KNOWING AND ERRING: THE CONSOLATIONS OF ERROR

Essay On Developmental Epistemology (Excerpt)

by

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THE LOSS OF CERTAINTY

The title of this book partially stems from a translation of Ernst Mach's work, Erkenntnis und Irrtum, written at the end of the nineteenth century. Mach attempted to base knowing on the physiological processes of perception, perhaps as part of the effort to make the study of epistemology scientific. Epistemology--the study of knowledge or how humans come to know--is to modern philosophy as the study of ethics was to Greek philosophy. Mind is replacing action and soul as the human center and therefore as the center of philosophy. At the same time, many today long for certainty in knowledge and find "error" to be as corrupting as the snake in the garden of Eden. Error, it is felt, poisons human existence, the admission that error is an inevitable part of life is intolerable. Our culture's watchword is, "Know the truth and the truth shall make you free." To question this, many feel, is to render life as a shadow or illusion, and to question the foundation of existence. At first, religion promised to slay the dragon of error; then science promised it would do it; then philosophy tried. The present century opened with the logical optimism of Bertrand Russell's analytic philosophy, but it is closing otherwise.

A specter haunts the mathematical/logical community since the late nineteenth century, which has developed into a crisis in the study of epistemology. The specter stems from the undigested contradictions of Russell et al to the deeper dissonances of Gödel, which precluded hope of the absoluteness of consistency and truth. The incontrovertible possibility of ineradicable error has invaded mathematics/logic. The Russell-like paradoxes had opened Pandora's box, and even if we manage to resolve one such paradox, the possibility of others is present. Gödel's theorems--that there are true, unprovable statements and that there is no absolute proof of the consistency of sufficiently complex mathematical systems--furthered the inevitable possibility of error and paradox in classical mathematics. Consonant problems in science arose around the same time, 1900, particularly in physics, which likewise shook the paradigm of cause-effect relations among phenomena. This increasingly led to ideas of propensities and statistical connections linking complex phenomena.

The major response to the dissonance posed by the possibility of inevitable error has been to encapsulate and effectively to ignore it. To be sure, Formalists like Hilbert tried to delimit mathematics to more finite methods to avoid the paradoxes of the infinite and to restrict what is mathematics to symbols, thereby vitiating the idea of meaning in mathematics. Hilbert's drive to divest mathematics/logic of error derived from a need for certainty:

"If mathematical thinking is defective where are we to find truth and certitude?" (Hilbert, "On the Infinite" in Philosophy of Mathematics ed. Benacerraf and Putnam p.134)

This need for certainty leads to a view of mathematics as an immaculate conception, unsullied by the earthly dialectical interaction of the cognitive being and the world. It has led to the avoidance of powerful tools in mathematical reasoning, such as Aristotle's law of the excluded middle.

Intuitionists like Brouwer limited logic and mathematics to finite processes without recourse to the law of the excluded middle in infinite domains, and determined that existence be based on the direct production of the existent.

Both these tendencies to banish error from mathematical/logical thought are rooted in a need to preserve the faith in the certainty of mathematics as a pristine heaven walled off from uncertainty and error. A variant of this attitude may be found in the attempts to separate pure from applied mathematics. Fear of error-making and uncertainty have played a subterranean and deleterious historical role in human thought. Mathematics/logic is nonetheless no worse off than the physical sciences which are, together with their mathematical underpinnings, fraught with the same potential for error and inconsistency, in addition to that of their physical models. So, mathematics/logic remains a source of relative certainty for the more empirical sciences, while it remains in a deductive/empirical dialectic while probing the nature of things. In amplification of this view that error, carefully nurtured, can foster the development of important metaphors and deepened understanding, one can cite the examples of the Dirac delta function and divergent infinite series, each arising from the powerfully productive but cognitively ambiguous world of Quantum Theory. The delta function, zero almost everywhere but infinite at one point was decried by many mathematicians as meaningless, used by many physicists to understand and apply the theory pretty adequately, and ultimately understood mathematically as a functional. The "error" evolved and deepened our understanding both mathematically and physically. Similarly, the divergent asymptotic series of Quantum Theory were held onto despite their divergence until they were re-understood (normalized), made convergent and proved fruitful in understanding the phenomena of the theory.

Piaget's developmental cognitive/epistemological models of knowing suggest that error making, awareness of dissonance, and the development of cognitive structures are important bases of creatively understanding problems of consistency and paradoxes. Mathematics is rooted in the human cognitive structure, and therefore is and should be treated as a developmental discipline with an empirical component intertwined with its essentially theoretical and speculative bases. Consonant with the views of Lakatos and Popper, mathematics should function somewhat parallel to the other sciences: that is, proofs should, wherever possible, adduce models of validation, counter examples should be actively sought, and understanding of error-making in mathematical/logical processes should be promoted as ways of deepening our knowing processes through acknowledging the role error plays in our cognitive processes.

The paradoxes and errors resulting from infinite processes and the law of the excluded middle should not be the occasion for ridding ourselves of the culprits. Let us use them carefully and creatively, conceiving of mathematics/logic as rooted in an empirical mode in our minds and the outer universe, but as always based in a developmental psychological/cultural framework.

TOWARD A METAPHYSICS OF KNOWING

All knowledge is filtered through metaphors, and all knowledge is metaphoric. The remainder of this book will explore the ramifications of this view, its validation, and what appear to be its consequences. The metaphors of our knowing processes are not like free floating atoms: they form structures, adhere to our thought processes and interact in ways that characterize what we say we know. These structures, in our deliberations about the world, help account for our knowledge about it and for the errors we make in understanding the world. They form connecting links between the minds of, say, children, and those of sophisticates such as philosophers and scientists. One of the inevitable problems resulting from this is to characterize the nature of these developmental structures and the errors which they may lead us to commit. It is also necessary and interesting to understand how the errors become known as errors to other people or to the erring mind itself: How does cognitive dissonance occur and how can it be resolved?

The other side of the coin of erring is the structural network involved in our knowing processes. The taxonomy we form in science, the language structures we use in understanding are developmental in nature; since they emanate from humans they are, at their roots, both psychological and dynamically connected to the world about us. Our task is to elicit these structures of knowing, both in their psychic manifestations, in the cognitive structures which develop in our interaction with our world, and as these structures map out the nature of our world.

A paradigm which illustrates and deepens our metaphysics of knowing may be found in the following which concerns erring and knowing in a mathematical and physical context:

A root of the aphorism attributed to the ancient Greeks, that nature abhors a vacuum may be found in Aristotle's physics, where he analyzes the motion of a falling body. Aristotle said, in effect, that an object falls slowly in a highly viscous medium, more quickly in thinner oil, and even more quickly in air. He concluded that the speed of an object is inversely proportional to the viscosity of the medium in which it falls. Ergo an object would fall with infinite speed in a vacuum (which has zero viscosity), which is impossible, therefore there can be no vacuum. (His famous dictum was "Nature abhors a vacuum.") This error occurs in scientific literature up to the 17th century and is found in Kepler's writings before his elliptical orbit theory appears. Kepler, while investigating the motion of Mars with respect to the Sun, noted that its speed at its furthest point from the Sun is slower than its speed at its closest point. He concluded that the speed of Mars was inversely proportional to its distance from the Sun. Students in today's schools (including colleges) when presented with Aristotle's and Kepler's analyses, without being forewarned that they are both errors, generally agree with their analyses and conclusions. In other words, both historically and psychologically, assumptions about numerical connections among things tend to be linear or inversely linear. This suggests a psychological preference for certain modes of cognition that span the centuries. Furthermore, among sophisticated scientists this kind of error does not occur after the 17th century. The key idea needed to clarify these phenomena was that of function. As the analytical notion of function developed in the 17th century and thereafter, and the multiple possibilities of the ways in which ideas like $y=f(x)$ could manifest themselves, then the immediate assumption of linearity or inverse proportionality was no longer tenable when investigating relations between two variables. When the notion of function broadened to that of transformation, fixed points and invariants as foci of stability gained in physical, mathematical, and psychological importance. In the last hundred years, ideas of invariants grew, which led to Felix Klein's geometric theory based on the invariants of various geometric transformations and to Einstein's theory of relativity, which could be re-named the theory of absolutes, because it focuses on invariant laws and properties of the universe. Similarly the stability and the invariance of the atom can be understood as predicted by quantum theory based on discrete transfer of energy rather than on continuous transfer. Such ideas of invariance are importantly involved in Piaget's genetic epistemology, where knowing structures are understood by focussing on the invariants of knowing processes. In spite of important cognitive developments, the idea that "nature abhors a vacuum," persists.

AN EPISTEMOLOGICAL REBELLION

Cognition should be studied in its relation to both epistemology and psychology. The separation of the two disciplines of psychology and philosophy/epistemology early in the nineteenth century allowed the infant psychology to develop and gain independent maturity, but it should now return to a resumed dialectical interaction. This would be a fruitful interdisciplinary linkage, fostered by Piaget's not-always successful, but provocative and illuminating theory of developmental structures, by some of the non-behaviorist studies in artificial intelligence, and by the historico-philosophical approach of Thomas Kuhn.

The basic ideas which sustain this investigation into knowing as a developmental process are structure and error. All knowing, from that of children to that of adults, from early primitive societies to philosophically sophisticated and scientifically based cultures, takes place within a structural framework and is subject to erring as an essential aspect of the cognitive process. The structures are both cultural (sociological) and individual (psychological) and may be characterizable mathematically (as in some of Piaget and Kalechofsky) or verbally. Erring is part of the developmental knowing structures. It is the other side of the coin of knowing.

That knowing, whether philosophical, mathematical, scientific or whatever, is a human activity, necessitates a psychological component. However, this common-sense statement, is anathematical to the classical view of science as "objective" and is fiercely denied by many "gatekeepers." C. G. Hempel's response to a question at a symposium on Kuhn's contribution to the History and Philosophy of Science (M. I. T., May, 1990) concerning a psychological underpinning for Kuhn's view of paradigms as used in science, was that he could not accept psychology as a basis for understanding the history and philosophy of science, exemplifies the opposition to the idea of knowing as a human activity.

A variation of Hempel's attitude is found in Wittgenstein. One would have thought that the experience of teaching children (in Austria), and experiencing the involvement with the daily problems of teaching-learning would have informed him of the psychological, developmental underpinnings of epistemology, because the epistemological, developmental enterprise is operational at all times during an individual's life. However, the idea of the epistemological style which proceeds as though one is an adult disembodied head was a sophisticated fiction foisted by aspects of Wittgenstein, the Vienna Circle, the Positivist tradition and their more modern forms.

In his later writings, Wittgenstein did not refer explicitly to the effect of his teaching experience on his understanding of the knowing process, but he did refer to psychological-like models of picturing knowledge. However, as with most philosophers, the psychological-developmental process of knowing was not considered by him and when, for example, he referred to the principle of identity (A=A) he denied it any psychological basis. From a Piagetian and developmental viewpoint, the principle of identity can be linked to the child's initial structuring of the invariance of the object and is an important cognitive underpinning in the development of ideas of invariance and knowing processes in the child and the adult.

The above is the first 15 pages of the book, *Knowing and Erring, The Consolations of Error: Essay On Developmental Epistemology*, Marblehead, Massachusetts: Micah publications, ISBN 0-916288-45-5 1. (Micah Publications, 255 Humphrey Street, Marblehead, MA 01945. http://www.micahbooks.com or micah@micahbooks.com)

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