedom that could be identified with fields measured by local observables inside the spacetime, or with such observables themselves. No observables have ever been constructed through either of the three methods mentioned which are local in the sense that they correspond to what an observer inside a relativistic spacetime would see.

The problem that arises in method i) is that to give such an observable explicitly as a functional on the phase space of the theory requires explicitly inverting the equations of motion of the theory over the whole space of possible initial data. This cannot be done for other than integrable systems. Similarly, method ii) has not been implemented because no one has a proposal for how to actually construct the reduced phase space. This also would involve an inversion of the equations of motion of the theory. The problem with method iii) is that while we have been fortunate enough to find an infinite number of quantum states which are exact solutions to the Hamiltonian constraints of theories of gravity[22, 23], finding operators which commute with the constraints has proved to be much harder, and only a few, rather trivial such operators have been found. No one has even proposed a practically implementable strategy for how to construct operators that both commute with the quantum Hamiltonian constraint and refers to local observations.

So whether or not there exist in principle observables of the second kind, there are no known methods to construct them for realistic theories.

3.2 Newman's worry: the implications of chaos

Ted Newman[18] and others have raised a further argument against the existence of observables of the second kind, constructed through either method i) or ii) which is that the fact that gravitational systems have chaotic behavior may in principle prevent their construction. It is well known that the motion of three bodies under their mutual gravitational attractions is chaotic in Newtonian mechanics. It is then hard to believe that the typical solution of general relativity is not also chaotic. It is also known that the generic cosmological model with a finite number of degrees of freedom, called the Bianchi IX model, is chaotic, at least if a physically realistic notion of time is used to in the criteria for chaos. If we do not yet have a proof of that the Einstein equations are chaotic it is mainly because of the problem of finding a definition of chaos suitable for the application to a field theory of gravity.

But if the equations are chaotic it means that physical observables are not going to be representable by smooth functionals on the phase space. The reason is that they must be constant under the trajectories defined by the hamiltonian constraints, as this must be satisfied by any physical, diffeomorphism invariant observable. But if the system is chaotic than any trajectory passes through any open set in the phase space. This means that non-trivial physical observables will take an infinite number of values in any open set. It is easy to construct examples of such observables which are not continuous anywhere inside at least certain open sets.

It will be difficult both to construct such observables explicitly and to evaluate them. It may also not be possible to define an algebra of such observables, as this requires taking derivatives to form the Poisson bracket. But if such observables do not have an algebra they cannot be represented in the quantum theory, as the quantum theory is generally defined in terms of a relationship between the algebra of quantum operators and the Poisson algebra of classical observables.

Newman's worry turns what may seem at first only a practical problem into a problem of principle. It may be not only beyond our power to construct observables for a formulation of cosmology in which time has been eliminated, it may be impossible in principle. If this is the case then the argument for the elimination of time fails, for a theory without observables that can be made to correspond to what we real observers can measure is not a physical theory, it is just a formal structure with no possible relation to the real world.

3.3 Markopoulou's argument: the configuration of the universe is not observable

In a recent article Markopoulou[4] pointed out that postulate C, observability of the configuration space is not likely to be satisfied by any quantum theory of gravity whose classical limit is general relativity. The reason is that in generic cosmological solutions to classical general relativity containing dust and radiation, with spacetime topology of the form $\Sigma \times R$, with Σ either R^3 or S^3, the backwards light cones of typical events do not contain the full Cauchy surface Σ. This means that Postulate C fails in these examples.

It is important to stress that the problem is not just the inability to measure enough observers to determine a trajectory in the fine grained configuration space $\mathcal{C}$. The problem is that as we see only a portion of every Cauchy surface, we cannot measure enough information to determine a history within any coarse grained configuration space. As George Ellis has pointed out[36] we cannot make use of a coarse grained cosmological model such as a minisuperspace model unless we supplement our assumptions with the *assumption* that the portion of the universe we see at any time is a typical region of the whole universe at that time. (Note that this also assumes sufficient homogeneity that it makes sense to talk of a simple cosmological time function.) This assumption may be dressed up into a principle, the so-called cosmological principle, but there can be no getting away from the fact that it is an assumption that does not, and very possible cannot have strong support from observation. Furthermore, the assumption is strictly false in many plausible cosmological models such as inflationary models.

Once one accepts the possibility that the cosmological principle may be false, Markopoulou's argument has force, because it implies that any internal observer is unable to gain from observation enough information to carry out a coarse graining which can satisfy either the classical or quantum version of Postulate C.

It is possible to avoid this conclusion in several ways. The first is to have a "small universe" in which the spatial topology is more complicated. Presently this appears to be ruled out by observation. The second is to weaken the requirement that the matter be described by dust and radiation. By doing so we can allow inflation to have occurred in our past. There are, however, two problems with saving the observability of the universe by inflation. The first is that without fine tuning inflation predicts that the universe is spatially flat. In this case we certainly do not see a whole Cauchy surface in our past. The second is that according to inflationary models the whole Cauchy surface would be seen only during the inflationary era when the quantum fields are close to their ground state. The inhomogeneities that drive structure formation in our universe are hypothesized to arise only at the end of the inflationary period, in a "quantum to classical transition" in which quantum fluctuations in the vacuum state are converted to classical fluctuations in the matter and geometry. This transition is believed to be akin to a measurement of the quantum state of the fields. The result is that the information necessary to determine the classical geometry of the universe is nevertheless not available in the backwards light cone of any one observer.

More could be said about these discussions, but the conclusion is that Postulate C is likely not satisfied in our universe.

We are then faced with the following choice: Give up postulate C and accept a classical or quantum theory of cosmology which is formulated in terms of quantities that are not observable or attempt to modify the theory so as to formulate it only in terms of quantities observable by real observers inside the universe. In [34] Markopoulou describes a framework in which general relativity and other spacetime theories may be reformulated completely so that the only observables involve information available to internal observers by means of information reaching them from their backwards light cones.

The potential power of Markopoulou's argument rests on the following simple observation: in classical general relativity the causal structure plays a significant role in delineating what is observable. To begin with Markopoulou points out that if we insist on the principle that any observable of a cosmological theory must be in fact observable by an observer inside the universe than it follows that our theory must be pluralistic in the particular sense that different observers have access to different information. However this is a structured pluralism because the causal structure implies certain logical relations amongst the observations made by different observers. Markopoulou then points out that the pluralistic logic required to define what is observable in general relativity is closely related to logics studied by mathematicians under the name of topos theory. The basic rules which relate observations made by different observers follow from the causal structure and the requirement that whenever two observers receive information from the same event they will agree on the truth value given to any propositions about that event.

Markopoulou's argument then formalizes the worry about novelty I mentioned in the introduction. The key point is that any given observer, located in some finite region of space and time, only receives information from a proper subset of the events in the history of the universe. Furthermore, since no observer sees a whole Cauchy surface, they have no way to predict what information will be received from systems of the universe they will come into causal contact with at a later time. As a result, the logic of observables in a cosmological theory must not satisfy the law of the excluded middle. No single observer can assign the values **TRUE** or **FALSE** to all of the propositions which describe the history of the universe. There must be other truth values such as, **cannot yet be determined by this observer**. In turns out that one has to be able to describe different degrees of ignorance, so that there is actually a whole algebra of possible truth values besides true or false[4, 5, 6].

The result is that the logic of observables in a relativistic cosmological theory must be both non-boolean and pluralistic. For any single proposition, different observers will be able to assign different truth values. Whenever two are able to assign **TRUE** or **FALSE** they must agree. But they need not agree if one or both are only able to assign truth values which indicate some amount of inability to determine.

So far we have been discussing only the classical theory. There are also implications for the quantum theory which Markopoulou develops in further papers[5, 6]. The point is that the possible truth values of the classical theory must appear in the spectrum of projection operators in the quantum theory. But, she points out, one cannot realize such a non-Boolean pluralistic logic in terms of the spectrum of operators on a single Hilbert space. Consequently, a quantum theory of cosmology whose classical limit reproduces the algebra of observables of the classical theory explored by Markopoulou cannot be based on a single Hilbert space. As she describes in [4, 5], it can, however, be constructed from a presheaf of Hilbert spaces, which is a structure in which there is a Hilbert space assigned to every event in a causal set.

The result is a new framework for quantum cosmology in which the causal structure, and hence time, plays a fundamental role in the formulation of theory. The basic implication of Markopoulou's work for the present argument is then that to the extent that the causal structure plays a role in the identification of observable quantities in general relativity, it must also play an essential role in the construction of the corresponding quantum theory. If we are then restricted to the first kind of observables, which are causal observables, than time cannot be eliminated from quantum cosmology.

3.4 Kauffman's argument: the configuration space cannot be described in advance of the evolution of the system

We come now to a challenge to the argument that attacks the first postulate, which is the constructibility of the configuration space[12]. This challenge was inspired by two very interesting questions raised by Stuart Kauffman in the course of recent work on theoretical biology and economics[12]:

- *Is it possible that the configuration spaces relevant for mathematical biology, ecology or economics cannot be constructed by any finite mathematical procedure?*
- *Even if the answer to the first is in principle no, is it possible that the construction of the relevant configuration spaces are so computationally intensive that they could not be carried out by any subsystem of the system in question?*

There are good reasons to suspect that the answer to at least one of these questions is yes when we are dealing with such potentially large and complex configuration spaces such as the space of all possible phenotypes (as opposed to genotypes) for biological species, the space of all properties that might be acted on by natural selection, the space of all biological niches, or the space of possible kinds of businesses or ways of earning a living.

We refer the reader to [12] for discussions of the implications of this possibility for theories of biology and economics. For the present purposes we are interested to consider the analogous questions[10]:

- *Is it possible that there is no finite procedure by means of which the configuration space of general relativity or some other cosmological theory may be constructed?*

[12] This kind of worry goes back at least to a paper by James Hartle in which he points out that the impossibility of classifying four manifolds is a problem for the Euclidean path integral formulation of quantum cosmology[9].

- *Even if the answer is no is it possible that the computation that would be required to carry out the construction of the configuration space is so large that it could not be completed by any physical computer that existed inside the universe?*

The possibility that the answer to one or both questions is yes arises from the fact that, as pointed out in section 2, the physical configuration spaces relevant for cosmological theories are quotients of infinite dimensional spaces by the action of infinite dimensional groups. For the case of general relativity in $3+1$ or more dimensions the physical configuration space is defined to be the quotient of the space of metrics on some fixed compact manifold, Σ, by the diffeomorphism group of Σ. No closed form representation of the quotient is known, even in the simplest case in which $\Sigma = S^D$. The space of metrics on a manifold Σ is known to be an infinite dimensional Riemannian manifold. Coordinates are known which cover it and the tangent space, metric, connection and curvature tensor are known. So the issue is not just the infinite dimensionality. The problem is that the diffeomorphism group is very large and its action is quite complicated. It is known that the quotient has singularities and is not everywhere an infinite dimensional manifold.

To define the quotient we must also be able to have a procedure to answer the following question: given two metrics, g_{ab} and g'_{ab} on Σ, does there exist a diffeomorphism $\phi \in Diff(\Sigma)$ such that $g'_{ab} = \phi \circ g_{ab}$? By computing a large enough set of curvature scalars one can reduce this problem to the question of when there is a diffeomorphism that can bring one set of functions on a manifold to another. But no finite procedure is known which can answer this question in general even in the case that we restrict to the analytic category in which both metrics and diffeomorphisms can be defined by power series expansions in some set of coordinates. The problem is that while one may be able to show in a finite number of steps that g_{ab} and g'_{ab} are not diffeomorphic, there is no finite procedure which works in the general case to tell whether they are or not.

But if one cannot tell whether two metrics are in the same diffeomorphism class or not one is not going to be able to construct the quotient by an explicit construction of the equivalence class within the space of metrics on Σ. If this is the case, postulate **D** is not satisfied[13].

One might try to avoid this by use of the alternative Postulate **D′**, which replaces the requirement of constructing the space of metrics mod diffeomorphisms by the solution of certain functional differential equations. This is suggested by the fact that in one approach to quantum gravity a complete solution to the problem of solving the diffeomorphism constraints has been stated in closed form. This is the loop quantum gravity approach[21]. The basic result of loop quantum gravity is a precise recipe for the construction of an orthonormal basis for the Hilbert space of quantum general relativity on a spatial manifold with topology and differential structure Σ. The basis is in one to one correspondence with the diffeomorphism

[13]Note that if the exact configuration space is not constrictible, one may still *postulate* coarse grained configuration spaces such as mini-superspace models. However, these are then not strictly speaking derivable from general relativity, while they may be suggested by some heuristic considerations based on general relativity, they must be considered to be logically independent of full general relativity.

equivalence classes of the embeddings in Σ of a certain class of labeled graphs called spin networks. The problem we are concerned with then reduces to the question of classifying the diffeomorphism classes of embeddings of these graphs.

There is no problem enumerating and distinguishing finite labeled graphs as this comes down to distinguishing finite dimensional matrices with integer entries. However, classifying and distinguishing diffeomorphism classes of the embeddings of graphs in three dimensions is a tricky question. The problem of the classification of ordinary knots up to ambient isotopy[14] has only recently been solved and even that requires a very computationally intensive procedure which involves classifying the finite groups that appear as the homotopy group of the complement of the knot. The homotopy groups of complements of graphs present a greater challenge, and, to my knowledge, these remain unclassified.

Even if this problem is solved the problem of distinguishing diffeomorphism classes is a good deal trickier for graphs with arbitrary valence intersections than the problem of distinguishing ambient isotopy classes. The reason is that in the case of sufficiently high valence (5 and higher in three spatial dimensions) the diffeomorphism invariance classes are labeled by continuous parameters.

If it turns out that there is no finite procedure for solving either problem then it follows there will be no finite procedure to construct the Hilbert space for quantum general relativity and related theories[15].

The problem is made even more complicated if one believes, as many do, that the topology of Σ should also be able to change by means of quantum transitions. In this case one has to classify networks embedded in arbitrarily complex three topologies. But it is sufficient to note that the problem is already quite serious for a fixed simple topology.

Finally, we note that even if the answer to the first question is no, the answer to the second is certainly yes. The maximum computational power of a universe could not increase faster than its volume, which is roughly proportional to the number of nodes of the graph of the corresponding quantum state. But the number of steps required to distinguish the embeddings of two graphs is likely to go up faster than any power of the number of nodes. We may also note that the situation is likely worse than this if the holographic principle[32, 33, 3, 7, 8] is correct, as that bounds the amount of information a region of space could contain to its area rather than its volume in Planck units.

To summarize this part of the argument: it is quite possible that the answer to the first question raised above is yes, and almost certain that the second is. In the case that the configuration space cannot be constructed the argument for the elimination of time in classical cosmology cannot be run, for the whole framework falls apart without a prespecified configuration space. Similarly, if the Hilbert space of states cannot be constructed the argument for the elimination of time in quantum cosmology cannot be run.

In either case, if the spaces are constructible, but require more computational power than the universe could contain, then we are faced with an interesting situation. A mythical

[14]Ambient isotopy is a weaker equivalence, which requires only that the embeddings of the knots or graphs are homeomorphic. This does not, in particular, preserve differentiability at nodes of the graphs.

[15]including all known couplings to matter including supersymmetric theories.

extra-universe observer could run the argument for the elimination of time. But this is impossible for any real observers inside the universe. A quantum theory of cosmology that requires more processing power than the universe could contain to set up its Hilbert space and check whether two states were orthogonal or not would not be a theory that we who live inside the universe could use to do real computations. So it is not clear what relevance for our physics there may be for the possibility that some imagined being outside the universe could eliminate time from physics. What matters is that we cannot work with a physical theory without time.

4 Conclusions

Let me begin my summary of these arguments by emphasizing the role played by the requirement that a theory of cosmology must be falsifiable in the usual way that ordinary classical and quantum theories are. This leads to the requirement that a sufficient number of observables can be determined by information that reaches a real observer inside the universe to determine either the classical history or quantum state of the universe. Only if this is the case can we do cosmology within the standard ideas concerning the methodology and epistemology of dynamical theories.

It does appear that this is not the case. Interestingly enough, this statement requires input from both theory and observation. General relativity allows both possibilities. Presently observations seem to rule out the possibility that we live in a universe in which there is a complete Cauchy surface within the classical region of the past light cones of typically situated observers such as ourselves.

The implication seems clear: if we want to do cosmology as a science, we must restrict ourselves to theories in which all observables are accessible to real observers inside the universe. To do this we must invent a new framework for quantum cosmology which does not include notions like "the wavefunctional of the universe." One way to do this has been proposed by Markopoulou, which is called "quantum causal histories"[5, 6]. These are an interconnected web of Hilbert spaces tied to the causal structure in such a way that each act of observation, considered as a particular event in the history of the universe, is represented in terms of a Hilbert space constructed to represent the information available to be observed at that event. These together provide a representation of projection operators whose spectrum consists of the observer dependent truth values we discussed above.

This proposal may be seen in the light of a general issue which has been discussed a great deal in quantum cosmology, which is that of context dependence. This arises most generally in the consistent or decoherent history approaches to quantum cosmology[27, 28]. As pointed out by Gell-Mann and Hartle[28] and Dowker and Kent[29], the consistent histories formulation requires the specification of a context within which observations are to be made, prior to their interpretation. This is necessary to replace the quantum world/classical world division of Bohr's interpretation of quantum theory in a way that avoids the preferred basis problem of the Everett interpretation. It may also be argued that the consistent histories approach does evade many of the issues which face a hamiltonian approach to quantum cosmology, including those discussed here[16] Isham and collaborators[30] have studied the general mathematical structure of such contextual approaches to cosmology and found they are naturally formulated in terms of topos theory. Markopoulou's quantum casual histories[5, 6] can be understood from this point of view as the result of using the causal structure of the history of the universe to define the contexts.

However, the general issue of context dependence does not by itself refute the argument for the elimination of time. Were it possible for a single observer inside the universe to make sufficient measurements to determine either its classical history or its quantum state, and assuming that the other four postulates also held, the argument for the elimination of time could be run. In this circumstance it might still be convenient to express quantum cosmology in terms of histories, but it would not be essential. Barbour and others would be able to argue that they could do cosmology perfectly well with no notion of histories apart from what was necessary to recover the classical limit.

It is only by insisting that the context of real observers inside their universe is defined by the information that reaches them by means of radiation that propagated from their past that a link is made between the issue of observability of the universe and its causal structure. Of course, the notion of causal structure may be loosened quite a bit from that which arises in general relativity, as has been done in various causal set and evolving spin network models of quantum gravity. But it is hard to divorce the notion of causal structure from the idea that there is a finite speed for the propagation of information, and hence from some notion of time.

Is the notion of time then built into the argument from the beginning? No, the key point is the insistence on building a cosmological theory that makes references only only to observations made by real observers inside the universe. It is then an observed fact that the universe is very big compared to its observers. A combination of observation and theory then leads to the conclusion that the observations made by one observer at one "moment" are insufficient to determine the classical or quantum state of the whole universe. We may note that all that is required here is the notion of a moment at which a number of simultaneous measurements may be made. This is already all we need to argue that cosmology requires a different framework from a conventional quantum system because the postulates of quantum theory require that the state of a quantum system must be uniquely determined by measurements of a complete set of commuting observables which, by definition, can be made simultaneously.

This is already sufficient to refute the argument for the elimination of time. The proposal of Markopoulou builds up from here to propose an alternative way to do cosmology, which is in terms of a structured set of observations, which are not made at simultaneous moments. The positive proposal is that the structure which is imposed on the possible measurements is a partial ordering which is derived from the causal structure of the universe.

This is a good moment to recall Newman's worry. This may be put in the following form.

[16] I would like to thank James Hartle for correspondence on this point.

Even if the universe is governed by classical deterministic equations, these are likely to be chaotic. This is important because of the fact that all measurements we real observers can do are of finite precision. It means that, even if all five postulates are true, and the argument for the elimination of time can be run, we still may not be able to make reliable predictions of measurements that refer to "moments" when our clocks read different "times." Even if time can be eliminated in principle, it cannot be eliminated in practice.

There is a stronger form of Newman's worry, which has to do with the existence of a physical observable algebra for a chaotic but time reparameterization invariant system. But even leaving this aside we see that this argument has an interesting parallel with Markopoulou's argument. Both refer to the information available to a real observer inside the universe. They concern two ways in which real observers differ from abstract idealized observers in the real universe. First, that they are small and are limited to information available in a local region of spacetime. Second, they can only make observations of finite precision. It is very interesting that both arguments suggest it may be useful to think about measurement in information theoretic terms. In particular both suggest it is important to make a distinction between what can be done in principle in a mathematical model and what can be done by a real observers in a finite number of steps.

A related issue of constructibility underlies the third argument, introduced by Kauffman. The implication of this argument is that if we want to have a measurement theory which is relevant to what real observers inside the universe do we must require that all the constructions necessary to formulate and compute in the theory can be carried out in a number of operations which is not only finite but small enough to be carried out by an observer inside the universe.

In the cases of theoretical biology and economics, Kauffman goes on to propose a method for formulating a theory that does not depend on the constructibility of huge and complex configuration spaces. He proposes that it may be sufficient that the theory provide an algorithm that allows the construction of all configurations which differ by a small number of local changes from any given present configuration. This space of possible nearby configurations is large enough for a large complex system, but still infinitely smaller than the space of all possible configurations. Kauffman calls it *the adjacent possible[12].*

Could we do quantum cosmology in terms of such an adjacent possible? A start on such an approach has been given in [11]. Suppose that the state space of a quantum theory of gravity was not constructible. For example suppose this is in fact the case for the ambient isotopy classes of embeddings of spin networks in a three manifold. It is still possible to construct the space of all spin networks that differ from any given one, Γ, by a small number of local moves. The local moves can be moves that involve rearrangements of less than P nodes, which together with their common edges must make an n simplex for $n \leq P$. These are the forms of moves appropriate to P spacetime dimensions. Given Γ let Ω^r_Γ be the linear space spanned by all spin networks that are the result of r local moves made on Γ. This will be a finite dimensional space.

We may now use the following facts from the theory of causally evolving spin networks[34]: 1) The evolution moves are themselves local moves of the same type. 2) Each local change constitutes the analog of an event in a quantum spacetime. 3) Each such event will correspond in the classical limit to a small, but finite spacetime. We can then deduce that the space of quantum universes which begin with the initial state Γ and have finite spacetime volume, V, in Planck units, will live in some Ω^r_Γ, with r a finite function of V. But for any finite r and finite Γ, Ω^r_Γ is a finite dimensional space. Using this kind of construction it does appear that quantum cosmology could be formulated in terms of an adjacent possible type construction[11].

However, the resulting theories are still subject to the argument of Markopoulou: the states in Ω^r_Γ are not measurable by any observer inside the universe. To take into account both we should realize this kind of construction in a quantum causal history. This is in any case a natural thing to do as a quantum causal history is based on a partial order structure, and such a structure is naturally generated by local moves made on graphs. The result is a framework for quantum cosmology which escapes the worries raised here.

In these theories the notion of information plays a crucial role. This is forced on us by the combination of finiteness of the space of local changes and finiteness of the propagation of the effects of local changes. But the importance of the notion of information also arises from an apparently independent set of considerations having to do with the Bekenstein bound[31] and the holographic principle[32, 33, 3, 7, 8]. These suggest that measures of geometry may actually be reduced to measures of information flow. In its "weak form", the holographic principle asserts that the geometric area of any surface must be reducible, in a fundamental theory, to a measure of the capacity of that surface as a channel of flow of information from its causal past to its causal future[7, 8].

In closing we note the very interesting way in which notions of finiteness and constructibility are coming into fundamental theories of quantum cosmology. This may not be surprising to experts in philosophy and the foundations of mathematics, for whom these notions are closely related to ideas on time. But it is a new idea for some of us who come at it from physics: if the universe is discrete and time is real, and is itself composed of discrete steps, then time may be none other than the process which constructs, not only the universe, but the space of possible universes relevant for observations made by local observers. Beyond this, there is the possibility of a quantum cosmology in which the actual history of the universe up till some moment and the space of possible universes present at that "instant" are not two different things, but are just different ways of seeing the same structure, whose construction is the real story of the world.

ACKNOWLEDGEMENTS

This paper was mainly a commentary on ideas of Barbour, Kauffman, Markopoulou and Newman, and I would like to thank them for many discussions on these issues. I am also grateful to James Hartle for commenting on a draft of this paper and Chris Isham and Carlo Rovelli for discussions on these issues. I would like also to thank the theoretical physics group at Imperial college for their hospitality during this last year. This work is supported by the NSF through grant PHY95-14240 and a gift from the Jesse Phillips Foundation.

References

[1] J. Barbour, *The timelessness of quantum gravity: I The evidence from the classical theory, II The appearance of dynamics in static configurations* Class. and Quant. Grav 11 (1994) 2853.

[2] Julian Barbour, *The End of Time*, Oxford University Press, New York and Weidenfeld and Nicolson, London, 1999.

[3] L. Crane in, Categorical Physics, hep-th/9301061; hep-th/9308126, in Knot theory and quantum gravity ed. J. Baez, —(Oxford Uni- versity Press); Clocks and Categories, is quantum gravity algebraic? J.Math.Phys. 36 (1995) 6180-6193, gr-qc/9504038; Topological Field the- ory as the key to quantum gravity.

[4] Markopoulou F, 1998, *The internal logic of causal sets: What the universe looks like from the inside*, gr-qc/98011053, to appear in Commun. Math. Phys. (2000)

[5] Markopoulou F, 1999, *Quantum causal histories*, hep-th/9904009, to appear in Class. Quantum Grav. (2000).

[6] F. Markopoulou, *An insiders guide to quantum causal histories*, hep-th/9912137.

[7] F. Markopoulou and L. Smolin, *Holography in a quantum spacetime*, hep-th/9910146.

[8] L. Smolin, *The strong and the weak holographic principles*, hep-th/003056.

[9] J.B. Hartle, *Unruly Topologies in Two Dimensional Quantum Gravity*, Class. & Quant. Grav., 2, 707, 1985b.*Scientific Knowledge from the Perspective of Quantum Cosmology* gr-qc/9601046; R. Geroch and J.B. Hartle, *Computability and Physical Theories*, Found. Phys., 16, 533, 1986.

[10] S. Kauffman and L. Smolin, *A possible solution to the problem of time in quantum cosmology*, gr-qc/9703026,to appear in the Proceedings of the Karpaz 99 winter school.

[11] S. Kauffman and L. Smolin, Combinatorial dynamics in quantum gravity hep-th/9809161, to appear in the Proceedings of the Karpaz 99 winter school, 1999.

[12] S. Kauffman *Investigations*, Oxford University Press, in press.

[13] J. Stachel, *Einstein's search for general covariance, 1912-1915* in *Einstein and the History of General Relativity* ed. by D. Howard and J. Stachel, Einstein Studies, Volume 1 Boston: Birkhauser.

[14] C.Rovelli, Phys. Rev. **D43**, 442 (1991).

[15] L. Smolin, *The Life of the Cosmos*, Oxford University Press,NY and Wiedenfeld and Nicolson, London, 1997

[16] L. Smolin *Three Roads to Quantum Gravity*, Science Masters Series, Wiedenfeld and Nicolson, London, 2000, Basic Books,NY, 2001.

[17] J. B. Barbour, Nature 249 (1974) 328 (Erattum Nature 250 (1974) 606; Nuovo Cimento 26B (1975) 16; Brit. J. Phil. Sci. 33 (1982) 251; J. B. Barbour and B. Bertotti, Nuovo Cimento 38B (1977) 1; Proc. Roy. Soc. Lond. A 382 (1982) 295.

[18] E. T. Newman, personal communication.

[19] A. Sen, *Gravity as a spin system* Phys. Lett. B119 (1982) 89; *Quantum theory of a spin 3/2 system in Einstein spaces*, Int. J. of Theor. Phys. 21 (1982) 1.

[20] A. Ashtekar, Phys. Rev. Lett. 57 (1986) 2244; Phys. Rev. D36 (1987) 1587.

[21] For reviews L Smolin: in Quantum Gravity and Cosmology, edsJ Perez-Mercader et al, World Scientific, Singapore 1992; *The future of spin networks* gr-qc/9702030 in the Penrose Feshscrift; R. Gambini and J. Pullin, *Loops, knots, gauge theories and quantum gravity* Cambridge University Press, 1996; C. Rovelli, gr-qc/9710008; L. Smolin, *The future of spin networks* gr-qc/9702030.

[22] T. Jacobson and L. Smolin, *Nonperturbative quantum geometries* Nuclear Physics B299 (1988) 295-345.

[23] C. Rovelli and L. Smolin, Phys Rev Lett 61 (1988) 1155; Nucl Phys B133, 80 (1990);*The physical hamiltonian in nonperturbative quantum gravity*,Phys.Rev.Lett.72(1994)446; *Discreteness of area and volume in quantum gravity* Nuclear Physics B 442 (1995) 593. Erratum: Nucl. Phys. B 456 (1995) 734.

[24] C. Rovelli and L. Smolin, "Spin networks and quantum gravity" gr-qc/9505006, Physical Review D 52 (1995) 5743-5759.

[25] Petr Horava, *M-Theory as a Holographic Field Theory*, hep-th/9712130, Phys.Rev. D59 (1999) 046004.

[26] L. Smolin, *The cubic matrix model and a duality between strings and loops*, hep-th/0006137.

[27] Griffiths, *Consistent Histories And The Interpretation Of Quantum Mechanics*,J.Statist.Phys. 36 (1984) 219; Omnes, *Logical Reformulation Of Quantum Mechanics*, Journal-ref: J.Statist.Phys. 53 (1988) 893.

[28] Murray Gell-Mann, James B. Hartle, Equivalent Sets of Histories and Multiple Quasiclassical Realms, gr-qc/9404013;J. B. Hartle, gr- qc/9808070

[29] F. Dowker and A. Kent, J. Statist. Phys. **82** (1996) 1575 [gr-qc/9412067].

[30] J. Butterfield and C.J. Isham, *Some possible roles for topos theory in quantum theory and quantum gravity*, gr-qc/9910005.

[31] J. D. Bekenstein, Lett.Nuovo Cim.4:737-740,1972, Phys.Rev.D7:2333- 2346,1973, Phys.Rev.D9:3292-3300,1974

[32] G.'Hooft, *Dimensional reduction in quantum gravity*, gr-qc/9310006.

[33] L. Susskind, *The world as a hologram*, J. Math. Phys. 36(1995)6377-6396, hep-th/9409089.

[34] Markopoulou F, 1997, *Dual formulation of spin network evolution*, gr-qc/9704013.

[35] L. Smolin, A proposal for the background independent formulation of M theory hep-th/9903166.

[36] G.F.R Ellis, in the Proceedings of GR12, Padua Italy.