Resonant Third-Degree Diurnal Tides in the Seas off Western Europe

RICHARD D. RAY

Laboratory for Terrestrial Physics, NASA Goddard Space Flight Center, Greenbelt, Maryland

16 August 2000 and 2 July 2001

ABSTRACT

Third-degree diurnal tides are estimated from long time series of sea level measurements at three North Atlantic tide gauges. Although their amplitudes are only a few millimeters or less, their admittances are far larger than those of second-degree diurnal tides, just as Cartwright discovered for the M_1 constituent. The tides are evidently resonantly enhanced owing to high spatial correlation between the third-degree spherical harmonic of the tidal potential and a near-diurnal oceanic normal mode that is most pronounced in the North Atlantic. By estimating the ocean tidal response across the diurnal band (five tidal constituents plus nodal modulations), the period and Q of this mode and one nearby mode are estimated.

1. Introduction

This paper is a footnote to Cartwright's studies of the curious M_1 tide in the North Atlantic Ocean (Cartwright 1975, 1976; Cartwright et al. 1988). The M_1 constituent consists of a cluster of small spectral lines near one cycle per lunar day (period about 24.84 h). Table 1 lists the eight largest M_1 lines in the astronomical tide-generating potential (Cartwright and Tayler 1971). The three largest lines are separated in frequency by only 1 cycle per 8.8 yr. The middle of the three lines (155.555) is an exact subharmonic of the principal semidiurnal tide M_2 and arises from the third-degree term in the astronomical potential. Being dependent on the third, as opposed to the second, power of the moon's parallax (the parallax is of order 1/60), third-degree tides are always very small, and in sea level data they are normally dominated by second-degree tides. Cartwright (1975) found the opposite to be true in the northeast Atlantic where the ocean evidently magnifies the third-degree M_1 line an order of magnitude relative to the second-degree lines (see Table 1). Cartwright's explanation for this involves the ocean's normal modes and their varying responses to different spatial forcing patterns. Since it forms the basis for the work presented here, his explanation is worth recalling; it requires a brief digression into the theory of normal modes.

Corresponding author address: Dr. Richard Ray, Space Geodesy Branch/Code 926, NASA Goddard Space Flight Center, Greenbelt, MD 20771.
E-mail: richard.ray@gsfc.nasa.gov

A tide of frequency ω has a complex tidal admittance $Z(\omega, L)$ at location L, which can be written as an expansion of oceanic normal modes (e.g., Platzman, 1991):

$$Z(\omega, L) = \sum_k (1 - \omega/\sigma_k)^{-1} S_k H_k(L). \tag{1}$$

According to this prescription each mode k contributes to the tidal admittance an amount depending on three factors: a "frequency factor" involving the mode's complex frequency σ_k, a "shape factor" depending on the spatial coherence between the tidal potential and the mode elevation, and a "location factor," which is simply the modal amplitude H_k at L.

Normal modes for the World Ocean have been computed by several groups. According to calculations by Platzman (1985) there are three modes within or very near the diurnal band, having periods 23.7, 25.7, and 28.7 h. The latter mode is primarily an Antarctic Kelvin wave and Pacific half-wave, with hardly any influence in the Atlantic Ocean. In contrast, the other two modes are primarily Atlantic modes. The relevant portion of the 25.7-h mode is shown in Fig. 1. It has two amphidromes rotating cyclonically in each basin and largest amplitudes in the North Atlantic, especially the northeast sector. The 23.7-h mode is not unlike the 25.7-h mode in being largest in the North Atlantic and relatively small elsewhere; it does, however, have an additional amphidrome in the equatorial Atlantic. See Platzman et al. (1981) and Platzman (1985) for a comprehensive set of mode diagrams.

As Cartwright (1975) pointed out, the 25.7-h mode is spatially well correlated with the third-degree com-

TABLE 1. The M_1 tidal amplitudes at Newlyn, England.

Doodson number	Degree	Frequency (°/h)	Potential[a] (mm)	Equilibrium[b] (mm)	Observed[c] (mm)
155.445	2	14.48520	1.37	0.359	
155.455	2	14.48741	7.41	1.943	
155.545	3	14.48985	0.59	0.188	
155.555	3	14.49205	3.99	1.268	4.8
155.565	3	14.49426	0.52	0.165	
155.645	2	14.49449	0.59	0.155	
155.655	2	14.49669	20.62	5.407	1.2
155.665	2	14.49890	4.14	1.086	

[a] Potential normalized by acceleration of gravity, extracted from tables of Cartwright and Tayler (1971).
[b] Equilibrium amplitude equals potential $\times\ \gamma_n\ Y_n^1$, where γ_n accounts for elastic body tide.
[c] Sea level amplitudes estimated by Cartwright (1975).

ponent of the diurnal tidal potential, but not with the second-degree component. The relevant spherical harmonics of the potential are (Cartwright and Tayler 1971)

$$Y_2^1(\theta,\ \phi) = \sqrt{(5/24\pi)}\ 3\ \sin\theta\ \cos\theta\ e^{i\phi}$$

$$Y_3^1(\theta,\ \phi) = \sqrt{(7/192\pi)}\ 3\ \sin\theta(5\ \cos^2\theta - 1)e^{i\phi}$$

for spherical coordinates (θ, ϕ). Both functions are of order 1, as is appropriate for diurnal tides; they are shown in Fig. 2. The Y_3^1 harmonic, like the 25.7-h mode, is symmetric about the equator, and both the harmonic and the mode have local maxima near latitudes 0° and ±60°. In contrast, the Y_2^1 harmonic is antisymmetric about the equator and it cannot be expected to easily excite a symmetric mode. Platzman (1984) confirmed this argument by calculating the correlations: he found that the shape factor S_k is 0.30 for Y_3^1 but only 0.05 for Y_2^1. (Note that the faster 23.7-h mode is also symmetric, but with an additional amphidrome near the equator it is less well matched to Y_3^1 than is the 25.7-h mode.)

Eventually, Cartwright et al. (1988) produced estimates of the M_1 tide at 13 stations widely distributed over the entire Atlantic, and they compared these with

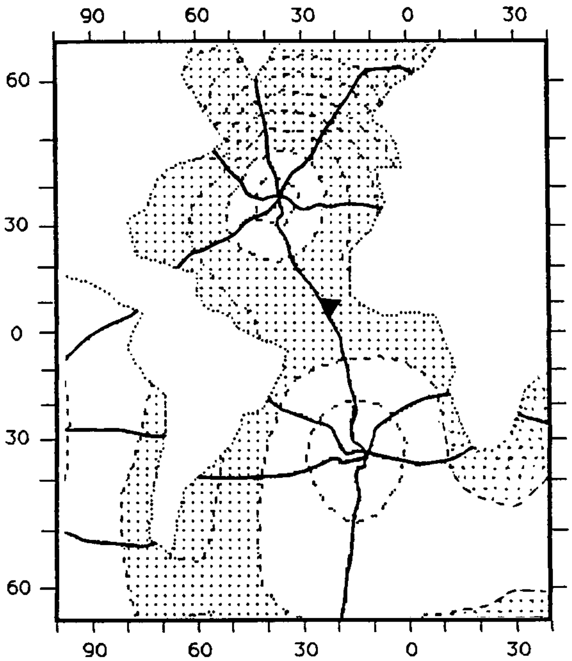

FIG. 1. The 25.7-h mode in the Atlantic Ocean as calculated by Platzman et al. (1981). Solid lines are 60° phase contours, with the arrowhead indicating direction of rotation. Dashed lines are amplitude contours, with shading denoting areas with amplitudes exceeding the global rms amplitude. Reproduced from Platzman (1991) with permission.

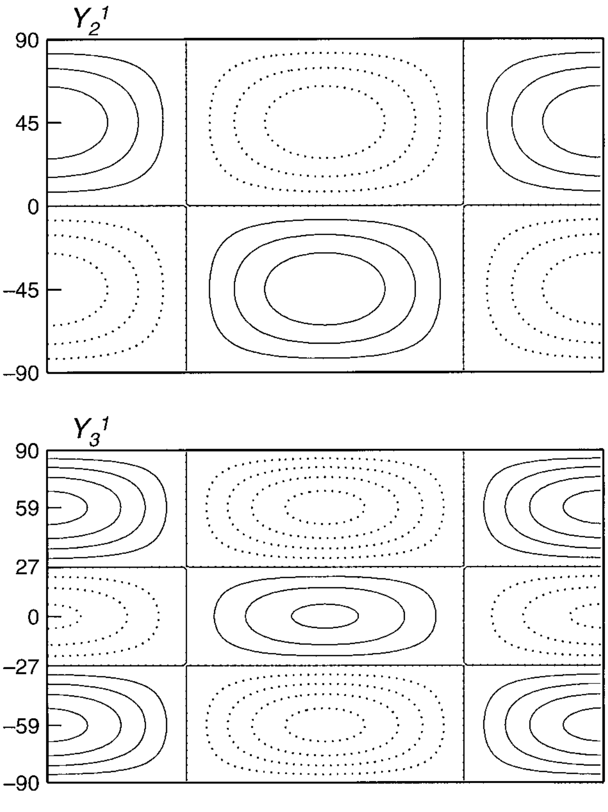

FIG. 2. Global contours of the second-degree (top) and third-degree (bottom) spherical harmonic components of the diurnal tidal potential. Solid and dotted lines denote regions of opposite sign.

TABLE 2. Third-degree diurnal tides at Newlyn, England.

Tide	Doodson number	Frequency (°/h)	Potential* (mm)	Amplitude (mm)	Phase lag (deg)	$\|Z\|$
$2Q_1$	125.655	12.84964	0.35	0.81 ± 0.22	205 ± 16	2.31 ± 0.62
	135.545	13.39181	0.50	1.00 ± 0.22	247 ± 12	1.99 ± 0.43
Q_1	135.555	13.39402	1.28	2.82 ± 0.22	249 ± 5	2.20 ± 0.18
O_1	145.655	13.94768	0.65	0.93 ± 0.22	276 ± 14	1.42 ± 0.33
M_1	155.555	14.49205	3.99	4.67 ± 0.22	275 ± 3	1.17 ± 0.06
J_1	175.555	15.59008	1.46	1.84 ± 0.22	294 ± 7	1.26 ± 0.15
	175.565	15.59229	0.59	0.77 ± 0.22	325 ± 16	1.30 ± 0.37

* Potential normalized by acceleration of gravity, extracted from tables of Cartwright and Tayler (1971).

Platzman's (1984) synthesized M_1 tide, which appears very similar to the single mode shown in Fig. 1. The comparisons are remarkably good, with the largest observed M_1 amplitudes in the northeast Atlantic, as Fig. 1 would suggest.

If this explanation of M_1 is correct, then one naturally suspects that other third-degree diurnal tides are similarly enhanced in the northeast Atlantic. It is the purpose of this note to show that they indeed are. In addition, by studying the third-degree tidal admittances across the diurnal band, one may deduce further characteristics of the normal modes and their responses, namely their periods—to be compared with Platzman's—and their specific dissipation Q^{-1}.

Like M_1, the other third-degree diurnal tides are separated from second-degree tides by only one cycle in 8.8 yr. The other third-degree tides are also significantly smaller than M_1, which is itself only a few millimeters. Both facts suggest that very long time series of hourly data are required before reliable estimates can be obtained. The data employed below, from three tide gauges, span at least two complete cycles of the lunar node, that is, at least 37 yr.

2. Estimates of third-degree tides

Hourly sea level data were analyzed from three stations: Newlyn, England (50°6′N, 5°33′W), Lerwick, Scotland (60°6′N, 1°8′W), and Vigo, Spain (42°14′N, 8°44′W). The data time spans are 1960–97 (Newlyn), 1959–97 (Lerwick), and 1943–90 (Vigo). The data are of high quality and remarkably complete; the longest gaps are a 3-week period during 1977 at Lerwick and two 2-week periods during 1953 at Vigo.

The tidal analyses were performed by least squares fits to sinusoids of known frequencies. All major constituents, including long-period tides and nonlinear compound tides, were accounted for, although in the tables given here (Tables 2–4) only the relevant third-degree diurnals are listed. For these third-degree diurnals, solutions were attempted for all lines whose potential amplitudes exceed 0.3 mm in the Cartwright–Tayler expansion. There are five such constituents spanning the diurnal band between $2Q_1$ and J_1. Although $2Q_1$ is more than 10 times smaller than M_1, it is included because of its importance at the edge of the band. The five constituents are separated in frequency by one cycle/month, with a single gap where one normally expects the K_1 constituent (its largest third-degree line is only 0.22 mm, even smaller than $2Q_1$).

Because of the extraordinarily long lengths of the time series, relevant nodal side lines in constituents can be included as separate estimates, independent of the main lines. For the third-degree diurnals, nodal lines above the 0.3-mm cutoff exist for constituents Q_1, M_1, and J_1. The nodal lines of Q_1 and J_1 appear to yield genuine third-degree estimates, but the nodal lines of M_1 are problematic: one of them clearly cannot be separated from second-degree lines, since (see Table 1) lines 155.565 and 155.645 are of almost identical frequencies, and the other line may be affected by nodal modulations in a weak nonlinear interaction between L_2 and K_1 (equivalent to a line at 155.465). Only nodal lines from Q_1 and J_1 are included in Tables 2–4 along with the five main lines of each constituent.

The tables give amplitude and Greenwich phase lag and the magnitude of the admittance Z, defined in this context as the ratio of the (complex) elevation to the

TABLE 3. Third-degree diurnal tides at Lerwick, Scotland.

Tide	Doodson number	Frequency (°/h)	Potential (mm)	Amplitude (mm)	Phase lag (deg)	$\|Z\|$
$2Q_1$	125.655	12.84964	0.35	1.35 ± 0.18	266 ± 8	3.84 ± 0.51
	135.545	13.39181	0.50	1.66 ± 0.18	307 ± 6	3.31 ± 0.35
Q_1	135.555	13.39402	1.28	4.08 ± 0.18	307 ± 3	3.18 ± 0.14
O_1	145.655	13.94768	0.65	1.66 ± 0.18	337 ± 6	2.54 ± 0.27
M_1	155.555	14.49205	3.99	7.25 ± 0.18	332 ± 2	1.82 ± 0.05
J_1	175.555	15.59008	1.46	2.54 ± 0.18	354 ± 4	1.74 ± 0.12
	175.565	15.59229	0.59	1.22 ± 0.18	15 ± 9	2.06 ± 0.30

TABLE 4. Third-degree diurnal tides at Vigo, Spain.

Tide	Doodson number	Frequency (°/h)	Potential (mm)	Amplitude (mm)	Phase lag (deg)	$\|Z\|$
$2Q_1$	125.655	12.84964	0.35	1.08 ± 0.15	179 ± 8	3.09 ± 0.42
	135.545	13.39181	0.50	1.02 ± 0.15	229 ± 9	2.04 ± 0.30
Q_1	135.555	13.39402	1.28	3.11 ± 0.15	213 ± 3	2.43 ± 0.12
O_1	145.655	13.94768	0.65	1.29 ± 0.15	236 ± 7	1.98 ± 0.23
M_1	155.555	14.49205	3.99	5.48 ± 0.15	232 ± 2	1.37 ± 0.04
J_1	175.555	15.59008	1.46	1.83 ± 0.15	250 ± 5	1.25 ± 0.11
	175.565	15.59229	0.59	0.68 ± 0.15	264 ± 13	1.15 ± 0.26

normalized potential. All three quantities include standard error estimates, based on the diagonal elements of the least squares covariance matrix. The admittance errors for $2Q_1$ are understandably largest, owing to its small amplitude in the astronomical potential. The given error estimates appear realistic according to two tests: 1) The original time series were partitioned into year-long segments and the statistical variability of the yearly estimates was compared with the tabulated standard errors (allowing for a $\sqrt{n}$ factor for n yearly estimates); this test, of course, can only be performed for major second-degree constituents, but the agreement in these terms appeared satisfactory. 2) The error estimates appear to account adequately for the small differences in $|Z|$ and Greenwich phase lag between the Q_1 and J_1 nodal lines and their main lines, in keeping with an expected smooth oceanic admittance.

The tidal estimates for M_1 at Newlyn and Lerwick agree well with those obtained by Cartwright (1975) for these same two stations, and station Vigo similarly shows an enhanced M_1 admittance. In fact, all three stations display a strong enhancement across the entire diurnal band. This lends considerable weight to Cartwright's original explanation of a resonance between the tidal potential and a single operating normal mode (of Q not too high). Moreover, all five constituents show phase lags increasing toward the north, consistent with the normal mode phases seen in Fig. 1. Both the en-

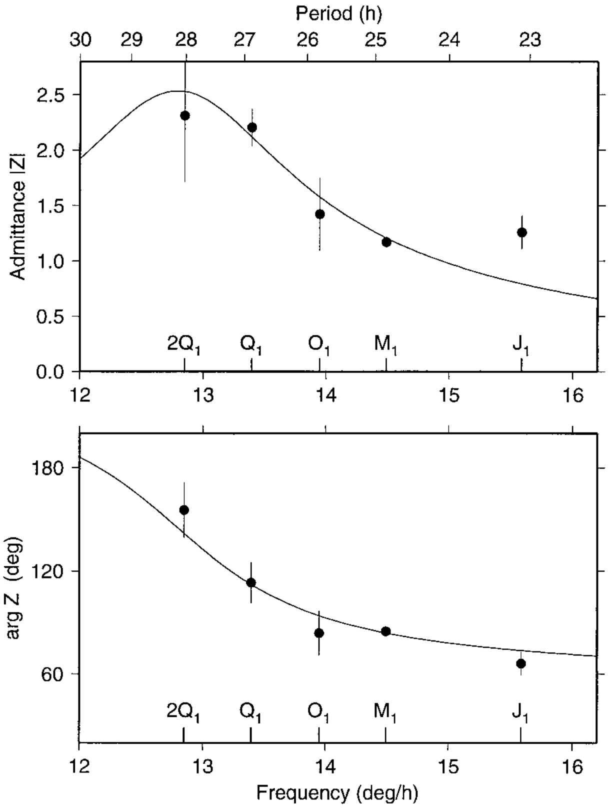

FIG. 3. Amplitude (top) and phase (bottom) of the third-degree diurnal tidal admittance at Newlyn, England, corresponding to constituent estimates tabulated in Table 1. The smooth curve represents a one-oscillator fit to the data. The amplitude peaks at the modal period 28.1 h. The poor fit to J_1 presumably indicates the influence of one or more additional modes.

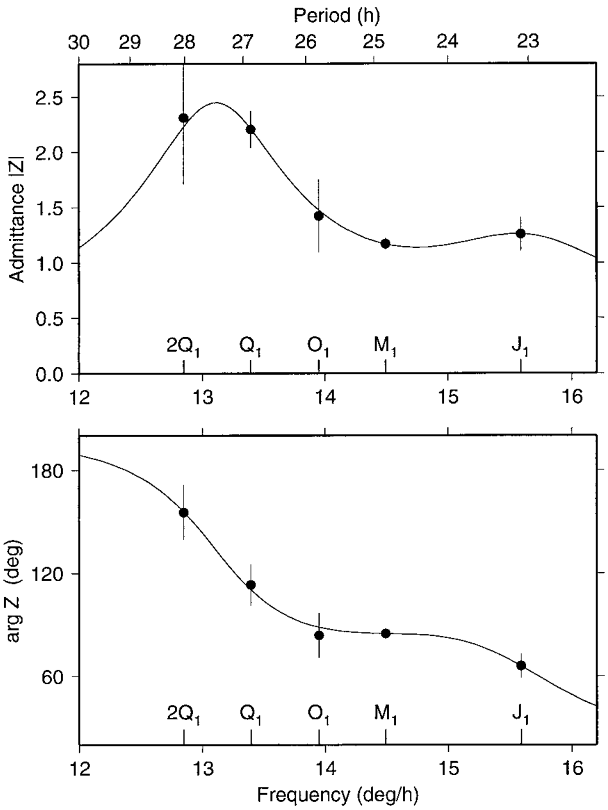

FIG. 4. As in Fig. 3 but for a two-oscillator fit to the data.

TABLE 5. Period and Q from modal fits. Periods are in hours.

	One-mode fits		Two-mode fits			
Station	Period	Q	Period	Q	Period	Q
Newlyn	28.1	6.9	27.5	11.4	23.0	9.2
Lerwick	28.2	6.3	27.6	9.2	23.4	9.9
Vigo	28.7	6.7	27.8	6.9	24.1	11.2

hanced amplitudes and the consistent phases must be counted as yet further evidence for the existence of oceanic normal modes, supplementing various other evidence discussed by Platzman (1991).

At all three stations, the admittance at J_1 is comparable to or slightly larger than that at M_1. More interestingly, all three stations show admittance amplitudes increasing as the frequency decreases away from M_1, with $2Q_1$ having the largest admittance in all cases. If Platzman's single 25.7-h mode is the primary contributor to these tides, which is the case according to his tidal synthesis (Platzman 1984, Fig. 10), then the admittances strongly suggest that his period for this mode is too low. A period below that of Q_1, and possibly below that of $2Q_1$, appears more consistent with the observed tidal admittances. An error of 10%–20% in Platzman's period is not unreasonable given various approximations made in the original calculations (e.g., coarse spatial resolution, missing shallow seas, and neglect of self-attraction and crustal loading).

3. Modal fits

With the tidal admittance defined across the diurnal band, it is straightforward to use the formalism of Eq. (1) to fit hypothetical normal modes (e.g., Platzman 1991). In particular, if one dominant normal mode is presumed to account for the behavior across the band, then we may fit the admittance Z by a simple equation of form

$$Z(\omega) = (1 - \omega/\sigma)^{-1}b, \tag{2}$$

where the modal strength b and frequency σ are treated as free parameters. Here Z, b, and σ are complex, while ω is real. Of primary interest is the frequency σ, which determines the modal period $T = 2\pi/\mathrm{Re}\sigma$ and specific dissipation $Q^{-1} = 2\ \mathrm{Im}\sigma/\mathrm{Re}\sigma$. Estimates of T and (especially) Q are, of course, strongly dependent on the assumption of a single excited mode.

Figure 3 shows the result of fitting the tidal admittances at Newlyn, using a weighted, nonlinear, least squares algorithm. In this case, there are two complex parameters to be determined from five complex admittances. The resulting frequency corresponds to a modal period of 28.1 h and Q of 7. This estimate of Q is about twice the value determined for the diurnal-band Antarctic Kelvin wave but less than the Q determined for the semidiurnal tides of the North Atlantic (e.g., Platzman 1991). It is clear that the fitted period depends sensitively on the admittance at $2Q_1$; since its amplitude falls well short of the linear trend between O_1 and Q_1, the fitted curve is forced downward, causing a peak (and hence estimated period) in the general vicinity of $2Q_1$. In this region of the diagram, the fit is satisfactory given the sizes of the error bars. On the other hand, the amplitude fit at J_1 is clearly poor and suggests the presence of one or more additional modes.

Equation (2) has therefore been augmented with a second oscillator. The resulting fit to the diurnal admittances is shown in Fig. 4. The estimated periods in this case are 27.5 and 23.0 h, somewhat closer to Platzman's 25.7 and 23.7 h. The corresponding Q, around 10, is higher than the single-mode fit. One should not, however, put too much credence to this augmented fit; we are fitting four complex parameters to only five complex admittances, and the very good agreement shown in Fig. 4, in fact, suggests overfitting. Unfortunately, there are no additional tidal frequencies that can be used to constrain a two-mode fit.

Fits to the data at Lerwick and Vigo are quite similar to those at Newlyn. The estimated periods and Q values for all stations are summarized in Table 5. I am disinclined to make a choice as to the "best" estimated period for the fundamental mode, except to say that the available evidence clearly suggests a period longer than Platzman's original 25.7 h, a period most likely longer than the period of Q_1, and perhaps in the vicinity of 27 to 29 h.

Acknowledgments. The tide gauge data employed here were obtained courtesy of the British Oceanographic Data Centre (Bidston Observatory) and the University of Hawaii Sea Level Center. The Vigo data were originally contributed by the Instituto Español de Oceanografía, Madrid, Spain.

REFERENCES

Cartwright, D. E., 1975: A subharmonic lunar tide in the seas off Western Europe. *Nature,* **257,** 277–280.
——, 1976: Anomalous M_1 tide at Lagos. *Nature,* **263,** 217–218.
——, and R. J. Tayler, 1971: New computations of the tide-generating potential. *Geophys. J. Roy. Astron. Soc.,* **23,** 45–74.
——, R. Spencer, J. M. Vassie, and P. L. Woodworth, 1988: The tides of the Atlantic Ocean, 60°N to 30°S. *Philos. Trans. Roy. Soc. London,* **324A,** 513–563.
Platzman, G. W., 1984: Normal modes of the world ocean. Part IV: Synthesis of diurnal and semidiurnal tides. *J. Phys. Oceanogr.,* **14,** 1532–1550.

——, 1985: Normal modes of the world ocean: Maps and tables. Dept. of Geophysical Sciences, University of Chicago, 97 pp.

——, 1991: Tidal evidence for ocean normal modes. *Tidal Hydrodynamics,* B. B. Parker, Ed., John Wiley and Sons, 13–26.

——, G. A. Curtis, K. S. Hansen, and R. D. Slater, 1981: Normal modes of the world ocean. Part II: Description of modes in the period range 8 to 80 hours. *J. Phys. Oceanogr.,* **11,** 579–603.