# THE CONTRADICTIONS IN THE CONSTRUCTIVIST DISCOURSE

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## Introduction

Constructivism is the most influential 'theory' of learning in education. In mathematics education in particular, many and if not most educators would claim to be constructivist in one form or another. The word 'theory', however, is inappropriate, for constructivism is not a structured framework of ideas or concepts that can determine an approach to mathematics education (a notable exception is the work done by David Tall, 1991, and colleagues on the learning of advanced mathematical concepts within a Piagetian framework). The point is that constructivism is more a belief system than it is a theory. It is a 'world-view' (*weltanschuung*) and a Broad Church (Matthews, 2000) that encompasses ethics, politics, ethnography and curriculum design and development.

The difficulty in writing about constructivism is that it is very hard to define or 'pin-down'. Despite the (literally) thousands of scholarly articles on mathematics and science education informed by a 'constructivist perspective', there are no text-books on constructivism as such – no A-B-C of its fundamental principles – just thousands of articles with hundreds of 'buzz-words' (e.g. *attentional frame* by von Glasersfeld, 1995; *label* by Mason, 1999) that are rarely referenced or duplicated. Of course, there are a few exceptions (e.g. David Wheeler's *mathematization* in Jaworski, 1994) but even some of these have been interpreted differently by different people (perhaps this would be a confirmation for the radical constructivist!).

Constructivism's popularity seems largely due to the consensus that the learner is not a passive recipient of knowledge but that knowledge is 'constructed' by the learner in some way. Von Glasersfeld (1995) calls this 'trivial constructivism' and many articles in mathematics and science education claim to be 'constructivist' because of the view

that learning is an active and not a passive affair. Indeed, many of the few critics of constructivism (e.g. Irzig, 2000) have argued that you don't have to be a constructivist to reject the passive recipient (or empty vessel) metaphor. Constructivism as a psychology – as a field of study that views learning as a construction process - may be uncontroversial. The controversy begins when constructivism confuses what it is that is being constructed (*mental representation*) with knowledge itself. One of the most striking features of constructivism is its denial that knowledge is anything over and above either mental representation or consensus.

If an A-B-C textbook on constructivism were possible, then one would expect knowledge defined in terms of mental representation under 'Radical constructivism' and knowledge defined in terms of consensus under 'Social constructivism. Unfortunately the situation is not so simple, which may account for why such an A-B-C textbook has not been forthcoming. As will be demonstrated below, in their consideration as to what knowledge is, many radical constructivists switch from 'mental representation' to 'consensual domain' and back again without realising that there are contradictions in doing so. It seems as if these contradictions are overlooked because of an incessant drive to deny the status of knowledge – to deny that knowledge is anything over and above what is subjective (mental representation) or intersubjective (consensual domain). Even social constructivists are unsure as to what constitutes the 'social' and how it relates to the individual. As Lerman (1994, 1999), has indicated, social constructivism in the main is merely radical constructivism's interpretation of the social – the 'consensual domain' added as an appendage to the way the individual interprets the world – and Lerman (1994) points out that if the subject is seen as the autonomous meaning-maker then any reference to the 'social' has no 'bite'. We suggest that if knowledge is seen in terms of something over and above mental representation and consensus then the contradiction between the individual meaning-maker and the 'social' might be resolved.

To understand how the existing ideas of the student can accommodate new ideas taught by the teacher would involve examining in some way the subjective ideas of the student. Similarly, to understand how a group of students arrived collectively at a solution to a problem would involve what may be described as an intersubjective consideration. In both cases, however, *consideration must be given to the content of*

*what is being learnt or solved, such as what concepts are involved and their relation to other concepts in the body of knowledge, or what inferences need to be made, etc.* That is, if we wish the students to make the mathematics their own (without reference to content would be like Woolgar and Latour observing the behaviour of two scientists in the laboratory without any reference to the content of the science that must have some influence on their behaviour. See Phillips, 1998, and Irzig, 2000, on this). There are three approaches to examining a discipline such as mathematics: a subjective, intersubjective and objective approach. The subjective approach may examine the ideas of a mathematician, the intersubjective approach may examine the response of the mathematician's peers to his or her ideas, but if both approaches are to make sense and not collapse into contradiction and misapprehension, then both have to refer to the mathematical concepts involved.

Education is to do with knowledge and any consideration as to what knowledge is has far reaching pedagogical implications. For example, if knowledge is nothing more than what is constructed by the individual, then the learner is never wrong - whatever has been constructed has made sense and whatever makes sense is knowledge! If truth is whatever the learner considers to be the case, then there is no body of knowledge, no 'subject-matter' that can be taught as such.

**There is no world 'out there' we can know; none we can talk about.**

Traditionally, epistemology – the 'theory of knowledge' that goes back to the Ancient Greeks – was a subject exclusive to philosophy and attempted to answer the question as to what is knowledge of the world and how is this knowledge possible. Although epistemology is still a philosophical subject, it has become a topic central to education research. If teaching and learning has to do with knowledge, then epistemology has to be involved. What is knowledge and how it is possible has implications for teaching, learning and the content of what is being taught/learnt. Perhaps the most dominant (or at least the most referenced) epistemology in education is the one expressed by von Glasersfeld:

1a. Knowledge is not passively received either through the senses or by way of communication;

1b. Knowledge is actively built up by the cognising subject.
2a. The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability;
2b Cognition serves the subject's organization of the experiential world, not the discovery of an objective ontological reality. (von Glasersfeld, 1995, p.51)

Of the first, von Glasersfeld (1998) states that this is 'trivial constructivism' and because it doesn't challenge 'traditional epistemology' he added the qualifier *radical*. Jaworski states that this is an expression of the Ausubelian form 'that the learner's new understandings are dependent on prior knowledge and experiences', (Jaworski, 1994, p.16). In one sense, this is quite obvious: you cannot expect a class or a pupil to understand a topic in mathematics without having the necessary mathematical background. Unfortunately, just how new understandings are dependent on prior knowledge has not been made at all clear in the literature. 'Prior knowledge and experiences' seems to be recall of previously learnt mathematics in order to solve a current problem but with no explanation as to how the prior knowledge was learnt in the first place. All we seem to get is basic recall. For example, After needlessly spending days on an activity involving measurement:

> I held up the ruler that Joan had been holding a few days before and asked if anyone knew what it was or how to use it. Mark said it was called a ruler and that his dad used one to measure things (Hendry, 1996, p.12).

Praise be for Mark and his dad, for without them the target-concept may never have been reached and the class may never have guessed what was in Hendry's head!

Although understanding may be regarded as an active process, *knowledge* as being actively built by the learner *is* controversial (although some constructivists, e.g. Staver, 1998, and some of its critics, e.g. Irzig, 2000, say that it isn't controversial). This will be discussed below in the context of whether knowledge can be communicated. What is agreed generally amongst constructivists and non-constructivists alike is that the second principle is controversial. A comprehensive philosophical analysis of the second principle can be found in Nola (1998), summarised by Irzig (2000) and a deconstruction of von Glasersfeld's radical constructivism can be found in Suchting (1998). The point we would like to stress

here is that the second principle seems to give license to replacing the 'top-down' curriculum (for example, a mathematics syllabus that is taught in school) with a child-centred 'bottom-up' pedagogy in which mathematics becomes, with all its entirety, *that* which is constructed by the child. Any reference to mathematics as an objective body of knowledge independent of consensus is to be condemned as 'absolutist' (see Rowlands et al, 2001).

In the defence of constructivism in science education, Staver elaborates the four principles of von Glasersfeld. The first three, Staver states, are non-controversial:

> Knowledge is actively built up from within by a thinking person: knowledge is not passively received through the senses or by any form of communication. Second, von Glasersfeld described the importance of social interaction in the construction of knowledge. Social interactions between and among learners are central to the building of knowledge by individuals. Third, the character of cognition is functional and adaptive. Cognition and the knowledge it produces are a higher form of adaptation in the biological context, in which the functional concepts of fit and viability - two concepts which we know well and embrace in evolutionary theory - also characterise knowledge. Fourth, von Glasersfeld described what the purpose of cognition is, and what it is not. Cognition's purpose is to serve the individual's organisation of his or her experiential world; cognition's purpose is not the discovery of an objective ontological reality (Staver, 1998, p.503).

If knowledge is not received by any form of communication (first principle) then how is social interaction central to the building of knowledge (second principle)? If knowledge is a product of cognition (third principle) then how can mathematics simply be a product of the student's cognition if mathematical knowledge has taken centuries to develop? As of the fourth principle: in what possible sense can we say that cognition's purpose *is* the discovery of an objective ontological reality? What does 'discovering' an ontological reality mean, and would we recognise an ontological reality once we have discovered it? The proposition 'There is an external world that exists prior to consciousness' separates the realist from the idealist and has been the subject of much heated debate in the history of philosophy (see Suchting 1986). This is also the central issue in constructivism and one that is central to Staver's article:

> The continuing debate about constructivism as an epistemology and, more fundamentally, about truth as correspondence versus coherence provides an opportunity to examine the relative importance of the two issues that Osborne mentioned. Scientists value parsimony and building on prior work. Which do

> scientists value more? As an epistemology, constructivism should be preferred on grounds of parsimony because it contains one less assumption - the realist presupposition; it does not assume a priori the existence of an external world which is separate from human perception (p.514).

Do scientists value parsimony? The Copernican system is often cited by many as an exemplar of 'simplicity' because it contains fewer assumptions than the Ptolemaic one (the latter contains assumptions such as epicycles in order to sustain it). Actually, for centuries, this was not the case and was never the reason why the latter replaced the former! There were more difficulties besetting the Copernican system then there ever were besetting the Ptolemaic one (initially, for example, the former still had to employ epicycles in order to give correct predictions) and these troubles were persevered with for centuries until the system was perfected (Chalmers 1982). So should we accept constructivism on the grounds of 'parsimony' - that it does not contain the assumption of an external world prior to consciousness? Despite any love of parsimony that the scientist might have, the *practice* of science presupposes the existence of the external world that is prior to consciousness *simply because science is an attempt to say something about the world and is not an attempt to give coherence to our experiences* (Chalmers 1982; Matthews 1994, 1998b; Nola 1998; Thomas 1994). In fact, the concepts and *theoretical objects* (see Suchting 1986) of science are contrary to making sense of experience. For example (and one that is inspired by Suchting 1986), a ball that is thrown upward is on the one hand 'going up' (unproblematic as regards making sense of experience) but is also in *free-fall* ('free-fall' is a theoretical object contrary to making sense of experience). On the one hand we have student 'preconceptions', 'misconceptions', 'alternative frameworks' or 'intuitive ideas' of force and motion (making sense of experience) as reported in hundreds of research papers (see Rowlands et al. 1999) and in contradiction we have the *well-defined* meaning of force in Newtonian mechanics (which models the physical world. See Hestenes 1987, 1992; see also Wittgenstein's *Tractatus*, propositions 6.341-6.361).

The removal of the assumption that the external world exists does not make constructivism simpler, in fact it does quite the reverse: constructivism will continue to be debated with the likelihood of more and more outrageous statements being added to what presently exists.

Von Glasersfeld's (1995), and Staver's (1998), attack on realism is in essence an attack on the correspondence theory of truth. The correspondence theory goes back to Aristotle and defines truth in terms of the adequacy of thought to reality. According to the correspondence theory, knowledge of the world is structured according to the state of affairs that it depicts. For example, if I were to say 'the cat is on the mat' then I am not just expressing a thought but saying that things are as I say they are - and the proposition is true *iff* the cat is actually on the mat! The correspondence theory implies a belief in the external world. For example, according to Searle (1995) the proposition 'there is no money in my pocket' only makes the sense it does because of the belief in the existence of money. To deny the existence of the external world, or to deny any possibility of knowing the external world, would be to deny the correspondence theory as plausible. However, to imply the converse would be to commit an error: *the denial of the correspondence theory is not necessarily a denial of the existence of the external world and, which is more to the point, nor is it necessarily a denial that knowledge of the external world is possible*. Von Glasersfeld and Staver do not deny the *possibility* that the external world exists, but they deny that knowledge of this world ('ontic' or 'ontological' reality) is possible. *Von Glasersfeld and Staver's fundamental error is the belief that a refutation of the correspondence theory is also, or must entail, a refutation of realism.* Following Nola, Irzig (2000) makes the point that a 'minimal realist' could believe that there exists a world independent of consciousness without necessarily holding onto the correspondence theory of truth.

The correspondence theory *may* be tenable with respect to everyday propositions and everyday states of affairs such as cats on mats, but the correspondence theory as a theory of the way science explains the physical world is fraught with difficulty (For a critique of the correspondence theory see Chalmers 1982, and Suchting 1986. A defence of the theory can be found in Searle 1995). For example, what state of affairs does the proposition 'an object in the absence of force moves in uniform motion' depict? No one on Earth has ever seen an object in the absence of force. The proposition 'the hydrogen atom has one electron' is embedded in the theoretical model of the hydrogen atom rather than describing an observed state of affairs (of course, an experiment may be seen as a creation of a state of affairs to show that

nature can be described by these means). Implicit in von Glasersfeld's main argument is that it is *because* the correspondence theory is untenable then the external world ('ontological reality') is unknowable (hence legitimising the claim that knowledge is of experience and not of the world). If the correspondence theory is untenable when it comes to science, then some account must be given as to what the theoretical objects of science actually refer to. For von Glasersfeld (1995), however, they don't refer to anything but are merely 'fictions' that can 'explain anything you want to explain' – but then how would von Glasersfeld reconcile Chalmers's (1982) point that such 'fictions' can be seen colliding with smoke particles in the phenomenon of Brownian motion, or that Kekule's molecular rings can be seen almost directly under an electron microscope or that Hertz was able to express Maxwell's equations 'in a visible and

Chalmers (1982) explains, *acceleration* is on the one hand an abstract mathematical concept and yet *it is what objects do!* If I were to drop a pen what would it do? For one thing it will not go up, for another it will accelerate downwards. *The pen obeys the laws of nature and cannot be explained in terms of making sense of experience* (unless perhaps you are Aristotle, but then Armstrong walking on the moon was calculated using Newtonian physics, not Aristotle's). What laws of nature? The laws of nature as expoused in many physics textbooks. *If the physical world was other than it is, then the concepts of science would be different to what they are*. So-much-so that if an alternative culture were to come up with an inverse-cubed law for gravitation, *then it can be shown that nature is such that it will not obey this law* (unlike the inverse-square law)!

Von Glasersfeld's two principles are much quoted in the mathematics education literature and, it seems, with the downplay of mathematics as a discipline very much in mind. For example, with reference to the second principle, Jaworski (1994) argues that *if* there are any 'absolutes' regarding triangles, then developing experience and knowledge tells us nothing about what they are - cognition has to do with viability and 'not the discovery of an objective ontological reality'. However, this is quite simply wrong! My experience of the proof that the area of all flat triangles is half base length times height tells be something about any possible flat triangle which is 'absolute' (fixed and unchanging). In fact, Jaworski is committing a fundamental contradiction: any intervention by the teacher (for example, a teacher constraining a student's thinking that there are two possible answers to the area of a triangle by

posing a contradiction [see ch.7]) would be from the teacher *knowing* that the student is wrong and what it is for the student to be right (that any flat triangle has one and only one value for the area). According to Jaworski, if we each know 'only what we have individually constructed' (p.17), then we can never know of anything outside what we have individually constructed, which includes other people (what they say and do being a question of how we perceive it). This is *solipsism*, the theory that either the self is all that exists or it is only the self that can be known, because reference cannot be made to anything outside of personal experience (many radical constructivists, e.g. Steffe, 1999, deny solipsism by stating that they do not deny the existence of the external world. Von Glasersfeld, 1995, argues that solipsism is a theory of being whereas radical constructivism is a theory of knowing. However, if knowledge is defined solely in terms of making sense of experience, then the external world might as well be denied). But Jaworski wants her cake and eat it because experience 'includes the interactions with others' (p.17). On the one hand, 'others' are merely part of my experience yet because of my interaction (with parts of my experience, which can be labelled 'others') *we no longer have solipsism but apparently a world containing other people*. Jaworski leaps from the world of the individual to the social world without realising the implications of such a leap – it is merely taken for granted that such a leap is unproblematic. For example, 'If the second principle implies there is no world outside the mind of the knower, it could, as Lerman (1989a) points out, imply that ''we are certainly all doomed to solipsism''. Lerman reassures us that this is actually not the case since the second principle recognises experience, which includes the interactions with others in the world around us' (Jaworski, 1994, p.17). As you will see below, Lerman has changed his mind since 1989 and no longer 'reassures us' with respect to radical constructivism.

In the introduction to the influential book 'Radical Constructivism in Mathematics Education', von Glasersfeld states what may be regarded as fundamental to the radical constructivist perspective:

> Language frequently creates the illusion that ideas, concepts, and even whole chunks of knowledge are transported from a speaker to a listener. This illusion is extraordinarily powerful because it springs from the belief that the meaning of words and phrases is fixed somewhere outside the users of the language. Perhaps the best way to dismantle the illusion is to remember or reconstruct how one came to form the meanings of words and phrases when one was acquiring language in the

> first place. Clearly it could only be done by associating bits of language one heard with chunks of one's own experience – and no one's experience is ever exactly the same as another person's. Thus, whatever another says or writes, you cannot but put your own *subjective* meanings into the words and phrases you hear. Given that we live in a community of other language users, our subjective meanings tend, of course, to become *intersubjective*, because we learn to modify and adapt them so that they fit the situations in which we interact with others. In this way we manage to achieve a great deal of compatibility. But to prove compatible, individual meanings do not have to be identical. Indeed, throughout our lives we now and then discover that the meaning we have associated with a certain word is not quite compatible with the use others make of that word. This may serve to remind us – especially when we act as teachers – that new concepts and new knowledge cannot simply be passed to another person by talk, because each must abstract meanings, concepts, and knowledge from his or her own experience. (von Glasersfeld, 1991, p.xiv)

Many pertinent points and criticisms could be made about this passage. For example, the appeals to our memory of personal experience (*introspection*, something that von Glasersfeld appeals to quite a lot. For example, 'A dictionary will in many cases resolve the problem – and, in doing so, confirm the illusion that meanings are, after all, fixed entities that do not depend on individual usage. But a moment's thought on how anyone acquires the meaning of a word would reveal that it is an illusion', von Glasersfeld, 1996, p.6. An exercise would be to count the number of introspective appeals in von Glasersfeld's, 1995, book - the best way to 'dismantle the illusion' that we can communicate being just one of them). This appeal to personal experience assumes that personal experience is the only legitimate basis upon which knowledge claims can be made, which it is if knowledge is defined as making sense of experience. What is striking about this passage is that it contains an irony and a glaring contradiction.

The irony is that the meanings of words and phrases within mathematics education and indeed other disciplines *are* fixed (many 'schooled' or 'academic' concepts are well-defined, Howard, 1987), but the so-called 'illusion' that we can communicate does not spring from the belief that meanings are fixed outside the user – it springs from the belief that we share a common meaning – and a common meaning is only possible *because* of the existence of the external world. For example (and one taken from Searle, 1995), you can understand what I mean if I say 'there is no money in my pocket' because of our shared belief in the existence of money.

Meanings within science and mathematics *are* fixed (with respect to some domain). However, what I mean by 'force', for example, may be different to what you mean by 'force' and both may be different to what is meant by 'force' within the Newtonian system – and any difference can only be ascertained by comparison with how force is defined and understood within the Newtonian system (an excellent example of the difference between how force is conceptualised by text-book authors and how force is defined within the system can be found in Warren, 1979. A rare examination of the logical structure of Newtonian mechanics together with student intuitive ideas can be found in Hestenes, 1992). The essential point here is that the passage above contains a glaring contradiction, a kind of switch from one statement straight into it's opposite. We have, on the one hand, the association of language with our own subjective experience – and no one's experience, we are told, is ever quite the same as anyone else's - yet on the other hand we live in a community in which our subjective meanings tend to become intersubjective. But then, how does von Glasersfeld know this? The contradiction is stark: we have private experience and subjective meaning as if you are the only entity in existence (or rather, you might as well be, since all we have here is private experience and no external world *to speak of*) and in one fell swoop we have on the other hand the existence of a social world (presumably one that can be said to exist independently of private experience) in which we have *intersubjective* meaning. How can radical constructivism refer to the intersubjective when it reduces *everything* to the subjective? However, if everything is reduced to how *you* perceive it and as an addition we have the social domain (the intersubjective), then you might as well include the physical as an addition as well. However, *it is the physical world – the world that lies outside of the individual' sense perception – that von Glasersfeld claims we can have no knowledge of!*

If knowledge, language, concept and meaning are all reduced to private experience, then the intersubjective becomes problematic. Yet many constructivists will begin with private experience and glibly bring in the social aspect and not realise that they have undermined their reduction to private experience. All this simply because the authority of truth has to be personal or social. An exemplar is Confrey (1991): 'In rejecting the idea of Platonist truths whose existence is independent of humanity, the constructivist relies on explanation based in the interplay between social negotiation of meanings and individual creativity and genius' (p.114) and is quite happy to speak

of meanings as lying within individual experience and, at the same time, created within a 'culture of mathematicians' or even society as a whole. Cobb et al (1991) even go so far as to state that they find it impossible to give an adequate explanation of how students cognitise independently of social norms that are negotiated and constructed: 'The norms (and consensually constructed mathematical knowledge) constrain the activity that creates the norms. Conversely, individuals' activity creates the norms that constrain that activity.....We thus acknowledge that social context is an integral aspect of an individual's cognitions without reifying mathematics as a ready-made body of cultural knowledge that is somehow internalised from without by individuals.' (p. 163). Despite any sense that can be made of individual activity 'restrained' by the 'norms' it creates, if the social context is an integral aspect of an individual's cognition, then why can't the social context also include mathematics as a 'ready made' body of cultural knowledge?

A consensual domain presupposes a domain outside private experience and once both are evoked then it must be stipulated how one relates to or 'impinges' on the other. The contradiction is that *if* knowledge is reduced to private experience (mental representation) then you *cannot* refer to the consensual domain *as if* it were outside what Lerman (1994) refers to as the 'autonomous meaning-maker'. If everything within the consensual domain is interpreted subjectively, then the notion of intersubjectivity becomes redundant.

In many publications, von Glasersfeld emphasises the following: 'The meanings of words – and this also applies to every sign and every symbol – must be constructed by each user of the language individually, and this construction is based solely on the subjective experience of the particular person. Hence it stands to reason that the interpretation of a word or a text will always remain an essentially subjective operation' (von Glasersfeld, 1998, p.26/27). From this particular quote von Glasersfeld then stresses that 'shared' meanings are only an 'impression'. However, if the meanings of words are our own subjective construction, then how is communication possible? No problem it seems for the radical constructivist! For example, Jaworski (1994) argues that viability and fit (as opposed to describing or referring to an 'ontological reality') applies not only to the construction of meaning but also to shared meaning. But if meaning is a private construction, then is shared

meaning possible? She states ‘A match in meaning between teacher and student can never be known, even if it were achieved. It is quite surprising that meanings are shared at all, yet in many cases people do appear to understand each other’ (Jaworski, 1994, p.23). If meanings are reduced to personal constructions then it is not only surprising that meanings are shared, it would be impossible to ascertain whether we do in fact share the same meanings. If meanings are reduced to personal constructs then a shared meaning implies an infinite regress within each personal construct. For example:

‘How could I ever know that what I understand by what you say is the same as what

For the radical constructivist:

‘I do not actually know if my interpretation of what you say is the same as what you mean; however, I can always assume that my interpretation is the same as what you mean because languages are learnt through similar experiences. I can always change my assumption if there is a perturbation between what I assume you are saying and what I previously understood as being said. In other words, *understanding what is being said is dependent upon the way the individual constructs a meaning of what is being said*, and if what is being said is unfamiliar then an idiosyncratic interpretation is possible’.

Now, this is fine if this is restricted to whether an individual understands another individual, but a ‘shared meaning’ also entails the other person assuming his or her interpretation of what I say is the same as what I mean, and here is the regress: if we are not to talk in ‘parallel’ then, in turn, I must assume that your interpretation of what I say is the same as what I mean and you must assume that my interpretation of what you say is the same as what you mean and so on *ad infinitum*.

Can you imagine what this would be like if a shared meaning were possible in a conversation between more than two people! The only way out of the regress is to assume that any conversation shares a common content. For example, and one adapted from Irzig (2000): I might believe that today is Sunday and you might deny that today is Sunday. We have two different mental representations but they share the same content expressed by the proposition ‘Today is Sunday’ (which is true or false depending on what day it is). For Irzig, mental representation is private, subjective and has to do with the individual’s intentional state, whereas content is public and

intersubjective. He makes the point that we might disagree on the content of a particular sentence, but it does not follow that 'my meaning' is any different to yours. With respect to intentional states, Searle (1995) argues that, in many cases, we do not have a collection of 'I' intentional states but a 'we' intentional state that is atomistic – it cannot be broken down into a number of 'I' components.

In the context of shared meanings, von Glasersfeld states 'The conceptual structures that constitute meanings or knowledge are not entities that could be used alternatively by different individuals. They are constructs that each user has to build up for him- or herself. And because they are individual constructs, one can never say whether or not two people have produced the same construct. *At best one may observe that in a given number of situations their constructs seem to function in the same way, that is, they seem compatible*' (von Glasersfeld, 1996, p.5, emphasis added). In other words, it may just so happen that we have formed compatible conceptual structures, although there is no way of knowing this apart from perturbations. This is scepticism and does not offer in a way to see how shared meaning is possible!

Jaworski (1999) adds the 'social dimension' to radical constructivism by stating the 'third principle' which 'derives from the sociology of knowledge, and acknowledges that reality is constructed intersubjectivity, that it is socially negotiated between significant others who are able to share meanings and social perspectives of a common lifeworld' (p.156). Adding a 'social dimension' is qualitatively different to asserting that a shared meaning is possible. Here 'reality' is something that is socially negotiated, not privately constructed. Yet Jaworski states that the third principle *is not an 'extra' but rather a qualification of the second*! The third, however, is not a qualification but a contradiction! To state this is to have apparently missed out on an ongoing debate between Leslie Steffe, who argued that radical constructivism is compatible with the socioculturism of Vygotsky, and the socioculturist Steve Lerman who argued that the social dimension cannot be reduced to the autonomous maker of meaning. Steffe is a radical constructivist who views learning as the capability of the individual to change his or her conceptual structures in response to perturbation, yet he also emphasises 'social interaction as a primary means of engendering learning' (Steffe and Tzur, 1994, p.8). With reference to Steffe's attempt to reconcile the individual with the social, Lerman states:

Larochelle and Bednarz, 1998, p.5, emphasis added). We seem to live in an age of ‘flat refusal’ – to deny what you will at all costs despite what has been said and the compelling arguments that happen to be inconvenient. ‘Of course, the role of prior knowledge in learning is only half the picture. If I believed that there were no mechanism by which the external environment could impose itself in some way upon a person’s conceptions, I certainly would not be writing this page, nor would I be concerned with any other social, collective activity. I

would be condemned, or delivered, depending on one's point of view, to my own Konold, 1991, p. 140). One can *believe* that there is a mechanism by which 'ontological reality' can impose itself, but von Glasersfeld's point is that we can never know (or refer) to what that mechanism is. He does not deny an ontological reality, he denies we can ever have knowledge of it – so we are left with our own internally isolated world (I say 'we', of course, according to this dictum, you may not actually exist as far as *I* know).

And of course, any sceptic who doubts the existence of the external world can be invited to drive into a brick wall at a speed proportional to his or her disbelief (Chalmers, 1982)! Lerman's main argument is that constructivists do not recognise that meanings are carried in practices and that cognition is 'situated' – a sociocultural view that does not acknowledge the separation between the subject and the world.[1] However, according to Steffe (1999) in a reply to Lerman, social interaction enters into radical constructivism 'at its very core' (p.2) and is not incorporated as an afterthought. But this is exactly how the social is incorporated! [2] If everything is reduced to subjective judgement and interpretation, then that reduction must include social interaction *because, for the radical constructivist, nothing is over and above how the individual has made sense of experience – including experience of social interaction*. Steffe (1999) criticises Lerman (1994) for the claim that the subjectivity of radical constructivism makes it a solipsistic position. For Steffe (1999), radical constructivism is not solipsism because (a) perturbations make it possible for the individual to distinguish between what is 'in here' from what is 'out there' (p.4) – without perturbations, everything would be the knowing 'I' and (b), it recognises 'a reality other than one's own' with its feet 'firmly on the ground' (p.7). But if knowledge is making sense of experience, as the radical constructivist claims, then how is it possible to distinguish between making sense of experience and the world that is independent of experience? If perturbations have made it possible for Steffe to distinguish between 'in here' from 'out there' then perhaps he ought to tell us what is 'out there'. Remember that for von Glasersfeld, no distinction can be made – for if it could, then we have a 'discovery of an ontological reality' which he denies. However, just when you think that Steffe is adopting realism in the defence of constructivism

('Because the other subject is not unlike ourselves, we are obliged to attribute a reality to that other that is distinct from our own reality' p.4/5), he uses relativism (the *intersubjective* as the most reliable level of 'experiential reality') to distinguish between knowledge ('that we want to trust as though it were objective' p.5 - i.e. let us *pretend* knowledge is objective) and illusion. [3] So Steffe wants us to *pretend* that knowledge is objective, to regard it as subjective and to use the intersubjective to dispel illusion – not a simple (coherent) theory at all!

**'Post-epistemology'**

According to Jaworski (1994), constructivism is 'post-epistemological' because it says nothing about the 'status' of knowledge: it says nothing as to whether a statement is true or false. Consequently, a teacher cannot 'challenge' a student's 'mis-conception', for there is only the student's conception.

Now, in case that you are thinking that this may be a clever teaching strategy that enables the facilitation of the student's awareness of what is true or false, Jaworski is making a claim about the status of knowledge itself. Her claim, and one that is argued by many prominent constructivists such as von Glasersfeld, is that knowledge itself cannot be predicated true or false because it is *context specific*. This is tantamount to saying that knowledge isn't possible because it doesn't actually exist. For example, 'the way we segment the flow of our experience, and the way we relate the pieces we have isolated, is and necessarily remains an essentially subjective matter. Hence, when we intend to stimulate and enhance a student's learning, *we cannot afford to forget that knowledge does not exist outside a person's mind*' (von Glasersfeld, 1996, p.5, emphasis added). Of course, knowledge does imply a knower, but von Glasersfeld is claiming much more than this! According to Matthews (1998a), 'these philosophical aspects of constructivism are frequently taken for granted, or asserted without argument or awareness of the tradition or depth of debate that has occurred around them. For instance two leading constructivists have recently written that ''the authority for truth lies within each of us''. This claim, which goes back at least to Protagoras in the 4$^{th}$ Century BC, if true, is truly breath taking in its cultural and epistemological ramifications. But the claim is made without any argument, or any

consideration of its obvious flaws' (p.x). Brown (1999), for example, makes a similar claim

> Mason follows Gattegno in seeing truth gravitating around personal awareness, that is, truth is located in the mind of the individual. (p.14),

but does not locate this claim in the history of philosophy. 'The history of philosophy is a footnote to Plato' (A. N. Whitehead) and the issues that surround relativism, that central ingredient of constructivism, have been debated since Plato and Protagoras. Brown (1999), for example, shifts the emphasis from the traditional view of mathematical meaning as independent of individual human performance to the personal awareness of mathematics, but without any reference to any tradition in the philosophy of mathematics. In so doing, *logical necessity* (that certain conclusions follow from the premises) is swept aside and student error becomes a question of language and activity (for a critique of relativism and logical necessity see Rowlands et al, 2001). It therefore comes as no surprise that, for Brown (1994), mathematics is a style of activity rather than something to be learnt.

'Does the external world exist and if so, how is knowledge of this world possible?' is a fundamental question of philosophy in which the various philosophical schools over the centuries tried to resolve (Suchting 1986). Constructivism, on the other hand, is a fairly recent and influential view of learning that asserts answers to this fundamental question but with little reference made to the history of research that went into this question. Of course, a constructivist could always claim that constructivism is a theory of learning and not a theory of knowing: that it is a psychological theory about how beliefs are developed rather than what makes beliefs true, that it makes no ontological claim concerning the external world and that it is 'post-epistemological'**.** But any learning-theory must have a core epistemology (something that Piaget well recognised) and this core outward must necessarily involve philosophical considerations (Matthews 1998b). Thomas (1994) is a radical constructivist who has 'no desire to attack radical constructivism' (p.33) yet recognises 'the denial of the possibility of knowledge of the world' (p.33) as a 'radical' deficiency:

> With a properly circumscribed claim, we can claim to know the world. What else? If we had only our own observations to go on, then we might reasonably be concerned whether our knowings were indeed of the world. But since our scientific

> knowledge is concordant with that of others within common understandings of the world, there is no practical room for doubt that it is the common world that we understand and have knowledge of. It is all very well for von Glasersfeld to be 'post-epistemological', but to insist that it is not the world that we know is more like the post-modernist cutting off the branch on which one sits. Amusing as creative writing, but not to be taken seriously or taught to children. The very discussion of constructivism relies on common understanding; it is important for consistency and for teaching that it does not lapse into a self-contradictory absurdity comparable to that of a proselytizing solipsism (Thomas, 1994; p.33).

Of course, if von Glasersfeld was sincere in his post-epistemological view then he would not be writing articles or trying to argue any position at all. Radical constructivism is a belief system, the paradox is that a true belief in radical constructivism would compel its believers to fall mute. While its advocates are legion and vociferous within the world of education academe, they would wish those at the 'chalk-face' to fall mute. Consider the following: A boy is shown a 45, 45, 90 triangle and a 30, 60, 90 triangle and he responds that one is not a triangle. Now, if we were to say that the boy is wrong, then Jaworski would say that 'this is to make judgements about truth without taking into account the circumstances which the statements (that this is not a triangle) fits. Consider, for example, a geometrical object with angles adding up to more than 180. We might be tempted to say that this could not be a triangle, meaning a plain triangle. However, a triangle on the surface of a sphere could fit the criterion. The context in which the statement is made is crucial to the validity of the statement, and it is very difficult to say therefore when any statement is true without knowing the context' (p.20). Unfortunately, Jaworski does not give the context why the boy thought one of the triangles isn't a triangle, but we can safely assume that the boy wasn't thinking of non-Euclidean triangles. The boy's *misconception* (with respect to all Euclidean triangles, or if that sounded too Platonic, then with respect to any possible triangle can be constructed within the Euclidean plane) has nothing to do with the context of triangles on surfaces on spheres, despite Jaworski's insistence that we have to know the context. One the one hand, Jaworski 'demands' that the context be known, yet there is no hint of specificity in this example. *The boy is wrong, and constructivists would disempower the teacher to explain why*.

**Impossibility of transmitting 'knowledge'**

We may think of many examples that would render the idea that 'new' knowledge is constructed from previous knowledge acceptable. Solving quadratic equations would make little sense without a knowledge of linear equations and teachers may often refer to previously 'learnt' material as a cue to a new concept, but constructivists have consistently failed to make explicit the process of utilising familiar concepts in the construction of new concepts With the exception of the work done by Tall (1991) and colleagues, 'utilising familiar concepts' has been *the teacher asking the class or a child to recall some fact so as to solve a problem* (e.g. Hendry, 1996). To ask how new knowledge relates to previous knowledge is not only a cognitive question but also a question of specificity regarding the domain of knowledge – how the concepts within that domain relate to each other (e.g. see Hestenes, 1992, on mechanics education). In other words, to ask how a newly acquired concept relates to an already existing cognitive structure (or 'schema') is a psychological question, but any specificity regarding the learning of an academic or 'schooled' concept transforms the psychological question into an epistemological one. A consideration of the question 'how is knowledge possible?' can enable us to link the learner with a discipline, a link between the psychology of learning with the way the discipline relates to the world. Although some constructivists have explored this link, the majority, it seems, would deny such a link as (shock! horror!) 'Platonist'!

According to Jaworski (1994), constructivism challenges the transport metaphor that underlies much of educational discussion. The idea that knowledge cannot be transmitted can be a very useful heuristic. If knowledge is seen as a construction process then there is a possibility of approaching a solution to the *learning paradox*: how can anyone get to know that x is the case if the learning process requires a *recognition* that x is the case. The problem is that constructivism has becomes radical – the construction process is no longer a heuristic but an *insistence*.

Schifter's (1996) introduction to the book 'What's Happening in Math Class? Envisioning New Practices Through Teacher Narratives', contrasts the practice of mathematics teaching of twenty years ago with the consensus for a 'new practice of mathematics instruction' based on 'changing social needs and two decades of research

in cognitive psychology' (p.1). Implicit throughout is the assumption that nothing can be transmitted or explained by the teacher. This has been made explicit in the book by Simon (1996), who states that, according to the constructivism of von Glasersfeld, 'teachers (or students) cannot *give* their understandings to another person. This idea can be validated easily: Interview students who have all listened to the same lecture and you will hear quite discrepant ideas of what was discussed' (p.39, emphasis given). The absurdity of this quote is twofold. Firstly, if it is all a question of interpretation, then how can anyone distinguish between ideas are discrepant with those that are not? Secondly, if it is all a question of interpretation, then which idea of the author has been 'validated easily'?

The view that communication is *inherently* impossible is popular with constructivists, but it is patent nonsense. The majority of people in an audience may not interpret the meaning as intended by the speaker, but that is not to say that understanding what the speaker is trying to say is impossible. Consider the kind of communication that takes place when a large team of engineers at Boeing designs the 747 and the various manufacturing parts are subcontracted out. Working off the engineer's plans the parts are produced and later assembled. The thing flies. If there are problems, then they can be traced to particular mistakes and misunderstandings and this would involve verbal communication. To say that communication is impossible, or necessarily arbitrary may say more about the speaker than the referent in that particular conversation.

Constructivism seems to be the anxiety not to transmit anything. Yet concepts have to be defined, explained, elaborated etc. if there is to be a 'cognitive response' to what is taught. We would argue that the fundamental question for mathematics educationalists has to be 'what is the best way to enable a class to understand – to make their own – a framework of concepts that constitutes the subject-matter?' By 'understand' we mean in the manner of Skemp's (1971) *relational understanding* of mathematics as a body of knowledge.

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[2] Steffe (1999) justifies his claim by saying that cognitive processes are, at the same time, a result of autoregulation and interaction with the environment. He states that radical constructivism is compatible with the Vygotskian approach that individual learning is dependent on social interaction yet radical constructivism is not identical to Vygotskianism because the former also includes interaction with the physical environment. However, quite apart from avoiding the issue as to how the social impinges on the individual, Steffe makes reference to the physical environment – something that von Glasersfeld denies that we can ever have knowledge of!

[3] The 'social' in Steffe's (1999) radical constructivism is treated in terms of relativism (a lá consensus) *and* in terms of a 'second order model' of an observer constructing an observed subject's knowledge. For Steffe, knowledge is still making sense of experience, but in a second order model it is making sense of someone making sense of their experience (rather in the manner of a teacher making sense of a pupil's reasoning), and since this is done through a process of social interaction then second order models constitute intersubjective knowledge and are 'social models'. Lerman's (1999) reply is that an individual consciousness is not an observers' formulation and that Steffe offers a very weak sense of intersubjectivity – a 'between people' rather than how meaning is carried in practices.