Soliton Compton Mass from Auto–Parametric Wave–Soliton Coupling

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In this paper a self-excited Rayleigh-type system models the auto-parametric wave-soliton coupling via phase fluctuations. The parameter of dissipative terms determine not only the most likely quantum coupling between solitons and linear waves but also the most likely mass of the solitons. Phase fluctuations are mediated by virtual photons coupling at light-velocity in a permanent Compton scattering process. With a reference to the SI-units and proper scaling relations in length and velocity, the final result shows a highly interesting sequence: the likely soliton Compton mass is about 1.00138 times the neutron and 1.00276 times the proton mass.

Introduction. For nonlinear field theory models in 1+1–dimensional space–time the equations of motion admit finite energy and finite width solutions called solitons [1]. Solitary waves were discovered in the first half of the nineteenth century by Rusell, the word soliton was invented by Kruskal, the sine–Gordon (SG) model by Skyrme [2]. Solitons retain their identity after collisions, can annihilate with anti–solitons, many–soliton solutions obey Pauli's exclusion principle. In 1+1–dimensional space–time there are two non–trivial minimal quantum field theories which describe non–perturbative phenomena: the SG model and the massive Thirring model [3] (a self-coupled Dirac field, see the Lagrangians [4]), both are intimately related [5]. It is an interesting question how stationary solitons (like breather) get their absolute mass/energy. In this paper a self-excited Rayleigh-type system [6] will model an auto-parametric wave-soliton coupling via phase-fluctuations. This system can as a classical example model simplified music instruments (in the original work of Rayleigh a clarinet reed). In [7] it has recently been shown, that Rayleigh-type self–excited auto-parametric systems [8] can stimulate "whispering gallery modes" and model Coulomb interaction between sine-Gordon solitons. In this paper exactly the same model will be applied to determine the most likely Compton mass of the soliton.

Quantum field theory. In quantum field theory, particles correspond to fluctuations of a space-time function that minimizes the action for some Lagrangian L which is itself the space integral of a Lagrangian density $\mathcal{L}$. A low-dimensional (bosonic) SG field of a single hermitian scalar field θ in 1+1 dimensions with Lagrangian density $\mathcal{L} = \frac{\mu}{2}\partial_\nu\theta\partial^\nu\theta - V(\theta)$ is a function of one space dimension and time. The time independent field equations reads $\mu\partial_r^2\theta = \partial_\theta V$ which can also be written as

$$V(\theta) = \frac{\mu}{2}(\partial_r\theta)^2\,, \tag{1}$$

where μ is the soliton mass. In the quantum model, the expectation of the square of the phase gradient contributes to the soliton self-energy, where the phase gradient usually scales proportional to the wave number that is proportional to the frequency.

Rayleigh–type perturbed SG. The coupling between linear waves and non–linear solitons could be important for the understanding for scale-dependent electromagnetic coupling. The Rayleigh-type perturbed SG equation is given by

$$(\partial_{rr} - \partial_{tt})\,\theta - \sin\theta = \left[b^2(\partial_r\theta)^2 - a^2\right]\partial_t\theta\,, \tag{2}$$

where $\partial_{rr} - \partial_{tt}$ is the D'Alembert wave operator. On the left side are the usual SG terms, on the right side are with eq.(1) the typical Rayleigh wave dissipation and regeneration terms. These two terms balance wave dissipation and regeneration and can be found in self–excited auto-parametric systems [8]. The term $b^2(\partial_r\theta)^2 - a^2$ controls θ by weighting the dissipative first order term $\partial_t\theta$. The amplitude of the coupling wave can be assigned to a phase difference between two solitons.

Rayleigh–type auto–parametric system. Following [8] we characterize auto–parametric systems as a vibrating system which consists of at least two subsystems: an oscillator that is generally in a vibrating state and the excited system which is excited indirectly and coupled to the oscillator in a nonlinear way. A linear wave will be defined as a disturbance which transports energy from one location to another location without transporting matter evolving at light speed c with $c\,\partial_r\theta = \partial_t\theta$. In the auto–parametric system the dissipation and excitement parameter will depend on the phase statistics. The Rayleigh–type system

$$\partial_{tt}\theta + \theta = a^2\partial_t\theta - b^2c^{-2}(\partial_t\theta)^3 \tag{3}$$

is called self–excited auto–parametric, and can be exactly treated by phase averaging methods providing for the semi–trivial solution [8]. The excitation is here also balanced by the (soliton vortex) regeneration term $V(\theta)$ of eq.(1) with parameter b^2 and (linear mode) damping term $\partial_t\theta$ with parameter a^2. So a, b, and c control the amplitude θ of intermediating linear waves.

The coupling mechanism. The vertex solitons are assumed to be identical and not synchronized in phase, such that the excitement of coupling modes is based on the (statistical) phase difference. Let the SG solitons be coupled such, that the amplitude of the perturbed linear wave will be the phase difference θ of the perturbed nonlinear SG solitons controlled by the potential of the phase gradient $V(\theta)$. Usually the non-trivial equilibrium fluctuation amplitude θ_0 can be found with given parameter a and b or ratio $q = b/a$, but in our case the scattering phase fluctuation amplitude θ_0 is given by the relative phase fluctuation range of solitons. Regarding the kink-antikink solution there exists a characteristic time scale or oscillation frequency that can be related via coupling wave velocity to a characteristic length scale or wavelength λ_μ. The fermion–like behavior pointed out by Skyrme [2] suggests to define an additive phase–fluctuation range by $2\times 2\pi = 4\pi$. Therefore, the coupling amplitude for the mediating wave will be given by

$$\theta_0 = 4\pi\lambda_\mu \,. \tag{4}$$

Later λ_μ will be reintroduced as the Compton wavelength of wave–soliton coupling.

Phase averaging. Statistically, there are phase oscillations or fluctuations randomly modulated in a special phase interval, so we can assume in the most simple case a maximum entropy phase modulation and fluctuation between two vertex solitons and find the most likely energy flow and coupling ratio from phase averaged solutions [9]. The phase–amplitude modulation with random τ as a solution to eq.(3) can be written as [8]

$$\theta = \theta_0 \cos(\tau + \psi_0) \,, \tag{5}$$

averaging over τ yields [8, 9] with constant ψ_0

$$\partial_t \theta_0 \propto \frac{1}{2}\left(a^2\theta_0 - \frac{3b^2}{4\lambda_\mu^2}\theta_0^3\right), \quad \partial_t\psi_0 = 0 \,. \tag{6}$$

1-dimensional coupling soliton energy. With constant fluctuation strength $\partial_t\theta_0 = 0$, the averaged equations provide with eq.(4) and eq.(6) for a balanced non-trivial equilibrium

$$q = \frac{b}{a} = \frac{1}{4\pi\sqrt{\frac{3}{4}}}, \quad \frac{2\overline{V}}{E_\mu} = \overline{(\partial_r\theta)^2} = q^{-2} \,. \tag{7}$$

Consequently, the 1-dimensional coupling strength q will enter the energy definitions via mean unit energy E_{1d} of 1-dimensional coupling

$$E_{1d} = E_\mu = q^2\overline{(\partial_r\theta)^2} = 1\mu c^2 \,. \tag{8}$$

To compare our theoretical soliton coupling model to real existing couplings, mass/energy has to be quantified. Let the unit mass be the 1d coupling mass–energy of a soliton with $E_\mu = 1\mu c^2$. The mutual 1-d coupling to a photon with amplitude/wavelenght fluctuation λ_μ can be regarded as a permanent Compton scattering process with mass–energy value related to λ_μ via Compton relation

$$E_\mu = 2q^2\overline{V} = \frac{hc}{\lambda_\mu} \,. \tag{9}$$

Energy scaling. According to eq.(7) and eq.(8), both E_μ and $\overline{V}$ scale with c^2. E_μ can be assigned to the soliton energy and mass. The absolute energy reference is based on the human artificial energy unit E_u referring to the kinetic energy of a unit mass 1μ moving at unit velocity u (in SI 1J = 1kg m^2/s^2). We will now find the scaling relations of E_μ and $\overline{V}$ with respect to the energy unit E_u provided by the system of units (SI) on one end and by Planck units ($\hbar = c = \mu = 1$) on the other end of the scale. $\lambda_u = 1m$ will be the human artificial reference wavelength. Planck units demand that

- the light velocity equals the unit velocity $c = u = 1$,
- $2\overline{V(c=1)} = E_u(u=1)$,
- the the Compton length equals the unit length $\lambda_\mu = \lambda_u = 1$,
- $E_\mu(\lambda_\mu = 1) = E_u(\lambda_u = 1)$.

The algebraic form of the two scaling relations is given by

- the mean soliton coupling energy $\overline{V}$ scales with the square of the wave velocity

$$\frac{2\overline{V}}{E_u} = \frac{c^2}{u^2} = \Xi^2, \quad \Xi = 299792458, \tag{10}$$

- the 1-dimensional quantum energy of waves coupling to particles E_μ is inversely proportional to the wavelength, especially to the Compton wavelength

$$\frac{E_\mu}{E_u} = \frac{\lambda_u}{\lambda_\mu}. \tag{11}$$

Practical necessity motivates to choose a unit velocity $0 < u \ll c$ with $\Xi \gg 1$ [10].

Results. The main result could be written as a characteristic soliton wavelength of one-dimension coupling exactly given with eq.(7) - eq.(11) by

$$\lambda_\mu = \frac{\lambda_u}{q^2\Xi^2}, \tag{12}$$

and provides for the soliton mass μ via Compton relation $\mu = h/(c\lambda_\mu) = q^2\Xi^2 h/(c\lambda_u)$. Realized in SI units the values are

$$\mu = \frac{\hbar}{c}\frac{\Xi^2}{6\pi\text{m}} \approx 1.67724... \cdot 10^{-27}\text{kg},$$
$$\lambda_\mu = \frac{12\pi^2\text{m}}{\Xi^2} \approx 1,31777... \cdot 10^{-15}\text{m}. \tag{13}$$

The corresponding SI-values for the wave number k_μ, angular frequency ω_μ, and energy E_μ are given by

$$k_\mu = 2\pi/\lambda_\mu = 4.7680443... \cdot 10^{15}\text{rad m}^{-1},$$
$$\omega_\mu = k_\mu c = 1.4294237... \cdot 10^{24}\text{rad s}^{-1},$$
$$E_\mu = \mu c^2 = \hbar c k_\mu = 940.86369...\text{MeV}. \tag{14}$$

It turns out, that μ shows a highly interesting and significant sequence with the fundamental masses of the two prominent baryons (Neutron with mass M_n and proton with mass M_{p+}) [10]:

$$\frac{\mu}{M_n} \approx 1.001382..., \quad \frac{\mu}{M_{p+}} \approx 1.002762.... \tag{15}$$

The system-invariant soliton mass scale is 1.001382 times the neutron and 1.002762 times the proton scale and could be interpreted as a stability limit. The neutron is energetically lower than μ, this could support semi–stability. In this context it is interesting to note, that two neutrons provide for approximately the same mass gain as one proton with respect to μ. This mass differences could be relevant for approaching (nuclear) binding energies with many–soliton models.

Can fundamental baryons be solutions to the SG? eq.(1) obtained from the SG–Lagrangian is the central control term of auto-parametric resonance and has also a central role in [11, 12, 13], where generalized fine-structure constants coupling between topological phase fields have been proposed. Solitons keep their energy in a permanent scattering process, retain their identity after collisions, can annihilate with anti–solitons, many–soliton solutions obey Pauli's exclusion principle. As pointed out by Skyrme [2] this can be interpreted as a fermion–like behavior. The interest in such low-dimensional SG models arises from their integrability, duality properties, non-perturbative aspects, the electric-magnetic duality in gauge theories, and as an universal concept in nonlinear science. Solitons appear in almost all branches of physics, such as hydrodynamics, condensed matter phenomena, particle physics, plasma physics, nonlinear optics, low temperature physics, nuclear physics, biophysics and astrophysics. Coleman's "Quantum SG equation as the massive Thirring model" [5] establish an identity between the quantized SG model and the Dirac spin-1/2 equation in 1+1 dimensions. But probably because of the low dimensionality and the wide acceptance of the standard model (including quark model) the interest was limited. Except the Skyrmion baryon approach (see i.e. [14, 15, 16]), predicting that the lowest energy $E_\mu = B$ is more than a factor $q^{-2} = 12\pi^2$ lower than the energy given by the Lagrangian (Fadeev-Bogomolny bound) and confirming the result of auto–parametric resonance in eq.(7). The central role approaching the baryon mass scale is the shift of human artificial units to Planck units (at the Fermi scale!), this extrapolates the soliton energy by a factor Ξ^2. The deviations in eq.(15) of course suggest to play with the self-coupling Dirac equation and massive Thirring model [3] and include other particles as dual or partner soliton states. This could eventually fill the gap and build the bridge to a 4-dimensional integrable quantum field theory.

Relevance to the previous work. Extremely interesting is the topological charge q obtained from the auto–parametric model regarding topological phase fields and electromagnetic coupling. Via Gauss relation the 1-dimensional coupling parameter q can also be related to a 3-d coupling with spherical "whispering Gallery modes" (WGM) coupling with a 3-d potential $\phi_{3d} = \frac{1}{137r}$. Why integral? When the round-trip path fits integer numbers of the wavelength (single-valuedness), WGM are formed. The corresponding quantum number of orbital degeneracy is given by $M = \left[\frac{4\pi}{q}\right] = 137$ where [ ] means next higher integral value. In [11, 12, 13] generalized fine-structure constants based on a SG type pseudospherical coupling of topological phase fields have been defined. $M = 137$ with $q = e$ corresponds to the Sommerfeld fine structure constant $\alpha = e^2/(4\pi\epsilon_0\hbar c)$ according to the Dirac theory of magnetic monopoles [17] (a generater of the topological Berry phase [18]).

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