# NOTES AND CORRESPONDENCE 

# Jet Drops Produced by Bubbles Bursting at the Surface of Seawater 

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#### Abstract

Several empirical formulas were reported to describe the production of jet drops by bubbles bursting at the surface of seawater; they were, however, based on scanty data. Recent observations of Spiel have provided new data for intermediate-size bubbles; his results are further quantified here, aiming for their extrapolations to both small and large bubbles. A greater number of jet drops are shown to be produced by smaller bubbles; the production follows a combined exponential and linear decline for larger bubbles, rather than the exponential or linear decline proposed earlier. The largest number of jet drops produced by the smallest bubble may not exceed seven. The largest bubble that can produce jet drops may not exceed about 1.7 mm , a radius somewhat smaller than the analytical value. The ratio between the radius of jet drops and that of their parent bubble is always greater than the oft-quoted 1-to-10 value; it approaches a minimum value of 0.110 for zero bubble radius and reaches a maximum value of 0.147 for bubbles having a radius larger than 0.8 mm .


## 1. Introduction

Air is entrained into the near-surface ocean by breaking waves to produce bubbles (Wu 1994). Because of their buoyancy, these bubbles return to the sea surface and burst to produce film and jet drops. Those drops produced through the fragmentation of the bubble film cap are film drops (Day 1964), and those produced through the break-up of a water jet formed by the collapse of bubble cavity are jet drops (Blanchard 1963; Wu 1981). Geophysically, we need to quantify their productions for evaluating effects of marine aerosols on various phenomena in the atmosphere and near the sea surface (Blanchard 1978; Schacher et al. 1981; Andreas 1992).

Earlier parameterizations of Blanchard (1989) and Wu (1989) on the production of jet drops by bursting bubbles were based on very limited data consisting of those reported by Kientzler et al. (1954) and others as described in Blanchard. Systematic observations have subsequently been conducted by Spiel (1994, 1995, 1997) with bubbles of intermediate sizes. With these sets of comprehensive data, I refine parameterizations for the production of jet drops by bubbles of various radii bursting at the surface of seawater. Both number and size of jet drops produced are found to differ sig-

[^0]nificantly from oft-quoted values. Trends of the variation in jet drops with the bubble size are also greatly altered. Functional variations were deduced with an emphasis on the probable production of jet drops by small and large bubbles, of which no observations were reported. The extension to small sizes is especially important for quantifying the production of marine aerosols by bursting bubbles, as the smallest bubble in Spiel's studies was more than one order of magnitude greater in radius than that at the peak of the size distribution of oceanic bubbles.

## 2. Earlier results and recent observations

Observations of jet drops by Kientzler et al. (1954) and other unpublished works were reviewed by Blanchard (1989). The 1-to-10 rule between radii of jet drops and their parent bubble was often cited. Blanchard found, however, that the radius of jet drops was $5 \%$ less than the 1 -to- 10 rule for bubbles having their radius smaller than $250 \mu \mathrm{~m}$ and became increasingly greater than the 1-to-10 rule for bubbles larger than $350 \mu \mathrm{~m}$. The number of jet drops produced by a single bubble decreased as the bubble radius increased, following roughly the dotted line in Fig. 1 and expressed as (Wu 1989)

$$
\begin{equation*}
N=7 \exp (-2 R / 3) \tag{1}
\end{equation*}
$$

where $N$ is the total number of jet drops produced, and $R$ in millimeters is the radius of the parent bubble.


FIg. 1. Numbers of jet drops produced by bubbles of various radii. The dotted curve was proposed on the basis of Kientzler et al.'s (1954) results; open circles and the dashed line were reproduced from Spiel (1997); crosses are obtained from his reported probabilities of occurrence, and the solid line represents Eq. (2).

In Spiel's (1994, 1995, 1997) experiments, bubbles within the radius range of $0.35-1.5 \mathrm{~mm}$ were generated individually by forcing air through a glass capillary. A laser photo array and electronic system was used to record the process of jet drop production. The vertical speed of jet drops was determined by measuring the flight time between a laser beam and an impact detector. Spiel's $(1994,1995)$ earlier experiments concentrated on the speed of jet drops and also included both freshwater and seawater. We are interested in results reported in his later study (Spiel 1997) on the number and size of jet drops produced at the surface of seawater at ambient temperatures of $27-29^{\circ} \mathrm{C}$. His results on the size of jet drops also deviated from the 1-to-10 rule. As for the break-off height of jet drops, the first drop was observed by Spiel to be closest to the water surface, the second drop is the highest, and all subsequent drops at intermediate heights. In summary, observations by Spiel provided a much more solid and substantial database than previously available.

## 3. Parameterization of jet drops produced by bursting bubbles

As mentioned earlier, one of the reasons to quantify Spiel's (1997) data is for evaluating the production of
jet drops by small and large bubbles. For the latter quantification, we need to determine the largest bubble that can produce jet drops. Our principal interest, however, is extending investigations with artificially generated bubbles to oceanographic applications. For example, Wu (1981) associated earlier the production of marine aerosols by bursting oceanic bubbles, and Wu (1992) subsequently estimated this production from the spectrum of oceanic bubbles. Note that the smallest bubble in Spiel's experiments was $350 \mu \mathrm{~m}$ in radius, while the size distribution of oceanic bubbles peaks in the range of 20-50 $\mu \mathrm{m}$ (Degterev and Kolobaev 1994; Wu 1994).

## a. Total number

Spiel (1994) obtained the average number of jet drops produced by bursting bubbles of various radii; the number generally decreased toward a larger bubble. A major portion of these data were further analyzed by Spiel (1997); see Fig. 1 in which the dashed line representing a linear variation was proposed to describe the results. He also reported probabilities of occurrence for the first through sixth jet drops. For example, the first drop occurred $100 \%$ of the time, while percentages of succeeding drops decreased. The probability of producing a sixth drop was shown to be zero for all bubbles except the smallest of $353 \mu \mathrm{~m}$ in radius. Even for bubbles of this smallest radius, Spiel reported the probability of producing the seventh drop to be nearly zero. By summing up these probabilities of occurrence, we obtained another set of numbers; see Fig. 1. These numbers, differing somewhat from those used by Spiel, are more definitive, especially for small bubbles. Although both sets of results deviate from the formulation represented by Eq. (1), the set with probabilities of occurrence agrees better with the earlier results at the end of small bubbles.

Blanchard (1989) reported that jet drops were produced by bubbles as small as $5 \mu \mathrm{~m}$ in radius. This is much smaller than those bubbles tested in any of the investigations. A trend is clearly illustrated in Fig. 1 that the number of jet drops produced increases as the bubble size decreases. This trend still persists even for smallest bubbles tested. Inasmuch as more than seven drops were very rarely produced, I still adopt the exponential form shown in Eq. (1) for small bubbles and combine it with a linear type of relationship proposed by Spiel (1997) for large bubbles. This combined functional variation can be expressed as

$$
\begin{equation*}
N=7 \exp (-2 R / 3)-1.3 R, \quad R<1.715 \mathrm{~mm} \tag{2}
\end{equation*}
$$

and is seen in Fig. 1 to represent well the data. The upper limit here is imposed by $N=0$ at $R=1.715$ mm .

First of all, the number of jet drops produced is suggested herewith and shown in Eq. (2) to be generally less than 7, instead of 10 as suggested by Blanchard (1983). Both sets of data (Kientzler et al. 1954; Spiel


FIG. 2. Size distributions of jet drops obtained from Spiel (1997). In the figure, $n_{i}$ and $N_{i}$ are explained in the text, while the number indicates the order of generation and the vertical line indicates $r / R$ $=0.13$.
1997) are seen in Fig. 1 to display quite clearly this trend. There are no indications from the trend of these data that there is a sudden rise of jet-drop productions by small bubbles. Even if one selected only those four data points adopted originally by Spiel shown in Fig. 1 lying above the line fitted by him, we found from the linear line fitted to them: $N=6.28$ at $R=0 \mathrm{~mm}$. The maximum number of seven still appears to be the upper limit; needless to say that this is done in the interest of getting a whole droplet. For oceanographic applications mostly over small bubbles with the peak of their size spectrum falling in the range of $0.05-0.1 \mathrm{~mm}$, the difference is also substantial between the dashed line proposed by Spiel and the presently proposed solid line.

The dropoff of productions by large bubbles is also reasonable, as effects of surface tension weaken. In a numerical computation of gas bubbles bursting at a free surface, Boulton-Stone and Blake (1993) pointed out that a high-speed jet could not be produced by bubbles larger than 2.5 mm in radius. The exponential expression derived from Kientzler et al.'s (1954) results provides no maximum bubble size for the production of jet drops, while the linear expression proposed by Spiel (1997) places it at the radius of about 1.70 mm . The latter differs little from the present result shown in Eq. (2) of about 1.715 mm . In summary, although the difference between Spiel's and the present analyses is not


Fig. 3. Size variations of jet drops with (a) their order of generation and (b) the size of their parent bubble. The data are obtained from Spiel (1997).
large, the latter is believed to cast the representation into a better functional form.

## b. Size distribution

For each size of bubble, Spiel (1997) reported the number of jet drops produced at different orders over various radius bands of $5-\mu \mathrm{m}$ width. The data for each order of generation in each band, $n_{i}$, of which the subscript $i$ indicates the order of generation normalized with the total number of jet drops produced for a given order of generation, $N_{i}$, are presented in Fig. 2 versus the size ratio between jet drop and its parent bubble. The data point in the figure is marked by its order of generation. The results are seen to center quite nicely around an overall ratio of $r / R=0.13$, being greater than the oftquoted 1-to-10 rule.

We next evaluate two other variations: whether the size of jet drops varies with their order of generation and whether it changes with the size of their parent bubble. In these exercises, we first obtained from Spiel (1997), for a particular order of generation, the average radius of jet drops produced by bubbles of a given size, $r_{i}$. Then, we averaged over all bubble size ratios of $r_{i} / R$ separately for each order of drop generation from the first through the last drops; see the average ratio along with its standard deviation in Fig. 3a. Again, the
$\bar{r}_{i} / R$ data indicate an overall average of 0.13 , as illustrated by the horizontal line; but there appears to have finer variations indicated by rather large standard deviations.

For the most frequently produced drops, say the first five, the second jet drop is seen in Fig. 3a to be the largest; this was first pointed out by Blanchard (1989). Spiel (1997), on the other hand, provided the following quantitative variations for the first three drops:

$$
\begin{array}{ll}
\text { First drop: } & r_{1} / R=0.1432 R^{0.206} \\
\text { Second drop: } & r_{2} / R=0.1526 R^{0.316} \\
\text { Third drop: } & r_{3} / R=0.1659 R^{0.796} \tag{3}
\end{array}
$$

The above expressions indicate that the first drop is larger than the second drop with their parent bubbles smaller than 0.56 mm in radius, and the second drop is larger than the third drop with bubbles smaller than 0.84 mm .

As discussed above, the size ratio between jet drops and their parent bubble may vary in a rather complicated fashion with the bubble radius. For their overall approximate trend, we went on to average for each bubble size the average radius of jet drops regardless of their order of generation and designate it as $\bar{r}_{r}$. Results of $\bar{r}_{r} / R$ are presented versus the bubble radius in Fig. 3b. As discussed earlier, we are interested mostly in the trend for small bubbles. For large bubbles, we question whether the size ratio shown in Eq. (3) can increase with the bubble radius indefinitely, because the influence of surface tension certainly diminishes for larger bubbles.

Spiel (1997) suggested that the size ratio between jet drops and their parent bubble increased continuously with the bubble radius for large bubbles. On the other hand, the ratio between the drop and bubble radii shown in Fig. 3b appears to increase with the bubble radius for small bubbles and to reach a constant value for large bubbles. These trends can be approximated by

$$
\bar{r}_{r} / R= \begin{cases}0.110 \exp \left(R^{3.4} / 1.7\right), & R<0.8 \mathrm{~mm}  \tag{4}\\ 0.147, & R>0.8 \mathrm{~mm}\end{cases}
$$

In summary, all jet drops appear to have a size distribution larger than predicted by the 1-to-10 rule. The drop-to-bubble size ratio approaches 0.110 for its minimum value and settles on the value of 0.147 for large bubbles. Of two regions of variations, I am more concerned with that for small bubbles as the smallest bubble tested was 0.35 mm . Fortunately, the trend over the region $R<0.35 \mathrm{~mm}$ is rather flat to assure the determination of the ratio of 0.110 .

Blanchard (1989) reviewed the size of jet drops produced by bursting bubbles at a seawater surface. The data were obtained with water in two different temperature ranges of about $4^{\circ}$ and $22^{\circ}-26^{\circ} \mathrm{C}$; our interest is in the latter group. The line drawn in Fig. 1 of Blanchard appears to consist of two straight lines joined together
by a curve; the latter spanned the bubble-radius range of $0.25-0.60 \mathrm{~mm}$. In other words, he suggested constant drop-bubble radius ratios for two ranges: bubbles smaller than 0.25 mm and bubbles large than 0.60 mm . These trends are actually quite similar to those indicated by the line drawn in Fig. 3b. There were three data points presented by Blanchard over the small radius range of bubbles, $0.099-0.145 \mathrm{~mm}$. Most interestingly, the average drop-bubble radius ratio for these three points is 0.109 . I also determined from the line drawn in Blanchard's Fig. 1 that the drop-bubble radius is 0.141 at $R$ $=0.8 \mathrm{~mm}$ and 0.145 at $R=1.0 \mathrm{~mm}$. All of these are quite close to those described by Eqs. (4) and (5).

## 4. Concluding remarks

The number and size of jet drops produced by a bursting bubble are generally most critical. Here, I found from Spiel's (1997) data that the number of jet drops produced by bubbles bursting at the surface of seawater might not exceed seven and decreased as the bubble size increased. The decrease in the jet-drop number with increasing bubble size followed a combined exponential and linear variation rather than simple exponential or linear variation. This new form is important for extending the parameterization obtained over intermediate bubble sizes to small and large bubbles since there are no observations of these. The extension to small size is important as discussed earlier for oceanographic applications of these results, and the extension to large sizes leads to the determination of the largest bubble (around, say, 2 mm in radius) that produces jet drops. Overall, the number of jet drops produced is less than that suggested by Blanchard (1989). The size ratio between jet drops and their parent bubble decreases with decreasing bubble radius for small bubbles, approaching a minimum value of 0.110 for zero bubble radius. The ratio of 0.147 is proposed for bubbles larger than 0.8 mm . These trends are illustrated well by Spiel's data and quantified by Eqs. (4) and (5). The size of jet drops is larger than the oft-quoted 1-to-10 rule, especially for those drops produced by large bubbles.

In summary, the investigation of Spiel (1997) has provided a much more comprehensive set of data than what I used earlier to quantity the production of jet drops by bursting bubbles (Wu 1989). Not only are oft-quoted values modified, functional variations of these values with the bubble radius are also established. More observations are worthwhile to confirm the extension of present quantifications to small and large bubbles.

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