Reply

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1. Introduction

In the accompanying comment, Kalmykov expresses two concerns regarding the work of Lin and Perrie (hereafter LP) 1) that the numerical simulations of five-wave interactions by Kalmykov were not cited by LP and 2) that, because the results of Kalmykov are qualitatively different from those of LP, therefore the conclusions of LP cannot be justified.

The authors apologize for not citing the work of Kalmykov. These papers constitute important early attempts to compute the five-wave interactions. Unfortunately, the later Kalmykov paper did not become available until after the LP paper had gone to press.

The authors' major concern in this reply is to demonstrate that the conclusions of LP are *correct.* The five-wave interactions, as described by LP, are for finite amplitude waves with a narrow spectral spreading and large peakedness γ, rather than small amplitude waves with a broad spectral spreading and small $\gamma = 1$, which is the approach taken by Kalmykov's papers.

2. Scientific background

First, the dominance of the five-wave interactions direct or inverse cascades depends on the shape of the spectrum. Five-wave interactions can cause both energy transfer from low to high frequencies (the direct cascade) and energy transfer from high frequencies to low frequencies (the inverse cascade). These cascades are shown in Fig. 1. Following LP, a dominant inverse cascade is shown in Fig. 1a, assuming a narrow (Hasselmann–Mitsuasyu) spectral spreading, as motivated by Hasselmann et al. (1980) from JONSWAP analysis,

$$D(\theta, f) = I(s)\cos^{2s}[(\theta - \theta_{\max})/2], \tag{1}$$

where $I(s)$ is a normalization factor and s is parameterized as

$$s = \begin{cases} 6.97(f/f_p)^{4.06} & \text{for } f/f_p < 1.05. \\ 9.77(f/f_p)^{\mu} & \text{for } f/f_p \geq 1.05. \end{cases} \tag{2}$$

Here, μ is a weakly dependent function of wave age, U_{10}/C_p, which satisfies the relation

$$\mu = 2.33 - 1.45\left(\frac{U_{10}}{C_p} - 1.17\right),$$

where C_p is the phase speed at f_p. A similar form was proposed by Mitsuyasu et al. (1975).

However, if the spectrum is very broad in angle, for example $\cos^2\theta$, then the dominance of a direct cascade, as suggested by Kalmykov's comment and papers, is correct. In Fig. 1b, we give five-wave interactions direct and inverse cascades for the latter spreading, showing the dominance of the direct cascade. However, in this case, assuming deep water waves that are not steep, LP and Kalmykov's papers all suggest that four-wave interactions should dominate over five-wave interactions, implying that inverse cascades due to four-wave interactions would be a dominant feature.

Second, four-wave interactions are local interactions, which are proportional to d^3, where d is the width of the spectrum, whereas five-wave interactions are global interactions that are not significantly affected by d, as presented in LP. Therefore, the five-wave interactions allow the study of finite-amplitude narrow spectral interactions, which may be able to obtain significant three-dimensional patterns, as described by McLean's (1982a,b) instability analysis, Su's (1982a,b) experimental analysis, Long et al.'s (1994) observations, and the long-lived patterns of Shrira et al. (1996).

Third, as discussed in Zakharov (1991), the direct cascades cause the energy transfer from low to high frequency, whereas the inverse cascades cause the en-

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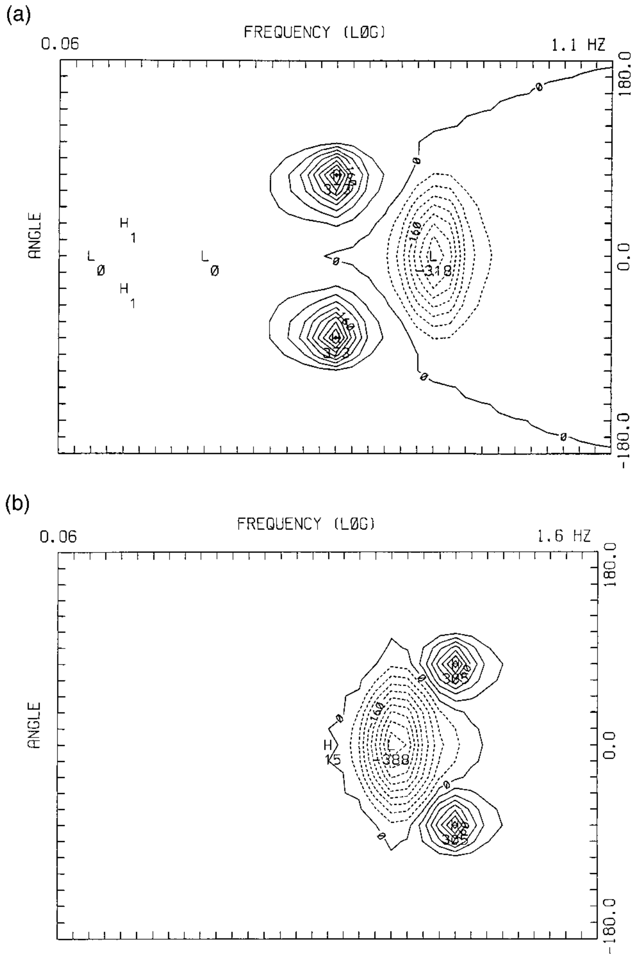

FIG. 1. (a) The nonlinear transfer for five-wave interactions, using JONSWAP input spectrum (with peakedness $\gamma = 3.3$) with Hasselmann–Mitsuasyu spreading as given in Eqs. (1)–(2), steepness 0.3 and water depth, 10 m. (b) As in Fig. 1a, assuming Pierson–Moskowitz input spectrum ($\gamma = 1$) with wide $\cos^2\theta$ spreading.

ergy transfer from high to low frequency. The latter can cause waves to grow and the peak frequency to downshift to lower frequencies, whereas the former causes waves to break. Therefore, inverse cascades, as given by the finite-amplitude, five-wave interactions of LP, are central to the long timescale, three-dimensional mechanics, as observed by Su (1982a,b), McLean (1982a,b), Long et al. (1994), and Shrira et al. (1996), whereas associated direct cascades are central to the short timescale mechanics for remote sensing and radar scattering. As four-wave interactions are primarily two-dimensional, whereas five-wave interactions are primarily three-dimensional, the four-wave interactions, by themselves, cannot provide a comprehensive description of these three-dimensional patterns of Su (1982a,b). By contrast, the five-wave interactions of Kalmykov (1999, p. 2) are only significant in shallow water. Unless waves are extremely steep, they are only associated with direct cascades and they are always much smaller than the corresponding four-wave interactions. Therefore, the five-wave interactions of the comment inhibit wave growth because of their dominant direct cascades, and they cannot account for the finite-amplitude three-dimensional patterns of Su (1982a,b).

Fourth, recently Lin and Chubb (1999) found significant direct cascades by weakly nonlinear four-wave interactions when the initial spectrums are split so the observed direct cascades at the tail of the spectrum may be due to four-wave interactions instead of five-wave interactions.

Finally, it is well known that five-wave interactions can be significant only when the wave steepness is greater than about 0.3, as given by Su (1982a,b), McLean (1982a,b), Shrira et al. (1996), and LP. At the bottom of p. 2, Kalmykov (1999) states that ‘‘This instability II (3D) was triggered by instability I (2D) or Benjamin-Feir instability.’’ This is inconsistent with the comment (i) at the top of p. 2, where Kalmykov suggests that these four-wave resonant conditions cannot be satisfied in shallow water and that five-wave interactions would result, and (ii) on p. 6 of the comment, where Kalmykov uses scaling arguments to suggest that even for very steep waves in very shallow water, five-wave interactions should be much less important than four-wave interactions. Moreover, we note that the mechanics by which four- and five-wave interactions are coupled to generate 3D wave–wave interactions patterns was suggested by Su (1982a,b) for deep water waves, not for shallow water waves.

3. Numerical simulations

We have some concerns regarding Kalmykov’s (1993, 1995, 1997, 1998) results.

1) A major problem is that in the first three of these papers, the nonlinear transfer due to five-wave interactions is strongly *asymmetric*. Figure 3 of Kalmykov (1997) notes that the energy is moved from the spectral maximum $\theta_o = 0$ to lateral waves at $\theta_2 = 60°$ and $\theta_1 = -90°$. By contrast Kalmykov (1998) gives *symmetric* lateral transfer to waves at $\theta = \pm 70°$, which is similar to the lateral transfer angle presented by LP. No discussion is given in regard to this lateral transfer. Thus, the results of Kalmykov (1998) appear inconsistent with the results of Kalmykov (1993, 1995, 1997).
2) It is difficult to quantitatively compare the magnitudes of Kalmykov’s results because of the nondimensionalization he has implemented. Even in nondimensional units, his results show some variation in his four papers cited above. For example, in dimensionless variables, five-wave interactions have a maximal magnitude of about 0.05 in Kalmykov (1995, 1997), which becomes about 18 in Kalmykov (1999) and Kalmykov (1998). Kalmykov (1998) does indicate the factor to convert these values to

dimensional values, which then gives maximal magnitudes of order 10^{-5}, similar to LP maximal values. However, a detailed direct comparison between LP and Kalmykov's results is difficult to make because Kalmykov uses a Pierson–Moskowitz input spectrum, which corresponds to very old sea state conditions, whereas LP use JONSWAP input spectrum with peak frequency at 0.3 Hz, corresponding to actively growing, evolving, young sea state conditions. Moreover, Kalmykov's directional spreading is $\cos^2\theta$ for $|\theta| \leq \pi/2$, which is significantly broader than the Hasselmann–Mitsuasyu spreading in LP, as described above. Although Kalmykov (1999) claims to use a JONSWAP input spectrum, it is, in fact, the old sea state spectrum (Pierson–Moskowitz) of Fig. 2a in Kalmykov (1998).

3) Young et al. (1996) suggest that shallow water spectra are more broad than deep water spectra, based on an extensive data analysis, comparing his shallow lake data to Donelan et al.'s (1985) deep water data from Lake Ontario. Using a broadly spreading spectrum, for example $\cos^2\theta$, Kalmykov (1998) suggests that the effect of shallower water is to *narrow* the spectrum, which is therefore contradictory to Young et al.'s (1996) observations.
4) Numerical simulation of the nonlinear energy transfer depends on both the transfer coefficient and the integration method. As Kalmykov claims his transfer coefficient is taken from Krasitski (1993), his simulation should be mostly correct. However, problems occur in his integration method and related assumptions. Kalmykov (1997) assumes the five-wave resonance conditions are given by $3k_o = k_4 + k_5$ and $3\omega_o = \omega_4 + \omega_5$, where $k_o = \omega_o = 1$, which is impossible for five-wave interactions. Unlike four-wave interactions, five-wave interactions are global interactions, which means that the five interacting waves do not need to have comparable wavelengths. This assumption makes Kalmykov's (1997) solutions qualitatively different from the real solutions.

4. Previous work

Kalmykov (1999) claims that all previous works, such as Dyachenko et al. (1994), Krasitski (1993, 1994), Meiron et al. (1982), Shrira et al. (1996), McLean (1982a,b), Stiassnie and Shemer (1984, 1987), and Su (1982a,b) support his results, including his claim that five-wave interactions support only direct cascades. Unfortunately, this is not correct. Dyachenko et al. (1994), Krasitski (1993, 1994), Meiron et al. (1982), and Stiassnie and Shemer (1984, 1987) basically studied the nonlinear transfer coefficient, not the numerical simulation. They concluded that five-wave interactions will generate three-dimensional wave–wave interactions. They did not discuss the ability of five-wave interactions to support inverse cascades or direct cascades.

Shrira et al. (1996) suggested that class II instability (five-wave interactions, seven-wave interactions, and so on) generate horseshoe patterns. However, inverse cascades and direct cascades are both present in horseshoe patterns, and Shrira et al. (1996) did not suggest that only direct cascades are present, as Kalmykov (1999) claims. Moreover, Shrira et al. (1996) did suggest that wave steepness needs to be greater than 0.33 in order for the generation of the three-dimensional patterns, which is approximately the same as LP's criterion for the dominance of five-wave interactions. Thus, five-wave interactions must support inverse cascades, as suggested by LP. Otherwise, if five-wave interactions only support direct cascades and if they dominate over four-wave interactions, as found by LP, then three-dimensional wave patterns will not be observable because it is impossible for waves to grow and evolve.

McLean (1982a,b) used a global linear method to study five-wave instability. He found that five-wave interactions can be greater than four-wave interactions when wave steepness is greater than 0.28. Su (1982a,b) first observed the three-dimensional wave–wave interactions in his data. These results agree with the finite-amplitude analysis of LP and disagree with Kalmykov (1993, 1995, 1997, 1998). Su and Green's (1984) analysis, based on their experimental results, suggests that the two-dimensional instabilities may trigger the three-dimensional instabilities in deep water. However, this approach does not apply to shallow water, because four-wave interactions become rapidly less important as water depth decreases, as shown in Lin and Perrie (1997b).

Recently, Lin and Su (1999) showed that a coupling of four- and five-wave interactions can cause three-dimensional wave–wave interactions in deep water, when the spectrum is narrow and wave steepness is significant. The results agree well with LP. When inverse cascades occur, transferring energy from high to low frequencies, then waves can only grow and the coupling of four- and five-wave interactions can occur.

The analysis of Long et al. (1994) considers three sets of observed wave spectra in shallow water. The dominant directions of the wave spectra distributions are about 60° from the downwind direction. The wave amplitudes are about 4 m high. These are finite-amplitude waves. While LP can provide a good explanation for these wave spectra phenomena, because their transfer to lateral waves by the five-wave interactions is at $\pm 60°$, other mechanisms such as the Phillips mechanism for wave generation or the five-wave interactions of Kalmykov (1993, 1995, 1997, 1998), cannot explain these phenomena.

Kalmykov (1999) states, "For any odd number of the wave interactions, the energy transfer is a direct cascade: from low frequency to high, while for any even number we get an inverse cascade (Zakharov 1998)." We could not find this stated or implied in Zakharov (1998). Both direct and inverse cascades are computed in Resio and Perrie (1991) for four-wave interactions, in accord with the theoretical study by Zakharov (1991). Moreover, Prof. Zakharov has discussed the nonlinear wave–wave interactions of LP several times with us: he has never suggested that odd (even) number wave–wave interactions should have only a direct (inverse) cascade.

5. Summary

We have shown that five-wave interactions should include both direct and indirect cascades, depending on the spectral spreading. This follows from the existence of the finite-amplitude long-lived, three-dimensional patterns, as observed by Su (1982a,b), and the associated analysis of McLean (1982a,b). Therefore, following Zakharov (1991), these features should be generated by inverse cascades, not direct cascades, as presented by LP. In this sense, we have shown that Kalmykov's results contradict the instability analysis of McLean (1982a,b) and the observational data of Su (1982a,b).

Kalmykov's (1993, 1995, 1997) earlier results are strongly asymmetrical whereas his most recent results, shown in Kalmykov (1998), are symmetrical. The former are unphysical, because there is is no obvious reason for asymmetry. Kalmykov gives no apparent discussion of this asymmetry/symmetry.

When the spectrum is broad, such as $\cos^m \theta$, and when peakedness γ is small, the direct cascade tends to be more dominant, as suggested by Kalmykov (1998) and presented in Fig. 1b. When the spectrum is narrower, as parameterized by the Hasselmann–Mitsuasyu spreading and when γ is large, the inverse cascade tends to dominate, as shown in Fig. 1a and in LP.

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