Photons uncertainty removes Einstein-Podolsky-Rosen paradox

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Abstract

Einstein, Podolsky and Rosen (EPR) argued that the quantum-mechanical probabilistic description of physical reality had to be incomplete, in order to avoid an instantaneous action between distant measurements. This suggested the need for additional "hidden variables", allowing for the recovery of determinism and locality, but such a solution has been disproved experimentally. Here, I present an opposite solution, based on the greater indeterminism of the modern quantum theory of Particle Physics, predicting that the number of photons is always uncertain. No violation of locality is allowed for the physical reality, and the theory can fulfill the EPR criterion of completeness.

INTRODUCTION

In 1935 Einstein, Podolsky and Rosen (EPR) provided one of the most severe criticism to the quantum theory with a paper entitled "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?", which has eventually become one of the most cited paper in the history of physics. In the abstract, they summarize their argument as follows: "In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete" [1].

As Laloë noticed in a recent review, for Einstein and collaborators "the basic motivation was not to invent paradoxes; it was to build a strong logical reasoning which, starting from well-defined assumptions (roughly speaking: locality and some form of realism), would lead ineluctably to a clear conclusion (quantum mechanics is incomplete, and even: physics is deterministic)" [2]. In fact, the EPR argument has been one of the main motivations for seeking a "complete" deterministic theory underlying quantum mechanics. In their (thought) "EPR experiment", the measurement of a physical quantity on a system A did influence instantaneously, and in a *perfectly deterministic* way, the result of a corresponding measurement performed on another spatially separated system B that had been interacting with A in the past. This fact, which was since called the EPR paradox, was considered to be a hint for a fully deterministic theory underlying the probabilistic quantum theory.

However, we will see that the modern Quantum Field Theory (QFT) description of Particle Physics, known as the Standard Model (SM) [3, 4], predicts a fundamental uncertainty about the number of photons that can be produced in any process involving an interaction. As a consequence, two distant measurements cannot influence each other in a certain, deterministic way, as required by the EPR argument.

THE EPR THOUGHT-EXPERIMENT AND THE EPR PARADOX

For our purposes, it will be sufficient to consider a class of "EPR experiments" defined as follows: two particles A and B [9] are emitted by a source; far apart, some conserved observable, such as a component of angular momentum (spin, helicity or polarization), is measured on particle A. According to the usual quantum-mechanical treatment, the measurement carried out on A reduces its state into an eigenstate of the measured observable, whose conservation immediately forces the second particle (B) to "collapse" into a corresponding eigenstate of this observable as well. For instance, let A and B be two spin 1/2 particles, produced with zero total angular momentum in a singlet spin state, described by the "entangled" spin vector

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|+\rangle_A |-\rangle_B - |-\rangle_A |+\rangle_B \right), \tag{1}$$

where $|\pm\rangle_A$ are the usual eigenstates of the spin component $S_z(A)$ of particle A with eigenvalues $\pm\hbar/2$ respectively (this example is usually called the "EPR-Bohm" experiment [2, 5]).

Assuming the state of Eq. (1), it is easy to see that the measurement of the spin component S_z performed independently on any of the two particles can give both values $\pm\hbar/2$, each with probability 1/2. On the other hand, if in a given single event $S_z(A)$ is measured on A and found equal to, say, $+\hbar/2$, it will then be possible to *predict with certainty* the result of the measurement of $S_z(B)$ on the distant particle B, that will give $-\hbar/2$. The quantity S_z observed on A would then instantaneously acquire "an element of physical reality" also on B, according to the original definition: *"If, without in any way disturbing a system we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there is an element of physical reality corresponding to this physical quantity"* [1] (the italics and the parenthesis also belong to the original paper). Moreover, the element of physical reality on B depends on the actual measurement that is done on A: for instance, if instead of measuring the component $S_z(A)$ of the angular momentum we decided to measure an observable incompatible with it, such as the component $S_x(A)$, then the state of the distant particle B after such a distant measurement would become an eigenstate of $S_x(B)$, rather than one of $S_z(B)$. Therefore, assuming that the two systems are no longer interacting, EPR deduced that two quantum-mechanically incompatible quantities ($S_z(B)$ and $S_x(B)$ in the example above) could be given a simultaneous reality. They then concluded that "the wave function does not provide a complete description of the physical reality";

otherwise, the definite values of $S_z(B)$ and $S_x(B)$ would have to "enter into the complete description, according to the condition of completeness" that they had defined as follows: "*every element of the physical reality must have a counterpart in the physical theory*" [1].

The EPR argument (later called paradox, see e.g. Ref. [6]) is so rigorous, that Laloë reformulated it in the form of a theorem: "*If the predictions of quantum mechanics are correct (even for systems made of remote correlated particles) and if physical reality can be described in a local (or separable) way, then quantum mechanics is necessarily incomplete: some 'elements of reality' exist in Nature that are ignored by this theory*" [2]. The importance of this argument, based on their objective definition of physical reality, is that it holds almost independently of the interpretation of the theory, with the exception of deterministic interpretations introducing hidden variables [2]. It is then impossible to find an "EPR-paradox-free" interpretation of the probabilistic quantum mechanics based on Eq. (1).

What EPR did not perhaps expect was that a way out was to be found in the modern version of the Quantum Theory itself. In fact, as we shall see, the SM description does not rely on the "entangled" state of A and B (whose spin part is Eq. (1) in the example that we have considered above), but it allows for the presence of an undetermined number of additional photons [10].

THE UNCERTAINTY ABOUT THE NUMBER OF PHOTONS

The EPR paradox, as described above, originates from the assumption of a state with a definite number of particles (two in our example), which is incorrect in Relativistic Quantum Mechanics. As we shall see, *the modern QFT description of Particle Physics predicts that it is impossible to prepare a state with a definite number of particles as the result of a given physical process, since additional real particles can always be created in the production process itself.* Which additional species can appear depends on the available energy. Since massless particles can have arbitrarily low energy, the possible presence of real "soft photons" (i.e. photons having a low enough energy) should always be taken into account in the theoretical treatment.

Here, I will prove this statement using QFT perturbation theory (i.e. Feynman diagrams). To be concrete, I will first discuss two kinds of ideal EPR experiments: i) those involving two charged spin 1/2 particles; and ii) those involving two photons. In both cases, I will
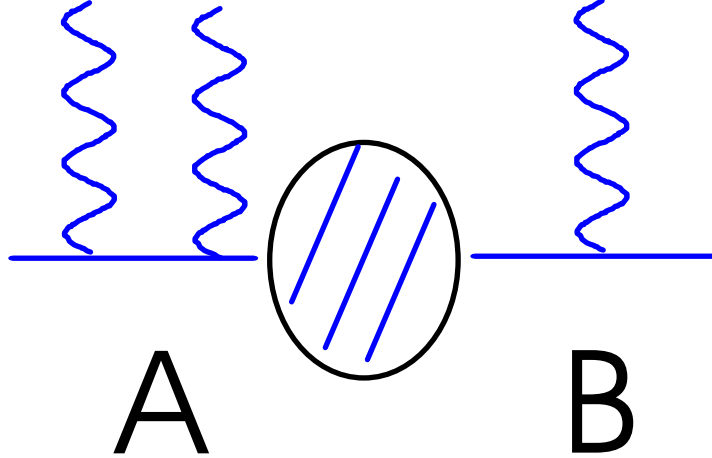
FIG. 1: Feynman diagram describing the production of an EPR pair of charged spin 1/2 particles, A and B, in coincidence with three additional photons. The dashed blob represents the part depending on the particular basic process and the initial particles that are considered.

give explicit examples predicting the creation of an arbitrary number of additional photons.

i) In Fig. 1, I have drawn a tree-level diagram where the "blob" represents the particular elementary process that produces particles A and B. Even without specifying that part of the diagram (involving some "initial" particles), we see that an arbitrary number of real photons (three in the particular case of the figure) can be attached to each of the external fermion legs (see also Chap. 13 of Ref. [3]).

ii) Since no three photon vertex exists at the tree level, in the "two-photon" EPR experiment we have to look for one loop effects. In Fig. 2, I show a "box" diagram for the production of two additional real photons [11]. The virtual particle in the loop can be any charged fermion (electron, muon, tau, quarks).

Note that additional photons can also be emitted by the other legs of the diagrams, those corresponding to the particles that are included in the "blob", or even by loop diagrams involving charged particles that can contribute to the "vertex part" of the "blob". All the relevant SM diagrams can be drawn, depending on the particular production process that is considered, although Feynman perturbation theory breaks down when the additional photons are attached to particles belonging to a bound system such as an atom [3]. Fortunately, for the purposes of the present paper I do not need an exact computation of all the possible contributions.

The rates for the production of a given number of additional photons should be compared
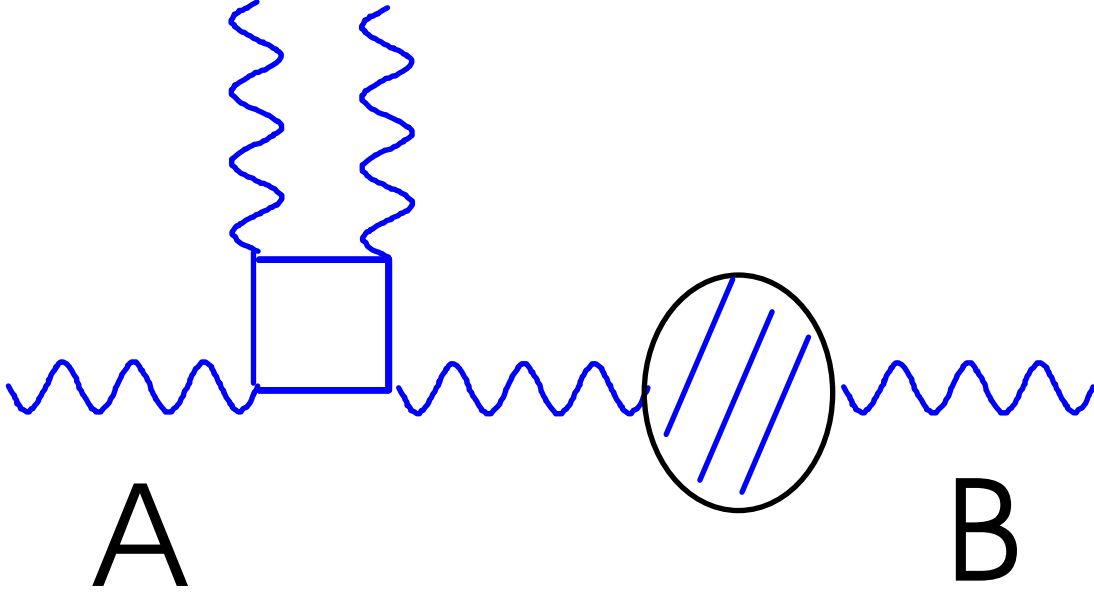
FIG. 2: Feynman diagram describing the production of an EPR pair of photons, A and B, in coincidence with two additional photons. The dashed blob represents the part depending on the particular basic process and the initial particles that are considered.

to the rate for the process in which only particle A and B are emitted, corresponding to the bare diagram without any additional photons attached (that would imply the same correlations as the old quantum-mechanical approach) [12]. In any case, such rates are suppressed by increasing powers of the fine structure constant $\alpha \simeq 1/137$, depending on the considered number of additional photons. Since particle A is detected when a measure (for instance of some angular momentum) is made on it, its energy E_A can also be measured. Therefore the upper limit for the total energy of the additional photons is $\Lambda = E - E_A - m_B c^2$ (where E is the total energy liberated in the basic production process). This limit reduces the phase space available for the diagrams involving an increasing number of additional photons. In the case of a two-photons EPR experiment, the probability of a diagram such as that in Fig. 2 is suppressed by four powers of the fine structure constant and by the electron propagators in the loop (which are larger than the available energy, unless one considers ideal EPR experiments with very energetic photons, at or above the MeV range); the additional photons radiation from the atomic system or the charged fermions that appear in the "blob" part of the diagram is then expected to be more important.

To summarize: the total probability for the creation of additional photons is usually

expected to be of order α with respect to the process involving only particles A and B, although the precise suppression factor depends on the particular case that is considered. For our purposes, it is sufficient to note that there is a non-vanishing probability for additional photons to be created, at least due to diagrams such as those of Figs. 1 or 2, and that they contribute to energy, momentum and angular momentum conservation.

The important fact is that this uncertainty principle can be generalized: *an undetermined number of photons is created in any experiment, in any step that involves an interaction,* and in particular in the process that originates our EPR particles [13]. In fact, all the known elementary particles, including the neutral ones, such as the photon, the neutrino (and even the possible Higgs boson), can radiate photons when they appear as external legs of an interaction process. For the charged particles and for the photon, this fact is shown in Figs. 1 and 2. For a neutrino (or a Higgs boson), it is easy to construct loop diagrams involving virtual W bosons and charged fermions to which a photon line can be attached. In the case of composite particles, such as the neutral K or B mesons, the production process involves the constituent quarks, charged particles to which external photon lines can be attached just as in Fig. 1 [14].

This generality is by no means accidental, but it corresponds to a well-known characteristic of QFT: any process that does not violate the fundamental symmetries is "allowed" and has a non-vanishing amplitude. Exceptions to such a "rule" are so rare, that they are thought to hide new symmetries. Here, it is sufficient to note that no symmetry forbids the radiation of additional photons in coincidence with a given interaction, since photons do not carry any conserved "internal" charge.

The uncertainty about photons radiation can also be related to a symmetry principle. In fact, it is based on two points: the existence of massless neutral particles, the photons; and of the fermion-photon vertex, that allows for the radiation of photons by any external line of the relevant Feynman diagrams (possibly through loops as in Fig. 2). But it is well known that both the electromagnetic vertex and the masslessness of the photon are the direct consequences of the local, unbroken (electromagnetic) gauge symmetry.

THE EPR PARADOX REMOVED

According to the previous discussion, the state arising from the interaction is *never* an eigenstate of the operator counting the number of photons: *the number of photons cannot be determined (it never gets a physical reality).* This implies that *it is never correct to use a state with a fixed number of particles, such as that of Eq. (1), as emerging from a given interaction.*

Now, in a given Feynman diagram, the conservation laws hold for the set including particles A and B together with all the additional photons that appear in that diagram. Therefore, *after the measurement on A in any given single event, the energy, momentum and angular momentum conservation laws do not hold for the two particle (sub)system, A and B.* The detection of particle A does not necessarily correspond to particle B appearing in the opposite direction. Moreover, *the measurement on A does not allow for a certain prediction of the value of the considered conserved quantity* (be it energy, momentum or angular momentum) on B (B is not put in an eigenstate of the observable that has been measured on A). For instance, in the EPR-Bohm experiment, S_z *will not be given a "physical reality" on B after it is measured on the distant particle A.* According to our previous discussion, this is sufficient to save the theory from the original EPR paradox. Note that this result holds even for a small probability of additional photons radiation.

Note that we can know that particle B appears in the given region, and that it is in a given eigenstate of the considered observable, only *after* detecting the particle B and measuring the considered observable on it. Therefore, the physical reality is given to the observables on B only after the measurement on its own location is performed. *In this sense, QFT respects local realism: the elements of physical reality of the theory can be obtained only after local measurements.* According to this definition, we can even say that QFT is locally realistic, although such a definition is usually reserved to hidden variables theories.

A general single event, where only particles A and B are detected, can show apparent symmetry violations. In particular, any violation of a discrete variable such as angular momentum is important, since it is a multiple of $\hbar$. These considerations suggest that a possible signature of the theoretical solution I am proposing would be the observation of an apparent symmetry violation event in an EPR experiment (actually due to the presence of additional unobserved photons).

It is worth pointing out that there is no possibility of getting rid of the uncertainty about the number of photons. Even if we filled the whole space with detectors, we would never catch all the possible photons involved in a single event, since they can have arbitrarily low energy [3]. Only a definite number of photons will be detected, while the state produced in the basic process has no definite photons content. After the measurement, the amplitude for the additional undetected photons spreads over the whole space, eventually overlapping with A and B; therefore there is no theoretical possibility to define two determined spatially separated subsystems as required by the EPR argument. Strictly speaking, in QFT it can be correct to say that A and B themselves are spatially separated only *after* measuring on both particles, since the measurement on A and global momentum conservation are not sufficient to ensure the "collapse" of B as a particle in a given direction as well (it is even possible that A and B are caught by the same detector!).

In other words, the modern QFT description is even less deterministic than the old non-relativistic Quantum Mechanics. In fact, the only predictions that it allows are on probabilities and average values. This greater uncertainty protects the theory from the EPR paradox. It seems that, to remove the paradox, one has to choose between the most extreme possibilities: determinism (hidden variables), or complete lack of determinism for the single event (QFT, the dice of God). We also see that the SM fulfills the EPR criterion of completeness that was cited above: for instance, in QFT the Fock space state vector after the measurement of a conserved charge on a massive particle is an eigenvector of that one-particle charge operator. The element of physical reality that appears after the measurement then has a counterpart in the theory, being the corresponding eigenvalue (i.e. the definite value) of this one-particle observable [15].

Ultimately, the recovery of local realism can be attributed to the local gauge symmetry that implies the uncertainty about the photons as we have seen. I think that this result is not surprising, since the local gauge symmetry implies local interactions. Note also that the possibility of radiating arbitrarily soft photons corresponds to the "infrared" behavior of the theory, i.e. to its long distance properties, which are precisely those that are expected to be relevant for the discussion of distant measurements.

ON BELL'S VARIANT OF THE EPR EXPERIMENT

The actual so-called EPR experiments [2, 7], that have been inspired by the work of Bell [8], do not test the EPR argument directly. First of all, the measurement on particle B is performed and only the events where particles A and B appear in coincidence at opposite directions are considered. The fact that we need to detect B in order to define such events already differentiates such experiments from the ideal EPR experiment. Moreover, the results are given in terms of the correlations between the polarizations of the two (or more) particles A and B, which are statistical averages over the products of the observed spin/polarization components for the different single events. For instance, in Bell's version of the EPR-Bohm experiment, the relevant correlations are the average values of the products of the components $S_{\vec{a}}(A)$ and $S_{\vec{b}}(B)$ of the spins of the two particles along arbitrary unit vectors $\vec{a}$ and $\vec{b}$ [8]. The old Quantum Mechanics prediction, based on the spin state of Eq. (1), was $\langle S_{\vec{a}}(A)S_{\vec{b}}(B)\rangle = -\frac{\hbar^2}{4}\vec{a}\cdot\vec{b}$, which is the maximal correlation (in absolute value) that can be achieved. In fact, the original EPR paradox requires strictly maximal correlations, since it is about completeness and determinism, being formulated for the objective "physical reality" (as we have seen, the measurement on A should imply a *certain* prediction of the measurement on B). These are theoretical problems, impossible to test directly due to the unavoidable experimental errors. On the other hand, Bell's version of the EPR experiment was explicitly aimed at testing the hidden variables solution of the paradox [8], and not the EPR paradox itself.

In fact, we have seen that the SM is EPR-paradox-free due to the photons uncertainty. This is a theoretical result. It is not difficult to see that the SM also predicts EPR correlations close to those calculated by the old quantum mechanics approach that agreed with the experimental data. In fact, allowing for the radiation of additional photons, the correlations will be smaller than maximal. However, we have already seen that the correction will imply some suppression factors at least proportional to some power of the fine structure constant. Moreover, the selection of the events where A and B appear in opposite directions implies a further, severe reduction of the phase space available for the additional photons, whose transverse momenta should add up to zero within the small solid-angle uncertainty given by the cross section of the detector divided by the distance from the production point [16]. For this reason, the contributions to the correlations from the diagrams involving additional

photons are expected to be small compared with the experimental errors, and will not spoil the previous agreement with the data (the "Bell's inequalities" [2, 8] will still be violated as observed). Such correlations, although they are not strictly maximal, may still be used in Quantum Computing and Quantum Information Theory [2].

SHOULD THE EPR CORRELATIONS BE CONSIDERED NONLOCAL?

The EPR correlations are usually interpreted as a sign of some non-locality in themselves, since they cannot be justified by a deterministic local theory based on additional "hidden variables" [2, 8]. Such a non-locality would now be less problematic, since I have shown that locality is respected by all the elements of physical reality (this allowed us for the removal of the EPR paradox). However, note that the original motivation for introducing hidden variables in the study of the EPR correlations was the EPR argument. Now this motivation has disappeared. Without introducing hidden variables, should QFT be considered non-local invoking "Bell's theorem" [2, 8]? Since the "objective" proof of the nonlocality of the theory, that was based on the EPR argument, has been removed by the photons uncertainty as discussed in the present paper, it may be expected that the decision about the local or nonlocal character of the theory has become a problem of the interpretation of the measurement process in QFT. This is not so obvious, since Bell's argument has been reformulated under very general hypothesis [2] [17]. I think nevertheless that this problem deserves further research, that could take also into account the views that I have presented here.

CONCLUSIONS

The QFT description of Particle Physics has been shown to be protected from the original EPR paradox by the local gauge symmetry. This corresponds to the fact that it allows for the creation of an undetermined number of photons in any interaction, in particular in coincidence with the observed particles in an EPR experiment. This result is particularly important since it does not depend on the interpretation of the theory, and it removes one of the most disturbing paradoxes of the quantum theory, the violation of locality in the "objective reality". On the other hand, the QFT description, which is not limited

to the "entangled" wave function with a definite number of particles, can fulfill the EPR criterion of completeness (although it can hardly be considered to be the ultimate "theory of everything", e.g. it does not describe gravity). This solution would be confirmed by the observation of an apparent symmetry violation in a single event in an EPR experiment. On the other hand, the EPR correlations are expected to be smaller than those calculated by ignoring the soft photons, but in the case of the actual experiments inspired by the work of Bell the correction is expected to be small, so that the agreement of the Quantum Theory with the present data is not spoiled. Such correlations are thought to be in themselves a sign of some "quantum nonlocality". This problem possibly depends on the interpretation of the quantum theory and deserves further research, that could profit from the views that I have presented here.

Acknowledgements

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[9] It is easy to generalize the present discussion to the case of three or more particles.

[10] Although this fact may be not surprising for Quantum Field Theorists, up to now it was not explicitly formulated and applied to the original EPR problem. As far as I know, all the existing treatments of the original EPR paradox have assumed entangled states with a definite number of particles, such as in Eq. (1).

[11] This box diagram has been studied in different contexts, e.g. in the theory of two photon scattering.

[12] The "Infrared Divergencies" related to the integration over arbitrarily small photons momenta can be handled as shown in Chapter 13 of Ref. [3], and they eventually cancel.

[13] An undetermined number of photons can also be created due to the interaction of any observed system with the particles belonging to the measuring apparatus. Although such an effect will not be used in the following discussion, it can be interesting for the Theory of Measurement.

[14] In all such cases, additional photons can also be radiated by the lines of all the possible charged particles (besides the EPR pair) that are involved in the relevant "blob" production process.

[15] Possibly the only observables that can actually get a (local) reality in QFT are the conserved charges. This would not prevent QFT to fulfill the EPR criterion of reality, that only requires the existence of a counterpart for all the physical reality of the theory.

[16] Note also that in the case of the EPR-Bohm experiment the diagrams allowing for parallel (rather than antiparallel) spins of A and B are suppressed by powers of $\left(\frac{\Lambda}{E}\right)^2$, where Λ is the energy available for the additional photons [3].

[17] The problem of the violation of Bell's inequalities is sometimes called "EPR paradox", although many authors still distinguish between the two problems. Here we have seen that they are different: the original EPR paradox was formulated for the objective "physical reality", and was removed by the photons uncertainty.