Propagation, Diffusion, and Decay of SST Anomalies beneath an Advective Atmosphere

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ABSTRACT

A simple conceptual model is used to illustrate aspects of how the midlatitude atmosphere, in the absence of ocean dynamics, responds to and feeds back on sea surface temperature (SST) anomalies. In the model, a dynamically passive ocean mixed layer of fixed depth exchanges heat with a single-level, energy-balance atmosphere with a constant mean wind, U. The temperatures of the two subsystems, T_O and T_A, respectively, strive to equilibrate through surface heat exchange, which is parameterized as $\lambda(T_O - T_A)$.

Atmospheric advection of heat has two important effects on the evolution of SST anomalies. First, the SST anomalies propagate downwind at the speed $(c_A/c_O)U$, where c_A and c_O are the heat capacities of the atmosphere and the oceanic mixed layer, respectively. Second, the damping rate of SST anomalies is scale dependent: the distance an atmospheric column travels before it equilibrates with the ocean through surface heat exchange (Uc_A/λ) introduces a length scale that discriminates between small-scale and large-scale SST anomalies. Local bulk formulas of surface heat exchange determine the damping of small-scale anomalies, which decay exponentially over the timescale c_O/λ. Large-scale anomalies, on the other hand, decay essentially diffusively and propagate downwind, until longwave radiation finally extinguishes them. The apparent diffusive decay results from the joint effect of atmospheric advection and surface heat exchange. And the effect becomes significant when the distance the atmosphere carries heat downwind is small compared to the scale of the SST anomaly. The kinematical diffusion coefficient associated with the phenomena is $c_A^2 c_O^{-1} U^2 \lambda^{-1}$.

1. Introduction

Compared to the ocean, the atmosphere is efficient in redistributing heat laterally. The distance over which dynamical processes rapidly achieve a smooth atmospheric temperate structure provides a length scale, which discriminates between small-scale and large-scale sea surface temperature (SST) anomalies. Bretherton (1982) and Frankignoul (1985) point out that, in absence of ocean dynamics, small-scale SST anomalies decay exponentially at a rate dictated by local bulk formulas of air–sea heat exchange (Haney 1971). In the atmosphere, the heat emanating from the SST anomaly is redistributed laterally. Therefore, the atmosphere warms outside the SST anomaly, which drives the heat back into the ocean. Large-scale SST anomalies, on the other hand, are weakly affected by atmospheric heat transport. Instead, emission of longwave radiation to space sets the decay rate of anomalies, which is significantly slower. The scale-selective damping of SST anomalies and its implications for ocean modeling have been discussed, for example, by Schopf (1983), Seager et al. (1995), Rahmstorf and Willebrand (1995), and Chen and Ghil (1996). Fewer studies, however, have focused on the spatial evolution of the SST field that arises through the interplay between the surface heat exchange and the atmospheric temperature response.

In an insightful review article, Frankignoul (1985) analyses how a linear midlatitude atmosphere responds to and feeds back on SST anomalies in the presence of a westerly mean wind. He considers a two-level as well as a continuous atmospheric model that exchanges heat with a dynamically passive ocean mixed layer of fixed depth. As a general rule, Frankignoul shows that the SST anomalies, due to interaction with the atmosphere, decay exponentially with time and are advanced eastward. The rate of decay and the speed of eastward propagation depend on the scale of the SST anomaly. Beneath the continuous atmospheric model, the rate of decay increases and the propagation speed decreases monotonically with wavenumber of the SST anomaly. The mathematically simpler two-level model produces a more complex relation between the rates of decay and propagation and the wavenumber of SST anomalies. This is attributable to the rigid upper boundary of the model, which admits a resonance between the heating and the atmospheric Rossby waves. At resonance the atmospheric response is equivalently barotropic and the damping of the SST anomaly is weak (e.g., Frankignoul 1985; Shutts 1987; Goodman and Marshall 1999).

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Marotzke and Pierce (1997) address a similar topic, but in a model where the atmospheric heat transport is diffusive rather than advective. They study the thermal interaction between an ocean mixed layer and a single-level energy-balance atmosphere that diffuses heat laterally. An interesting feature that emerges transparently in their study is that the air–sea heat exchange not only damps the SST anomalies but also modifies their structure in a systematic fashion. For large-scale anomalies, Marotzke and Pierce show that the SST field is diffused by the atmosphere, but with a kinematical diffusivity that is reduced by the ratio between the atmospheric and the oceanic heat capacities.

The aim of this study is to illustrate some key features of how the large-scale mean winds in combination with air–sea heat exchange would modify the structure of SST anomalies in the absence of ocean dynamics. The intention is to illuminate a fundamental aspect of atmosphere–ocean interaction in the context of an idealized model, rather that to faithfully simulate reality. As a conceptual representation of the atmosphere, we use a model of energy-balance type where heat is advected by a constant mean wind. A dynamically inert ocean mixed layer of constant depth exchanges heat with the atmosphere. It is easy enough to criticize the model, and it can be debated to what extent it captures the thermal interaction between the midlatitude atmosphere and SST anomalies. However, Frankignoul (1985) shows that this simple model, to a fair approximation, reproduces the near-surface air temperature of a linear continuously stratified atmospheric model on a β plane. As linear models tend to capture the gross features of the dynamics (Held 1983), we believe that the present model may be useful at a conceptual level. At any rate, our atmospheric model can always be interpreted as a simple proxy for a linear continuously stratified model of the midlatitude atmosphere. One caveat, however, has to be kept in mind: in a situation where the real atmosphere (or a sophisticated model) attains resonance, the present model probably fails even at a qualitative level.

Here, we consider the time evolution of one-dimensional SST anomalies of arbitrary shape beneath an advective atmosphere. Frankignoul (1985) and Schopf (1985) address aspects of this problem by analyzing the dynamics of the SST field in wavenumber space. As it turns out, the analysis in physical space reveal some interesting features that were overlooked in the previous studies: the mass center of an SST anomaly propagates at fixed speed simultaneously as the anomaly is diffused relative to the mass center. Also, we connect this work to the study of Marotzke and Pierce (1997) by studying how a combination of advective and diffusive atmospheric heat transport damp and disperse SST anomalies.

2. The evolution of SST anomalies in atmospheric flows

a. The model

The model is a conceptual representation of thermal interaction, in one horizontal dimension, between a dynamically passive oceanic mixed layer and a single-level atmosphere. In the atmosphere, heat is transported by a constant mean flow and by diffusion, and longwave radiation exports heat to space. Through the air–sea interface, the ocean and the atmosphere exchange heat at a rate proportional to the difference between their temperatures. Except for the advective heat transport, the model is identical to that of Marotzke and Pierce (1997). Saravanan and McWilliams (1998) use a mathematically similar model, in which the ocean currents are responsible for the advection of heat, to study the generation of SST anomalies.

The model equations are based on conservation of heat in the atmosphere:

$$c_A \frac{\partial T_A}{\partial t} + c_A U \frac{\partial T_A}{\partial x} = c_A D \frac{\partial^2 T_A}{\partial x^2} - BT_A + \lambda(T_O - T_A), \tag{1a}$$

and in the ocean:

$$c_O \frac{\partial T_O}{\partial t} = -\lambda(T_O - T_A). \tag{1b}$$

Here, T_A and T_O are the temperature anomalies of the near-surface air and the ocean mixed layer respectively, U is the wind speed, D is the atmospheric diffusivity, c_A and c_O are the heat capacities per unit area of the atmosphere and the ocean mixed layer, B is the longwave radiation coefficient, and λ is the surface heat exchange coefficient.

The list of assumptions and restrictions that goes with the model is long. Here, we merely point at some assumptions that are important for the interpretation of its results. First of all, we assume a correlation between anomalies in the near-surface air temperature and the temperature of the free atmosphere. In midlatitudes, such a correlation exists only in a statistically sense, when an average is made over many individual weather events. Therefore, we apply our model only to describe the evolution of the SST field, which is assumed to be slow enough for the atmosphere to be in statistical equilibrium with the ocean. In the subsequent analysis, we ignore the local time tendency of the atmospheric temperature in Eq. (1a). Second, we assume that the surface heat flux heats (or cools) the atmosphere locally. For the latent heat flux this is a questionable assumption (e.g., Kushnir and Held 1996), and it is discussed in section 3. Finally, it is important to emphasize that c_A and U should be interpreted as the heat capacity of the atmospheric temperature anomaly and an effective advection velocity, respectively. These parameters may vary and are admittedly difficult to estimate without resorting to a sophisticated atmospheric model.

To give an impression of the order of magnitudes involved, we consider a mixed layer depth of 50 m and assign the following values to the parameters in Eqs. (1) (e.g., Marotzke and Pierce 1997):

$$c_A = 10^7\ \mathrm{J\ m^{-2}}; \qquad c_O = 2 \times 10^8\ \mathrm{J\ m^{-2}};$$
$$U = 5\ \mathrm{m\ s^{-1}}; \qquad D = 2.5 \times 10^6\ \mathrm{m^2\ s^{-1}};$$
$$\lambda = 50\ \mathrm{W\ m^{-2}\ K^{-1}}; \qquad B = 2\ \mathrm{W\ m^{-2}\ K^{-1}}.$$

Ignoring for the moment the diffusivity D, we can in Eqs. (1) identify three timescales and one length scale that enter in the thermal adjustment process:

$$\tau_A \equiv c_A\lambda^{-1}(\approx 2\ \text{days}), \qquad \tau_O \equiv c_O\lambda^{-1}(\approx 50\ \text{days}),$$
$$\tau_R \equiv c_O B^{-1}(\approx 3\ \text{yr}), \quad \text{and} \quad L_A \equiv \tau_A U(\approx 1000\ \text{km}).$$

The first two timescales characterize the equilibration of the atmosphere and the ocean, respectively, with the restoring surface flux, and the last timescale characterizes the relaxation of the coupled system through emission of infrared radiation. The length scale, which can be called the atmospheric advection radius, represents the distance an atmospheric column is advected before it has equilibrated with the ocean through surface heat exchange. We stress that the precise values of time and length scales should not be taken too seriously.

b. *The evolution of a localized SST anomaly*

Here we isolate the effect of advective atmospheric heat transport by setting the diffusivity D to zero in Eq. (1a). To begin, it is instructive to briefly retrace Frankignoul's (1985) analysis of how a linear continuously stratified midlatitude atmosphere responds to and feeds back on the SST anomalies in a slab mixed layer. In his study, the zonal wind is independent of height, and the atmospheric heating per unit mass is distributed vertically as

$$q = \gamma \exp(-\gamma z) Q(x, t),$$

where $Q = \lambda(T_O - T_A)$ is the net upward heat flux at the sea surface. Frankignoul notes that for moderate friction, the near-surface air temperature of the continuous model to a good approximation is captured by

$$c_A U \frac{\partial T_A}{\partial x} = \lambda(T_O - T_A), \tag{2}$$

which is a special case of Eq. (1a). [This equation is also discussed by Power et al. (1995), who present some solutions of the atmospheric temperature for a simple and fixed distribution of the SST.] In this context, c_A is related to the depth of the atmospheric heating anomaly: $c_A = \rho_A c_p \gamma^{-1}$, where ρ_A is the density of near-surface air and c_p is the heat capacity of air at constant pressure.

By combining the advection equation with the SST equation [Eq. (1b)] and making an anzats of the form

$$T_O(x, t) = T_{\text{in}} \exp(-\sigma t) \cos[k(x - ct)],$$

Frankignoul (1985) obtains

$$c = U\frac{c_A}{c_O}\frac{1}{1 + (kL_A)^2}, \tag{3a}$$

$$\sigma = \frac{1}{\tau_O}\frac{(kL_A)^2}{1 + (kL_A)^2}. \tag{3b}$$

Here, T_{in} is an amplitude, c is a phase velocity, σ is an inverse damping timescale, and k is the wavenumber.

In fact, Frankignoul (1985) does not give these relations explicitly but illustrate them graphically (in his Fig. 27, which can be reproduced using $U = 15\ \mathrm{m\ s^{-1}}$, $\gamma^{-1} = 2500$ m, $\lambda = 40\ \mathrm{W\ m^{-2}}$, and $c_O = 4 \times 10^8\ \mathrm{J\ m^{-2}}$). Shopf (1985) derives a similar expression of the damping rate, but does not comment on the propagation of anomalies.

The wavenumber dependence of the phase speed suggest that an SST anomaly disperses with time. In fact, the group velocity for wavenumbers greater than L_A^{-1} are directed opposite to the wind. However, the damping rapidly attenuates the small-scale features, which yields an advective–diffusive behavior at large times. To illustrate this, we give an analysis of the evolution in physical space. Although no additional physics is introduced, this approach illuminates some features that are difficult to anticipate directly from Eqs. (3). We approximate and rewrite the governing equations (with $D = 0$, but allowing for radiative damping) as

$$\left(L_A\frac{\partial}{\partial x} + 1 + \frac{B}{\lambda}\right)T_A = T_O, \tag{4a}$$

$$\frac{\partial T_O}{\partial t} = -\frac{1}{\tau_O}(T_O - T_A). \tag{4b}$$

Below we present a general solution to these equations. At first, however, we discuss asymptotic versions of the equations that describe the limiting cases of large-scale and small-scale anomalies, respectively. To this end, we consider heat conservation in the coupled system, which is obtained by adding Eqs. (4a) and (4b):

$$\frac{\partial T_O}{\partial t} + \left(U\frac{c_A}{c_O}\right)\frac{\partial T_A}{\partial x} = -\frac{1}{\tau_R}T_A. \tag{5}$$

Clearly, if the ocean and the atmosphere temperatures are nearly equal, an SST anomaly propagates downstream at the speed

$$V_O \equiv L_A/\tau_O = (c_A/c_O)U, \tag{6}$$

which is exactly the value predicted by Eq. (3a) for $k = 0$. Simultaneously, it decays through radiative heat exchange with the space. The physical mechanism behind the propagation may be illustrated as follows: The atmospheric advection is able to carry heat downstream a distance of the order of L_A; to change the SST downwind (or on the windward side) of the initial anomaly takes a time of the order of τ_O. That the SST anomalies move slower than the air is related to the thermal inertia of the ocean, which has to be transported by the atmosphere. [If λ goes to infinity, which implies that $T_O = T_A$, then Eqs. (1) predict that the mass center of an anomaly moves at the speed $Uc_A(c_A + c_O)^{-1}$.] Hansen and Bezdek (1996) point out that surface heat exchange, in the presence of a mean wind, causes SST anomalies to propagate. Using scaling arguments, they obtain an expression for the propagation speed that is mathematically similar to the one derived here.

Depending on the ratio between L_A and the initial length scale of the SST anomaly, say L, Eq. (4a) yields two different, asymptotic relations between T_A and T_O. First, consider the situation when $L/L_A \ll 1$. Directly above the anomaly, the atmospheric temperature can be approximated as

$$L_A \frac{\partial T_A}{\partial x} \approx T_O,$$

which is correct to the order of L/L_A. In this case, the atmospheric temperature is nearly equal to the undisturbed upstream temperature, which results in a vigorous surface heat exchange. Substituting this relation into Eq. (5) and ignoring terms of the order of B/λ, we obtain the evolution of the SST within the anomaly

$$\tau_O \frac{\partial T_O}{\partial t} \approx -T_O, \tag{7}$$

which is shown to decay exponentially with the e-folding timescale τ_O. Physically, this is attributable to the advective atmospheric heat transport, which picks up the heat over the anomaly. Downstream, the heat is returned to the ocean over a distance of length L_A, approximately. A small-scale anomaly, accordingly, expands downstream and reach the length scale L_A after the time τ_O has elapsed. As discussed by Schopf (1985), not only localized, small-scale anomalies decay over the timescale τ_O, but also any small-scale structure in an arbitrary initial SST distribution.

We proceed now to the evolution of large-scale anomalies for which $L/L_A \gg 1$. As delineated by a solution to Eqs. (4a,b) in Fig. 1, the lateral growth of anomalies is slowed down but not halted when the length scale L_A is reached. Therefore, the limiting case of large-scale anomalies applies for any initial SST field provided that $\tau_O \ll t$. Treating $(L_A/L)^2$ and B/λ as small parameters, we approximate Eq. (4a) as

$$T_A \approx T_O - L_A \frac{\partial T_O}{\partial x}.$$

In this case, the temperatures of the ocean and the atmosphere are nearly equal, which implies a weak surface heat exchange. Inserting this relation into Eq. (5), yields an approximate equation governing the evolution of large-scale anomalies. Retaining terms of the order of $(L_A/L)^2$ and B/λ, we arrive at

$$\frac{\partial T_O}{\partial t} + \left(U\frac{c_A}{c_O}\right)\frac{\partial T_O}{\partial x} = \left(\frac{L_A^2}{\tau_O}\right)\frac{\partial^2 T_O}{\partial x^2} - \frac{1}{\tau_R}T_O. \tag{8}$$

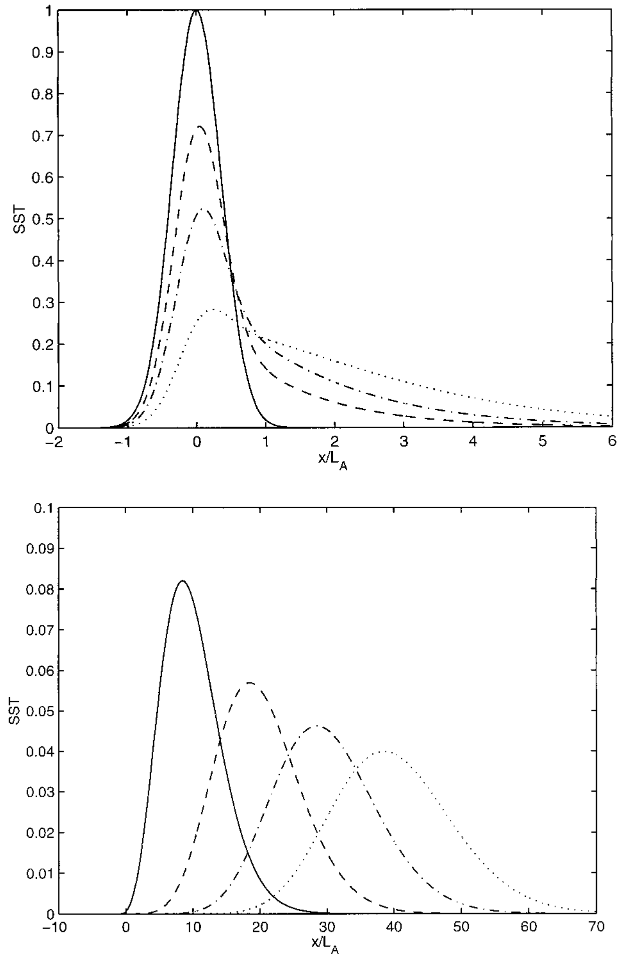

FIG. 1. The evolution of an SST anomaly in of the initial form $\exp(-4x^2L_A^{-2})$ calculated from Eq. (10). (a) The initial phase of downstream expansion; the SST-field is delineated at the times $t/\tau_O = 0$ (solid line), ½ (dashed), 1 (dash–dotted), and 2 (dotted). (b) The long-term behavior, characterized by advection and diffusive spreading; the SST field is delineated at the times $t/\tau_O = 10$ (solid line), 20 (dashed), 30 (dash-dotted), and 40 (dotted). Here B/λ is set to zero, which excludes the damping by longwave radiation. Note, that the anomaly initially becomes asymmetric but eventually regains a symmetric shape.

Anomalies are thus propagated downstream and diffused while decaying through longwave radiative exchange with space. It is important to stress that the apparent diffusivity of the SST field results from the combined effect of air–sea heat exchange and advection of heat in the atmosphere. The equivalent kinematic diffusion coefficient (say $D_\lambda \equiv L_A^2\tau_O^{-1}$) can be phrased as

$$D_\lambda = \frac{c_A^2 U^2}{c_O \lambda}. \tag{9}$$

By an analogy with turbulent diffusion, we may think of L_A and τ_O as the length scale and the turnover time, respectively, of the eddies. Fundamentally, the diffusive spreading of large-scale anomalies is related to the fact that the distance the atmosphere transports heat is small compared to the length scale of the SST field.

A general solution to Eqs. (4a,b), subject to the initial condition $T_O(x, 0)$, can be phrased as (see the appendix for the details)

$$T_O(x, t) = T_O(x, 0)\exp(-t_*) + \int_{-\infty}^{+\infty} G(x - x', t)T_O(x', 0)\, dx'. \quad (10)$$

Here, the Green's function G is defined by

$$G(x - x', t) \equiv \exp[-t_* - X_*(1 + \epsilon)]\sqrt{\frac{t_*}{X_*}}I_1(2\sqrt{X_* t_*})\frac{H(X_*)}{L_A}, \quad (11)$$

where I_1 is the modified Bessel function, H is the unit step function, and we have introduced

$$t_* \equiv t\tau_O^{-1}; \qquad X_* \equiv (x - x')L_A^{-1}; \qquad B/\lambda \equiv \epsilon.$$

Figure 1 portrays the evolution of a localized SST anomaly with initial dimension L_A, approximately. For times comparable to τ_O, the atmosphere picks up heat from the anomaly and returns it to the ocean over a distance of the order of L_A downstream. In this phase, the anomaly expands downstream rapidly while its amplitude declines in accordance with heat conservation. Beyond times greater than $5\tau_O$, roughly, the evolution of the SST field is characterized by downstream advection and diffusive spreading as suggested by Eq. (8). Indeed, this is verified by the asymptotic expansion of Eq. (10) (see the appendix), which for large times yields

$$T_O(x, t) \approx \frac{\exp\left[-t/\tau_R - \frac{(x - V_O t)^2}{4D_\lambda t}\right]}{2\sqrt{\pi D_\lambda t}}\int_{-\infty}^{+\infty} T_O(x', 0)\, dx'. \quad (12)$$

Here, the decay rate of the anomaly is inversely proportional to the square root of time. Marotzke and Pierce (1997) identify a, mathematically, similar decaying regime, which is attributable to the diffusive atmospheric heat transport in their model.

When the time becomes comparable to τ_R, the emission of longwave radiation commences to extinguish the SST feature. At this stage, the size and amplitude of the anomaly are of the order of

$$\frac{L_A}{\epsilon^{1/2}} = \frac{Uc_A}{\sqrt{B\lambda}},$$

and $\epsilon^{1/2}$, respectively. The anomaly has now propagated a distance of the order of

$$\frac{L_A}{\epsilon} = \frac{Uc_A}{B}.$$

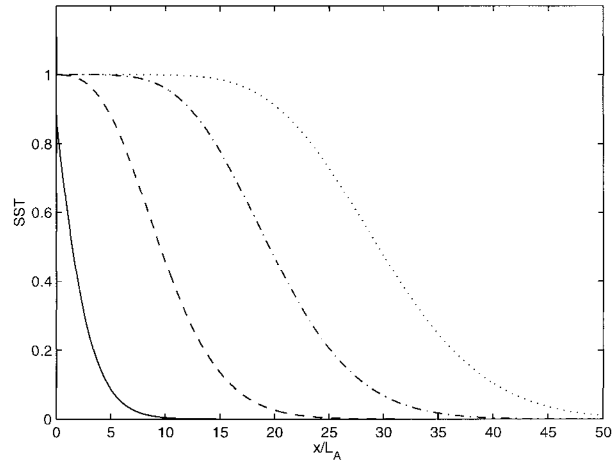

FIG. 2. The SST response in a semi-infinite domain to a sudden change in the atmospheric temperature at $x = 0$ (see the appendix for details). Initially, the SST field is zero. The curves illustrate the subsequent evolution at the times $t/\tau_O = 2$ (solid line), 10 (dashed), 20 (dash–dotted), and 30 (dotted). The damping by longwave radiation is suppressed by setting B/λ to zero.

It is interesting to note that these quantities are independent of the oceanic heat capacity.

Based on the results above, we can delineate the response of the SST field to a sudden, but persistent, change in the upstream atmospheric temperature. This scenario may illustrate some basic elements of the SST response to an outflow of cold air from the eastern margin of an ocean basin (e.g., Seung 1987). A solution to this boundary value problem is given in the appendix and is illustrated in Fig. 2. After the time τ_O (a few months), an exponentially decaying signal penetrates a distance of the order of L_A (of the order of 1000 km) into the ocean. When the time significantly exceeds τ_O (several months), the SST field takes the form of a front that broadens diffusively (with the square root of time) and propagates at the speed V_O. The front separates the upstream region, where the SST has adjusted to the new atmospheric temperature, from the undisturbed surface water ahead. The final state is an SST signal that decays away from the coast with e-folding length scale $(Uc_A)/B$.

c. Atmospheric diffusivity

As Bretherton (1982) points out, lateral atmospheric heat transport plays an instrumental role in determining the local surface heat exchange in a region of anomalous SST. Against this background, it is interesting to ask if there are any significant differences in how atmospheric advection and diffusion, respectively, damp SST anomalies. Also, this issue connects the present study with the work of Marotzke and Pierce (1997), who consider a purely diffusive atmosphere. As we use a one-dimensional model, we have to bear in mind that two dimensionally will alter the picture to some extent.

By including the diffusivity operator, Eq. (4a) transforms to

$$\left(L_A\frac{\partial}{\partial x} - L_D^2\frac{\partial^2}{\partial x^2} + 1 + \frac{B}{\lambda}\right)T_A = T_O, \qquad (13)$$

and heat conservation in the coupled system [Eq. (5)] transforms to

$$\frac{\partial T_O}{\partial t} + \left(U\frac{c_A}{c_O}\right)\frac{\partial T_A}{\partial x} = \left(D\frac{c_A}{c_O}\right)\frac{\partial^2 T_A}{\partial x^2} - \frac{1}{\tau_R}T_A, \qquad (14)$$

where we have introduced

$$L_D \equiv \sqrt{D\tau_A}.$$

This length scale is termed L_1 by Marotzke and Pierce and it represents the distance over which atmospheric diffusion can transport heat before it is returned to the ocean. For the present choice of parameters, L_D is O(700 km), which is comparable to L_A.

Also in this case, the SST dynamics at times comparable to τ_O is characterized by exponential damping of small-scale features in the SST field (Schopf 1985). The length scale that should be used to distinguish between large-scale and small-scale anomalies is the largest of the two lengths, L_D and L_A. When the latter is much greater that the former, the dynamics dominated by advection, as discussed above. In the opposite limit, the dynamics approach the nonadvective purely diffusive case treated by Marotzke and Pierce (1997).

Consider the dynamics of large-scale SST anomalies (or equivalently, the dynamics at times that are large compared to τ_O). Here we may approximate Eq. (13) as

$$T_A \approx T_O - \left(L_A\frac{\partial}{\partial x} - L_D^2\frac{\partial^2}{\partial x^2}\right)T_O.$$

Using this in Eq. (14) and keeping terms of the order of $(L_A/L)^2$ and $(L_D/L)^2$, we obtain

$$\frac{\partial T_O}{\partial t} + \left(U\frac{c_A}{c_O}\right)\frac{\partial T_O}{\partial x} = \left(\frac{c_A}{c_O}D + D_\lambda\right)\frac{\partial^2 T_O}{\partial x^2} - \frac{1}{\tau_R}T_O. \qquad (15)$$

In an advective–diffusive atmosphere, accordingly, the effective kinematic diffusivity acting on large-scale SST features is

$$\frac{c_A}{c_O}D + D_\lambda = \frac{c_A}{c_O}\left(D + \frac{c_A U^2}{\lambda}\right).$$

For the present choice of parameters, we have

$$\frac{c_A}{c_O}D = 1.25 \times 10^5\ \mathrm{m^2\ s^{-1}}, \qquad D_\lambda = 2.5 \times 10^5\ \mathrm{m^2\ s^{-1}}.$$

Thus, the contribution to the diffusion of the SST field from the combined effect of advection and surface heat exchange may exceed the contribution from the diffusive heat transport in the atmosphere, at least in this conceptual model. It is interesting to note that in a two-dimensional formulation of the model, the effective diffusion of large-scale SST anomalies would be anisotropic, with a tendency to elongate anomalies in the direction of the wind.

It is instructive to compare how our advective model and the diffusive model of Marotzke and Pierce (1997) describe the evolution of a localized anomaly. Given that the two length scales, L_A and L_D, are equal the SST field will evolve similarly in the two models. Essentially, the difference is the downstream propagation of the SST signature in the advective model.

Suppose that the initial anomaly is small compared to the length scales L_A and L_D. For times comparable to the oceanic adjustment time ($\tau_O \approx 2$ months), both models yield expansion and exponential decay of the anomalies. In the diffusive model, the expansion is symmetric while it is biased to the downstream side in the advective model (see Fig. 1a). For times between the oceanic adjustment time and and the timescale of radiative damping ($\tau_R \approx 3$ yr), the remainder of the anomalies are diffused and decay as the inverse square root of time; the effective kinematical diffusion coefficients of the advective and the diffusive model are $L_A^2\tau_O^{-1}$ and $L_D^2\tau_O^{-1}$, respectively. Except for the downstream propagation in the advective model, the two models yield a nearly identical evolution of the SST in this phase.

3. Discussion

The main virtue of the present model is its simplicity, which admits illustrative analytical solutions. However, its qualitative nature has to be emphasized. Potentially important effects have deliberately been ignored in the model, and the represented physics is highly idealized. Also, it is important to recall that we attempt to illuminate how the joint effect of air–sea heat exchange and advective atmospheric heat transport would modify the SST field in the absence of ocean dynamics. Thus, even if the simple atmospheric model is correct at leading order, ocean dynamics may obscure or alter the picture delineated here. Frankignoul (1985), Seung (1987), Saravanan and McWilliams (1998), and Goodman and Marshall (1999), for example, discuss how ocean currents and feedbacks between winds, SST, and currents affect SST anomalies in midlatitudes.

In attempting to identify features of this model with real world phenomena, two oversimplifications in the representation of the atmosphere have to be remembered. First of all, if resonance occurs between atmospheric Rossby waves and the induced heating of the SST anomaly, the present model may be wrong even at a qualitative level. Resonance (or an equivalent barotropic response) is attained frequently in two-level atmospheric models (e.g., Frankignoul 1985; Shutts 1987), but appears to be more elusive in atmospheric GCMs (e.g., Held 1983; Kushnir and Held 1996). It is beyond the scope of the present study to give an account of the dynamics near resonance. However, one feature of the two-level model study of Frankignoul (1985) deserves to be pointed out: Near resonance, the model response is sensitive to the two-dimensional structure of the SST anomaly.

Second, the assumption that the surface heat flux heats the atmosphere locally neglects the fact that the evaporated water vapor may condense remotely from its source. It is relevant to ask if this approximation is so severe that our model fails to capture the dynamics at leading order. In the GCM study of Kushnir and Held (1996), about half of the latent heat flux is realized as local atmospheric heating, while the remainder is exported to other regions. Assuming that this is a generally valid result, how does it affect the evolution of the SST field? It is instructive to focus on some limiting cases in connection with a large-scale warm SST anomaly. Consider first a scenario in which all of the surface heat flux heats the atmosphere far away over continental land. Clearly, the atmospheric temperature would be unaffected above the SST anomaly, which would decay exponentially over the timescale τ_O regardless of its size. The opposite extreme, in which all the surface heat flux is realized as local heating, is the case addressed in section 2. In a situation where some fraction of the air–sea heat exchange heats the atmosphere locally, we would observe a reduced tendency of downwind advancement and diffusive spreading, which now is accompanied by an exponential decay over a timescale that falls between τ_O and τ_R. Therefore, the result of Kushnir and Held (1996) suggests that remote condensation would obscure the simple picture of our model, but not remove it entirely.

Keeping the caveats of the model in mind, we recapitulate three important features and point at some physical phenomena to which they may pertain. First, the distance an atmospheric column is advected before it equilibrates with the ocean through surface heat exchange may be estimated as

$$\frac{c_A U}{\lambda}.$$

This length scale (L_A) is a key parameter for the SST dynamics beneath an atmosphere where the heat transport is dominated by advection. As noted by Schopf (1985), this length scale discriminates between small-scale rapidly decaying and large-scale long-lived SST anomalies. Over the oceanic adjustment timescale τ_O, roughly, a small-scale SST feature expands to a size of the order of L_A. During the lateral growth, conservation of heat causes the amplitude of the anomaly to decline. Also, large-scale atmospheric circulation anomalies of moderate persistence are expected to generate SST features with the scale L_A. For example, our analysis indicates that anomalously cold air moving off a continent creates an SST signal that penetrates a distance of about L_A in a few moths. It is not inconcievable that the eastward penetration of interannual SST anomalies in the western North Atlantic (e.g., Cayan 1992; Kushnir and Held 1996) partly reflects this length scale.

Second, the thermal communication through the air–sea interface causes the wind to advance SST anomalies downstream at the speed

$$\left(U\frac{c_A}{c_O}\right).$$

The reduced advection velocity, relative to the wind, reflects the oceanic thermal inertia that has to be transported by the atmosphere. In addition to currents and baroclinic Rossby wave dynamics in the ocean, this transport mechanism may play some role for the observed movement of interannual SST anomalies in the North Atlantic (Hansen and Bezdek 1996).

Third, after the initial fast adjustment, the evolution of the SST field is characterized by downstream propagation and diffusive expansion. Surface heat exchange in combination with advection of heat broaden large-scale SST anomalies in an essentially diffusive manner, with a kinematical diffusivity given by

$$\frac{c_A^2 U^2}{c_O \lambda}.$$

This diffusive feature is analogous the diffusive transport of heat in a gas for which the scale of the temperature field is much larger than mean free path of the molecules.

In conclusion, we believe that our conceptual model exposes some interesting and nontrivial features that may arise through the interplay between air–sea heat exchange and advective atmospheric heat transport. The extent to which our results are relevant to the real world has to be tested against more sophisticated models or data. However, we hope that some elements of this study may be pieced into a future conceptual picture of feedbacks between the midlatitude atmosphere and SST anomalies.

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APPENDIX

Green's Function

The starting point is the equation pair, (4a) and (4b), which are nondimensional using the scales

$$x_* = x/L_A, \qquad t_* = t/\tau_O.$$

By combining these equations to eliminate T_A (and dropping the asterisk on the nondimensional variables), we get

$$\left(\frac{\partial}{\partial x} + 1 + \epsilon\right)\left(\frac{\partial}{\partial t} + 1\right)T_O - T_O = 0, \qquad \text{(A1)}$$

where $\epsilon \equiv B/\lambda$. We seek the evolution of the SST field, $T_O(x, t)$, subject to the initial condition

$$T_O(x, t = 0) = T_{\text{in}}(x).$$

It is convenient to make the following ansatz:

$$T_O(x, t) = T_{\text{in}}(x) \exp(-t) + A(x, t),$$

where A satisfies the nondimensional form of Eq. (4b); that is,

$$\left(\frac{\partial}{\partial t} + 1\right)A = T_A,$$

and the initial condition

$$A(x, t = 0) = 0.$$

Applying the Fourier–Laplace transform (e.g., Arfken 1985), we arrive at

$$\tilde{A}(k, s) = \frac{i\hat{T}_{\text{in}}(k)}{(s + 1)^2\left[k + i\left(\epsilon + \frac{s}{s + 1}\right)\right]}, \quad \text{(A2)}$$

where

$$\hat{T}_{\text{in}}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} T_{\text{in}}(x) \exp(+ikx)\, dx$$

is the Fourier transform of the initial SST field.

The Green's function is obtained by representing T_{in} with a delta function $[\delta(x - x')]$ for which

$$\hat{T}_{\text{in}}(k) = \frac{\exp(+ikx')}{\sqrt{2\pi}}.$$

The inverse Fourier transform

$$\tilde{A}(x, s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \tilde{A}(k, s) \exp(-ikx)\, dk$$

is determined by the simple pole of $\tilde{A}$ in the lower complex half-plane. The result is

$$\tilde{A}(x, s) = \exp[-(x - x')(1 + \epsilon)]\, \frac{\exp\left(\frac{x - x'}{s + 1}\right)}{(s + 1)^2} \times H(x - x'). \quad \text{(A3)}$$

The time domain representation of this Laplace transform is the Green's function and is found in Beyer (1984):

$$G(x - x', t) = \exp[-t - X(1 + \epsilon)]\sqrt{\frac{t}{X}}I_1(2\sqrt{Xt})H(X), \quad \text{(A4)}$$

where $X \equiv x - x'$, I_1 is the modified Bessel function, and H is the unit step function. For an initially localized SST anomaly, the evolution at large times can be approximated as

$$T_O(x, t) \approx G(x, t) \int_{-\infty}^{+\infty} T_O(x', 0)\, dx'. \quad \text{(A5)}$$

This can be further simplified by using the asymptotic representation

$$I_1(x) \approx \frac{\exp(x)}{\sqrt{2\pi x}},$$

valid for large argument, which after some algebra yields

$$G(x, t) \approx \frac{1}{2\sqrt{\pi t}} \exp\left[-t\epsilon - \frac{(x - t)^2}{4t}\right]. \quad \text{(A6)}$$

It is straight forward to obtain the SST response to the boundary value problem

$$T_A(x = 0, t) = 1, \qquad T_O(x, 0) = 0.$$

The solution is

$$T_O(x, t) = \exp[-x(1 + \epsilon)] \int_0^t \exp(-t')I_0(2\sqrt{xt'})\, dt'. \quad \text{(A7)}$$

For large t, we have approximately

$$T_O(x, t) = \frac{\exp(-\epsilon x)}{2}\left[\text{erf}\left(\frac{x - t}{2\sqrt{t}}\right) + 1\right],$$

where erf is the error function.

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