The Connection between Bubble Size Spectra and Energy Dissipation Rates in the Upper Ocean

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ABSTRACT

A formula for the maximum size of a bubble for which surface tension forces can prevent bubble breakup by inertial forces, combined with the observed sizes of air bubbles in breaking waves, implies an energy dissipation rate. One dataset from the surf zone gives a dissipation rate of the order of 0.1 W kg^{-1}, but the large number of small bubbles, and the bubble size spectrum generally, are puzzling. A simple dimensional cascade argument suggests that injected air beneath a breaking wave is rapidly broken up by turbulence, producing an initial size spectrum proportional to $(\text{radius})^{-10/3}$ before modification by dissolution and rising under buoyancy. This spectral slope is comparable with data from the surf zone. The cascade argument does, however, predict that for a constant dissipation rate there is a rapid accumulation of a large number of bubbles at the scale at which surface tension prevents further breakup; it is possible that the observed size spectrum reflects the range of turbulent energy dissipation rates rather than the result of a cascade. If so, an estimate of about 40 W kg^{-1} is obtained for the dissipation rate implied by the surf zone dataset. Once an initial size spectrum is formed by the rapid action of differential pressure forces, it will evolve subject to dissolution and buoyancy. It is shown that the former will tend to flatten the size spectrum at small scales, whereas the latter will tend to steepen the time-averaged spectrum observed at large scales. The slope change and transition radius predicted by a very simple model are in reasonable agreement with observations.

1. Introduction

The measurement and interpretation of the sizes of air bubbles in the upper ocean are important for the evaluation of gas transfers between atmosphere and ocean (e.g., Woolf and Thorpe 1991; Keeling 1993; Melville 1996). Bubbles also affect the optical properties of the sea surface (Flatau et al. 1999; Terrill et al. 1998), the production of sea salt aerosols in the atmosphere by bursting bubbles (Monahan and Van Patten 1989), and sound transmission within the ocean (Carey 1997). For these and other reasons considerable observational and theoretical attention is being focused on the topic, but without the emergence to date of any clear and comprehensive picture.

Recent photographic studies by Deane and Stokes (1999) have illustrated the disintegration of entrained air cavities and filaments into bubbles, but there are peculiar and unexplained aspects of the problem, such as enhanced bubble populations in salt rather than freshwater (Scott 1975; Thorpe 1982). This is historically attributed to reduced coalescence rates in saltwater (e.g., Craig et al. 1993), though Slauenwhite and Johnson (1999) suggest that a bubble in saltwater may shatter into more, smaller, bubbles than a bubble in freshwater. The latter authors produced the parent bubbles through an orifice, however, so the relevance to oceanic bubble production is uncertain. Moreover, Loewen et al. (1996) found little difference in the bubble populations beneath mechanically generated surface waves in saltwater and freshwater.

In the face of this uncertainty, we have been motivated by recent data on bubble size spectra beneath breaking waves (Deane 1997; Farmer et al. 1998) to seek the connection between the size spectrum and the turbulent energy dissipation rate. We take the point of view that turbulent pressure fluctuations are the dominant mechanism for bubble breakup, and leave unresolved any differences between saltwater and freshwater. We will not propose a complete solution to the problem of bubble formation, and stress that our objective is merely to explore a number of basic physical

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concepts in an effort to build intuition and focus questions. Our starting point, in section 2, will be a discussion of Hinze's (1955) formula for the largest size that a bubble can be and still have surface tension forces holding it together against the disruptive pressure forces associated with turbulence.

Hinze's (1955) formula does not provide any insight into the expected size spectrum. We discuss this in section 3, based on the idea that air is injected at the sea surface, by a breaking wave, at a scale much larger than the Hinze scale, and is then rapidly broken up by the turbulence. Simple dimensional arguments, which have a clear physical interpretation, lead to a particular power law for the size spectrum, and also suggest that the breakup of entrained air into a bubble size spectrum occurs very rapidly compared with the timescales of wave breaking, dissolution, and buoyancy-driven rising. This justifies the concept of an "initial" size spectrum that might be observed before dissolution and buoyancy effects cause modifications. A drawback to this scenario is that, for a given dissipation rate, the air fraction is expected to accumulate as a large number of bubbles at the smallest scale rather than dissolving, which would be the analog of viscous dissipation in the turbulent kinetic energy spectrum. We thus examine in section 4 the alternative scenario that dissipation at a particular rate gives bubbles of a single size and that the bubble size spectrum is therefore a measure of the intermittency of the dissipation rate ϵ.

Any bubble size spectrum formed in the early stages of air injection will later evolve under the influence of dissolution and buoyancy. In section 5 we show that the average size spectrum at any depth is rendered steeper at large scales by the buoyant rise of the bubbles through the water, but flatter at small scales by dissolution, with a break in slope at a predictable scale.

We should stress that the bubbles we are concerned with are those directly associated with breaking waves, not the very much smaller residual population of chemically stabilized microbubbles that can occur and not the bubbles that can be produced in some situations by biological processes.

2. The Hinze scale

Following an earlier study by Kolmogorov (1949), Hinze (1955) pointed out that, at high Reynolds number, a gas bubble, or a droplet of any different fluid, is likely to break up under the influence of differential pressure forces on its surface if these exceed the restoring forces associated with surface tension. In the frame of reference of the bubble, the differential pressure is comparable with ρq^2, where ρ is the water density and q is the water speed differential over the scale of the bubble. Hence Hinze's (1955) criterion for breakup is that the Weber number $\rho q^2 a/\gamma$ should exceed some critical value, where a is the bubble radius and γ is the surface tension.

Although the initial void fraction (the fraction of air in the fluid) can be several tens of percent in a whitecap (e.g., Deane 1997; Lamarre and Melville 1992, 1994), we assume that it is small and does not affect the dynamical behavior of the flow. The velocity field is then assumed to be described by Kolmogorov's inertial subrange with $q^2 \propto \epsilon^{2/3}a^{2/3}$ so that the Weber number may be written

$$\mathrm{We} = (\rho/\gamma)\epsilon^{2/3}a^{5/3}. \tag{1}$$

For this to exceed a critical value We_c requires that $a > a_H$, where

$$a_H = c(\gamma/\rho)^{3/5}\epsilon^{-2/5} \tag{2}$$

with $c = \mathrm{We}_c^{3/5}$ a constant of proportionality. We shall refer to a_H as the Hinze scale, noting that (2) is dimensionally inevitable if one seeks a length scale based on γ, ρ, and ϵ. (One could add an unknown function of the density ratio of the two fluids. This would allow for less of a tendency for breakup if the droplet density were larger than that of the water.)

Hinze (1955) used the laboratory results of Clay (1940) to claim that $c = 0.363$ if a_H is defined such that 95% of the air is contained in bubbles with a radius less than a_H. The three significant figures in this value of c should not be taken too seriously; Hinze (1955) cites a standard deviation of 0.16. Moreover, the turbulent dissipation rate ϵ was estimated rather than measured directly.

The above formula for a_H can be used to evaluate ϵ from data on the bubble size spectrum $N(a)$, where $N(a)da$ is the number of bubbles, per unit volume, with radii between a and $a + da$. The definition of a_H then implies that it can be determined from

$$\int_0^{a_H} a^3N(a)\,da = 0.95\int_0^{\infty} a^3N(a)\,da. \tag{3}$$

Figure 1 shows an observed spectrum beneath a breaking wave in the surf zone (Deane 1997), in water of 2-m depth with a significant height of 0.9 m for the incident waves that broke in the transition between spilling and plunging. Over much of the range the spectrum is close to being proportional to a^{-3} (as is the spectrum found by Terrill et al. (1998) for the largest values of void fraction that they find, indicative of a new event), though Deane (1997) fits a slope of -2.5 for $a < 1$ mm and -4.5 for larger bubbles. The definition of a_H in (3) gives $a_H \simeq 3$ mm (Fig. 2) so that, for $\gamma/\rho = 7.3 \times 10^{-5}$ m^3 s^{-2}, $\epsilon \simeq 0.1$ W kg^{-1}. As suggested by a reviewer, it is interesting to put this in the context of energy dissipation rates that might be expected. The shoreward energy flux for waves of amplitude A in water of depth h is $\frac{1}{2}\rho g A^2(gh)^{1/2}$. If we suppose that this is being lost in the top d of the water over a distance L in the shoreward direction, due to dissipation over a fraction δ of a wavelength, then the energy dissipation rate per unit mass is $\frac{1}{2}gA^2(gh)^{1/2}(dL\delta)^{-1}$. For $A \simeq 0.9$ m, $h = 2$ m,

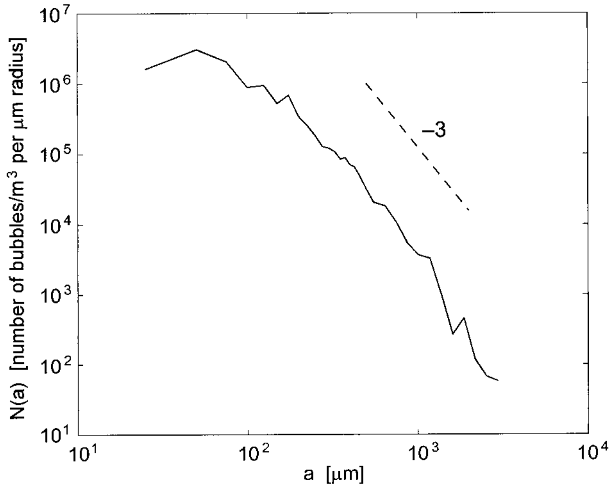

FIG. 1. The bubble size spectrum $N(a)$ in a breaking wave in the surf zone [redrawn from Deane (1997) with correction for a factor of 2 error in the original plot (G. Deane 1999, personal communication)]. The dashed line is proportional to a^{-3}.

and with somewhat arbitrary choices of 0.2 m for d, 50 m for L, and 0.1 for δ, this gives about 20 W kg^{-1} for ϵ. Larger or smaller values are possible for different choices of the parameters, but the expected dissipation rate does seem to be larger than the 0.1 W kg^{-1} estimated from (2) with Hinze's value for c.

We are taking advantage here of measurements in the surf zone as being representative of the breaking process occurring more widely in the ocean. The macroscopic aspects of surf and ocean waves undoubtedly differ, but we assume that the process of air entrainment and breakup under the influence of turbulence, although probably different in magnitude, is qualitatively similar.

The argument leading to (2) assumes that the Reynolds number for flows at the scale of the bubbles is large, or, equivalently, that the bubbles are much larger than the Kolmogorov scale $(\nu^3/\epsilon)^{1/4}$. For $\epsilon = 0.1$ W kg^{-1} and $\nu = 10^{-6}$ m^2 s^{-1}, the Kolmogorov scale is 60 μm, smaller than both a_H and the bubble radius at the peak of the spectrum in Fig. 2. We note that the Kolmogorov scale decreases more slowly than the Hinze scale with increasing ϵ. Thus at very large values of ϵ viscous rather than pressure forces may be important in the breakup, as discussed by Li and Garrett (1998) for oil droplets in the ocean, in a discussion motivated by the need to predict oil droplet size after an oil spill.

We note that, for typical size spectra $N(a)$ such as that in Fig. 1, there are many bubbles of considerably smaller radius than the Hinze scale a_H. It is possible that the smaller droplets observed by Deane (1997) are the partially dissolved remnants of previous events rather than the product of a fresh injection, or alternatively, that the number of bubbles with radii less than about 100 μm has been underestimated (G. Deane, 1999, personal communication). Disregarding these possibilities for now, the existence of bubbles much smaller than the

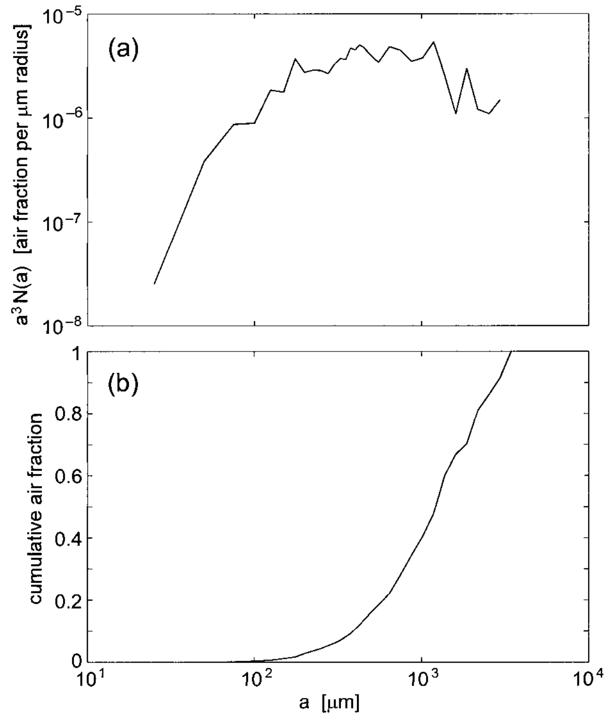

FIG. 2. (a) The scaled spectrum $a^3N(a)$ for the data of Fig. 1. The total air fraction is given by $4\pi/3$ times the integral of this. (b) The cumulative air fraction as a function of bubble radius. The radius at which this reaches 0.95 is the Hinze scale a_H.

Hinze scale a_H is curious, given that the latter is derived from an argument balancing pressure and surface tension forces. Smaller bubbles should be able to resist the distortion by pressure and not break up. Of course the constant of proportionality in the scale argument used is not precisely determined, though Martínez-Bazán et al. (1999a) cite a value of 0.63 for the value of c giving the smallest drop that can be broken up. It is possible that the breakup process of bubbles involves the production of numerous, much smaller, bubbles so that the size of the bubbles at the spectral peak is given by (2) with a smaller value of c, but Martínez-Bazán et al. (1999b) find that a bubble close to a_H in radius breaks into only two smaller bubbles and that these tend to be of comparable size. For these the appropriate value of c in (2) is $0.63 \times 2^{-1/3} = 0.50$, not much different from Hinze's (1955) value. Compatibility between the results of Martínez-Bazán et al. (1999b) and Hinze's (1955) result for the size of the bubble bigger than those containing 95% of the air would appear to require a much narrower distribution of the void fraction between different bubble radii than that shown in Fig. 2.

There is thus a possibility, to which we return later in this paper, that (i) for a given dissipation rate, bubbles are produced with a fairly narrow band of radii, perhaps centred on a_H from (2) with c from Martínez-Bazán et al. (1999b), and that (ii) the production of smaller bubbles is associated with the intermittency of ϵ, with the intermittency being greater in the ocean than in the laboratory experiments cited above.

At this stage of our discussion, though, we have the provisional conclusions that the value of ϵ for Deane's (1997) data using Hinze's (1955) formula is about 0.1 W kg^{-1} (though this seems rather low), and that the presence of the small bubbles is a puzzle. We next consider the possibility that the bubble size spectrum can be determined from the average dissipation rate ϵ using dimensional arguments.

3. The bubble size spectrum

The bubble size spectrum in the ocean is determined by the initial bubble formation, breakup, coalescence, dissolution, vertical motion caused by buoyancy forces, and turbulent mixing. We first discuss the processes of formation and evolution by breakup, leaving until later a consideration of the ways in which dissolution and rising under buoyancy affect the spectrum.

Several authors have considered the initial creation of a size spectrum of bubbles. Longuet-Higgins (1992) suggested that bubble size spectra can be related to the distribution of volumes that results from cleaving a cube with three random sets of planes, each orthogonal to one of the axes of the cube. He showed that, as the number of planes in each set becomes large, the bubble size spectrum tends to a limiting shape that resembles that of observations, though, of course, the actual size of the bubbles depends on the number of cleaving events. Longuet-Higgins evaluated the energy required to create the greater potential energy associated with the greater surface tension forces and higher pressure in the smaller bubbles into which an initial volume of air is broken, but he did not relate this energy to possible inputs such as the dissipation rate.

Baldy (1993) discussed the rate of formation $s(d)$ of bubbles of diameter d, with s defined as the number of bubbles generated per unit time, per unit radius, and per unit area of an air–water interface. He suggested that $s(d)$ depends only on ϵ and the bubble formation energy, which is proportional to $\gamma d^2/\rho$. He concluded, on dimensional grounds, that $s(d) \propto \epsilon(\gamma d^2/\rho)^{-1}$. He then found the source function $S(d, z)$ of bubbles of diameter d at depth z to be also inversely proportional to d^2, but decaying away from the sea surface. It is this source function that would give the initial size spectrum before modification by rising, mixing, and dissolution. We note, however, that if all of ϵ, d, and γ/ρ are relevant, then the source function, and hence the predicted size spectrum, can be multiplied by an arbitrary function of the turbulent Weber number defined by (1), using the bubble radius a instead of the diameter $d = 2a$. Baldy's result depends on assuming that the source function is linearly proportional to ϵ.

We envisage a different physical process along the following lines. Air is entrained into relatively large bubbles (uninfluenced by surface tension) as a wave breaks, with these bubbles then being acted upon by turbulent pressure fluctuations and rapidly broken into smaller bubbles at a rate that depends on the level of turbulence as represented by ϵ, with dimensions L^2T^{-3} (L represents length and T time). Assuming that the total fraction of air is not big enough to have a back effect on the turbulence, then the level of the bubble size spectrum is linearly proportional to Q, the average rate of supply of air to a given volume (Q has dimension T^{-1}). The bubble size spectrum $N(a)$ has dimension L^{-4} as it is the number of bubbles per unit radius and per volume and for dimensional consistency must be of the form

$$N(a) \propto Q\epsilon^{-1/3}a^{-10/3}. \tag{4}$$

There are many more small bubbles than large ones, but $a^3N(a) \propto a^{-1/3}$, showing, after integration, that most of the air fraction is in large bubbles during the cascade to smaller scales. The Hinze scale, defined as the radius such that 95% of the air is in smaller bubbles, then depends on the large radius cutoff of the spectrum.

This presents a puzzle as the Hinze scale is supposed to be the scale at which surface tension becomes important, so it certainly will be for smaller bubbles. This raises the possibility that surface tension is actually important throughout the breakup process so that, as for Baldy's (1993) source function, the size spectrum of (4) can be multiplied by an arbitrary function of the dimensionless parameter $(\rho/\gamma)\epsilon^{2/3}a^{5/3}$. A key question is whether surface tension is important at the Hinze scale, or only at the (smaller) scale at the peak of the spectrum. In pursuing an inertial theory we are essentially assuming the latter.

It is interesting to consider a mechanistic argument behind the dimensionally inevitable formula (4). Our scenario is that one bubble of radius a is broken up by the action of the differential velocity over a distance a. For the inertial subrange of turbulence this velocity is $(\epsilon a)^{1/3}$, which cleaves the initial bubble in a time $a(\epsilon a)^{-1/3}$. A bubble of radius a thus persists for a time of order $a^{2/3}\epsilon^{-1/3}$. We suppose that it is broken into m bubbles, each of radius $m^{-1/3}a$, which then persist for a time $(m^{-1/3}a)^{2/3}\epsilon^{-1/3}$. Given this reduced lifetime, we expect $m \times (m^{-1/3})^{2/3} = m^{7/9}$ times as many bubbles of radius $m^{-1/3}a$ as of radius a. The factor $m^{7/9}$ gives the increased number of bubbles per interval, and we must divide this by the ratio $m^{-1/3}$ of radius intervals to obtain the number density per unit radius. Hence $N(m^{-1/3}a) = m^{1/3}m^{7/9}N(a) = m^{10/9}N(a)$ and so $N(a) \propto a^{-10/3}$, just as in (4).

Martínez-Bazán et al. (1999a,b) also cite a breakup time of order $a^{2/3}\epsilon^{-1/3}$ for bubble radii greater than the Hinze scale. As noted earlier, they find that, for moderate values of the Weber number, a bubble actually breaks up into two bubbles, with equal volumes being the most probable occurrence, though they suggest that more bubbles are formed in the breakup at large Weber numbers. The size spectrum (4) is dimensionally inevitable, independent of the number m of bubbles formed (and of any nonuniform size distribution in the formation process), if we assume that the breakup process is purely inertial and depends only on ϵ.

We also note that the general bubble concentration equation of Garretson (1973) contains a source term $\partial[\upsilon a^3 N(a)]/\partial a$, which is the divergence of the flux of gas through the size spectrum with υ a speed associated with the cascade. On dimensional and physical grounds, $\upsilon \propto (\epsilon a)^{1/3}$, the differential speed cited earlier, so that, if the flux is nondivergent, we are again led to (4).

The predicted spectrum (4) is in encouraging agreement with the data shown in Fig. 1, but there is a major caveat. The argument leading to (4) is equivalent to that leading to Kolmogorov's inertial subrange in the energy spectrum of isotropic turbulence. In that case, however, the energy ultimately cascades into eddies at the Kolmogorov scale $(\nu^3/\epsilon)^{1/4}$ or less, and is then removed by viscosity. In the present case the input of air gets broken into smaller and smaller bubbles until surface tension becomes important at a certain scale and halts the cascade. Air would thus tend to accumulate in a large spectral peak that would only slowly disappear as the bubbles dissolved. Such a peak is not observed. Moreover, during this cascade process the lifetime of bubbles of any particular radius is, in fact, very short. For example, if $a = 3$ mm and $\epsilon = 0.1$ W kg^{-1}, the lifetime $a^{2/3}\epsilon^{-1/3}$ is only 40 ms, certainly much less than the time for bubble dissolution. The lack of evidence for a spectral accumulation thus seems to argue against taking (4) too seriously, though it is possible that the dissipation episodes are brief enough that a significant pileup of a large number of bubbles at the smallest scale does not occur and that we are just left with a size spectrum, given by (4), down to some larger scale.

Before concluding this section, we should comment on the general use of the concept of a turbulent cascade. This has been somewhat discredited as an interpretation of the Kolmogorov velocity spectrum and replaced by ideas of coherent structures at all scales (e.g., Frisch 1995); perhaps the filaments of entrained air observed by Deane and Stokes (1999) are related to these. Dimensional arguments, however, such as that leading to the $-5/3$ power law for the velocity spectrum and (4) here, still seem to have merit. Moreover, the rms differential velocity over a scale a still scales as $a^{1/3}$ even if higher-order structure functions behave in a manner that is inconsistent with early cascade ideas (Frisch 1995), so our interpretation of (4) as a cascade in bubble breakup may still be appropriate.

The form of (4) does, of course, depend on the assumption that it scales with the gas input rate per unit volume. This could be interpreted as a gas transfer rate across the sea surface divided by a depth over which it is mixed, but other forms of the source function, with different dimensions, could perhaps be proposed. This would lead to different power laws for the dependence of $N(a)$ on a, but the problem of bubble number accumulation at the Hinze scale would remain.

4. The bubble size spectrum as a measure of intermittency?

We have so far pursued the idea that a given dissipation rate gives rise to a size spectrum proportional to $a^{-10/3}$ and that this spectrum evolves under the influence of dissolution and buoyancy. We suggested, however, in section 3, that the intial breakup might lead to a pileup of bubbles all of a size determined by surface tension and the dissipation rate ϵ, and proportional to that in (2). If this peak were very narrow, then it is possible that the size spectrum observed in a breaking wave, before significant modification by dissolution and buoyancy, is a measure of the probability of different values of the intermittent dissipation rate ϵ, with the small bubbles being produced by large values of ϵ and large bubbles persisting in regions of small ϵ. Assuming that the same air fraction exists in each region, we are therefore assuming a correspondence between the probability $P(\epsilon)$ of the dissipation rate ϵ and the probability of the air fraction being in bubbles of radius a. This latter probability is proportional to $a^3N(a)$, so we are assuming that

$$P(\epsilon) \propto a^3 N(a) da/d\epsilon. \tag{5}$$

Now if the relationship between a and ϵ is $a \propto \epsilon^{-2/5}$, as in (2), then

$$P(\epsilon) \propto a^{13/2} N(a). \tag{6}$$

This is an interesting result. One might have feared that if $N(a)$, or even $a^3N(a)$, increased for decreasing (small) a, then this would imply an implausible increase of $P(\epsilon)$ for increasing (large) ϵ. The conversion formula (6) shows, however, that this is not the case unless $N(a)$ is a very steep function of a. In fact, if $a^3N(a)$ falls off more slowly than $a^{-7/2}$ then $P(\epsilon)$ decreases for increasing ϵ.

For the purposes of exploring the hypothesis that ϵ is lognormally distributed (e.g., Frisch 1995), we need

$$P(\ln\epsilon) = P(\epsilon)d\epsilon/d(\ln\epsilon) = \epsilon P(\epsilon) \propto a^4 N(a). \tag{7}$$

Figure 3 shows $P(\ln\epsilon)$ according to (7), based on $N(a)$ from Fig. 1. It does not look particularly Gaussian, but it does confirm that the numerous small bubbles do not imply an increasing probability of large values of ϵ. The ordinate of Fig. 3 has been scaled to give an integrated probability of 1. The abscissa depends on the actual relationship between ϵ and a.

The average dissipation may be written $\overline{\epsilon} = \int_0^\infty \epsilon P(\epsilon) d\epsilon / \int_0^\infty P(\epsilon)\, d\epsilon$. Using (5) and assuming the relationship (2) between a and ϵ, this leads to

$$\overline{\epsilon} = c^{5/2}(\gamma/\rho)^{3/2} \int_0^\infty a^{1/2}N(a)\, da \bigg/ \int_0^\infty a^3 N(a)\, da. \tag{8}$$

As discussed earlier, the results of Martínez-Bazán et al. (1999b) suggest that $c = 0.50$ for the two bubbles that are likely to result from the breakup of the smallest bubble that can be broken up. With this value, the data of Deane (1997) lead to a mean dissipation rate of $\overline{\epsilon} =$

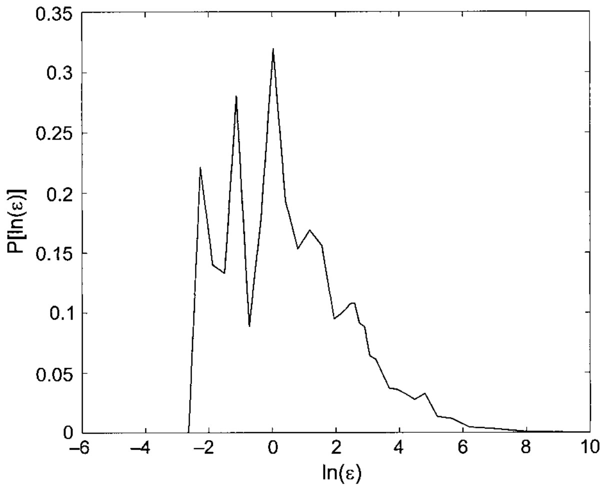

FIG. 3. The probability $P(\ln\epsilon)$ as a function of $\ln\epsilon$, using (7) and the data of Fig. 1, normalized to have an integral of 1. The abscissa is uncertain to within an additive constant determined by the coefficients connecting a and ϵ in (2).

40 W kg^{-1} from (8). This is comparable with our earlier very rough scale of about 20 W kg^{-1}, and probably more plausible than the value of only 0.1 W kg^{-1} obtained using Hinze's (1955) result directly.

5. The effects of buoyancy and dissolution

We consider a scenario in which an initial bubble size spectrum is produced by a breaking wave, and ask how the spectrum evolves as a function of time at any depth. As it is observationally difficult to determine bubble size spectra in both space and time, we will emphasize predictions for the spectrum expected at a fixed location after averaging in time over several events.

One of the main influences is buoyancy forces, which cause larger bubbles to rise to the sea surface faster than smaller ones, thus steepening the average spectrum. Another important process is dissolution, which makes all the bubbles smaller and hence changes the spectral shape. We first examine the consequence of each of these effects in very simple models and then combine the two effects in another simple model. The models should be regarded as an attempt to build intuition and guide data interpretation, rather than as precise predictive tools.

a. The effect of buoyancy

We consider the very simple scenario of an initial size spectrum $N(a) = N_0 a^{-n}$ for $a > a_c$, and zero for smaller bubbles. We also assume that this spectrum is uniform with depth down to some injection depth h and that it evolves only in response to buoyancy forces (ignoring turbulent diffusion and dissolution), with all the bubbles reaching the surface before the next injection event a time T later. We also assume that the rise speed is equal to Aa^2 where $A = (2g/9\nu)$ at low Reynolds number, as for small bubbles ($a < 500$ μm) and assuming that surface contamination causes a no-slip condition at the bubble surface (Thorpe 1982). At a distance z below the surface, bubbles of radius a thus exist for a fraction $H/(Aa^2T)$ of the time, where $H = h - z$, and so the average spectrum is just this fraction times the initial spectrum. This steepens the initial spectrum by a factor proportional to a^{-2}, with the average spectrum being given by

$$N_0(H/AT)a^{-(n+2)}. \tag{9}$$

This very simple result is easily extended to cover other rise rate formulas.

b. The effect of dissolution

It is easy to see that dissolution acting on its own is likely to flatten the bubble size spectrum. Consider, for example, a bubble size spectrum $N(a, t)$ as a function of radius and time, with $N(a, 0) = N_0 a^{-n}$ over some range, so that for $n > 0$ there are more small than large bubbles. Suppose that the dissolution rate is independent of radius [as seems applicable over at least a range of bubble radius from 20 to 350 μm (Thorpe 1982)] and reduces the radius of any bubble at a rate D. At time t, ignoring coalescence, the number of bubbles of radii between a and $a + da$ must be the same as the number of bubbles with radii between $a + Dt$ and $a + Dt + da$ in the original spectrum. Hence

$$N(a, t) = N_0(a + Dt)^{-n}, \tag{10}$$

which becomes independent of a if $Dt \gg a$. If small bubbles dissolve faster than large bubbles, this tendency to reduce the ratio of small to large bubbles will be enhanced, leading to more large than small bubbles. On the other hand, any tendency for small bubbles to dissolve more slowly than large ones, perhaps because of surface contaminants, would again raise the ratio of small to large bubbles.

Turbulent mixing may also play a role, but here we choose only to demonstrate and discuss the basic influence of buoyancy and dissolution, without attempting to develop a comprehensive model. To illustrate more clearly the effects of buoyancy and dissolution acting together, we start by assuming that, in a breaking wave, bubbly air is injected to some depth h and that the spectrum then evolves.

c. Combining buoyancy rise and dissolution

The previous two models suggest that the average size spectrum at a given depth will become steeper if the bubbles rise to the surface in a time that is short compared to the dissolution time, but will become flatter if there is time for dissolution to act. The change of behavior will occur at a bubble radius for which the rise time and dissolution time are comparable. We can illustrate this with a simple combination of the calculations above. As before, we start with a spectrum $N(a) = N_0 a^{-n}$ for $a > a_c$, and zero for smaller bubbles, and assume that the bubble density is uniform with depth to a distance h below the surface, that is, to a distance $H = h - z$ below the observation point at depth z. We also assume a size-independent dissolution rate D so that at time t after injection a bubble that started with a radius a_0 has a radius $a = a_0 - Dt$. We also assume a rise rate $-dz/dt = Aa^2$ as before (z is positive downward). This may be integrated to

$$z_0 - z = \frac{1}{3}(A/D)[a_0^3 - (a_0 - Dt)^3] \qquad (11)$$

$$= \frac{1}{3}(A/D)[(a + Dt)^3 - a^3] \qquad (12)$$

for a bubble that started with radius a_0 at depth z_0 and is now at depth z with radius a. The size spectrum observed at time t after injection is then $N(a) = N(a_0)(da_0/da)|_t$ and the average over several injection events a time T apart is

$$N(a) = \frac{1}{T}\int_{t_1}^{t_2} N(a_0)(da_0/da)|_t \, dt, \qquad (13)$$

where t_1 is the time of first appearance of bubbles of size a (so that $t_1 = 0$ if $a > a_c$), and t_2 is the time of appearance of bubbles that originated at the largest value of $z_0 - z$, H in this case. Thus from (12),

$$(a + Dt_2)^3 = a^3 + a_d^3, \qquad (14)$$

where $a_d^3 = 3DH/A$ and we note that there is no need to have $t_2 < T$ if we are averaging over many events.

In the present case, $a_0 = a + Dt$ and $da_0/da = 1$ so that for $a > a_c$ (13) gives

$$N(a) = \frac{1}{(DT)}\frac{N_0}{(n-1)}[a^{-(n-1)} - (a^3 + a_d^3)^{-(n-1)/3}], \qquad (15)$$

which varies with depth through the dependence of a_d on $H = h - z$.

For $a \gg a_d$ the spectrum is given by (9) as derived when dissolution is neglected. For $a \ll a_d$ (but still with $a > a_c$, with $a_c \ll a_d$ thus assumed),

$$N(a) = \frac{N_0}{DT(n-1)}a^{-(n-1)}, \qquad (16)$$

which is the average of (10) from the time of injection onward and is flatter by a factor a^3 than (9). The two approximations to the spectrum intersect where $a = [(n - 1)\,DH/A]^{1/3}$. Apart from the factor $(n - 1)^{1/3}$, this is the radius of a bubble for which the rise time (ignoring dissolution) is equal to the dissolution time. If $n = 3$ (perhaps a crude approximation to the data of Fig. 1), $D = 10^{-6}$ m s^{-1} (Thorpe 1982), $H = 2$ m, and $A = 1.7 \times 10^6$ m^{-1} s^{-1}, then we expect a transition radius of 130 μm between spectra proportional to a^{-2} for smaller radii and a^{-5} for larger radii. The transition radius would be less for smaller H, though not sensitively in the light of the 1/3 power dependence. A more elaborate model might allow for an initial distribution with depth that was not constant, but this could be thought of as arising from the superposition of several uniform distributions with different values of H, thus giving a broader transition.

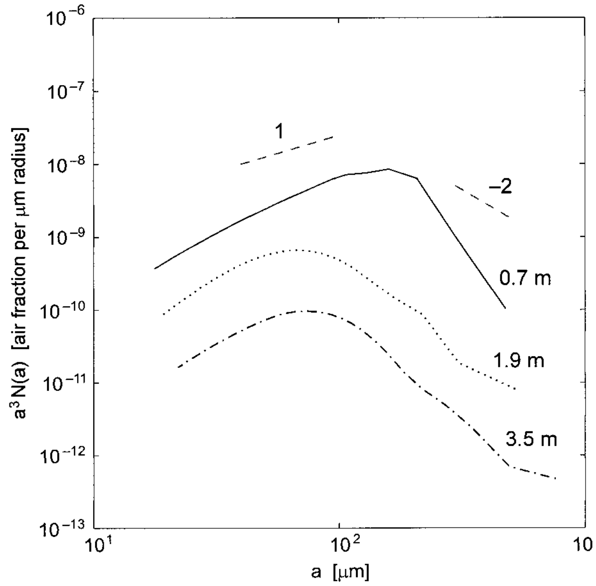

FIG. 4. The average bubble size spectrum at three depths below the surface, obtained at a wind speed of 11.9 m s^{-1} and averaged over 30 minutes [acquired during an experiment described by Farmer et al. (1998)]. The spectrum has been scaled as in Fig. 2a; the dashed lines with slopes of 1 and −2 correspond to $N(a)$ proportional to a^{-2} and a^{-5}, respectively.

Volume-scaled bubble size distributions averaged over 30 min, acquired in the Gulf of Mexico (Farmer et al. 1998) at a wind speed of 11.9 m s^{-1}, are shown in Fig. 4 for depths 0.7 m, 1.9 m, and 3.5 m. Observations at each depth have a characteristic shape that is broadly consistent in power laws and transition radius with the above expectations assuming an injected spectrum initially proportional to a^{-3}. The spectral roll-off at very small bubble radii may reflect a lack of small bubbles in the initial spectra as well as the effect of dissolution. However, for the simple model described above, the expected average spectrum $N(a)$ for radius a less than the small radius cutoff a_c of the injected spectrum is flat if the dissolution rate is constant as assumed. [Mathematically, the time of first appearance of bubbles of radius a is given by $a + Dt_1 = a_c$, giving t_1 for (13); physically the flux of bubbles to smaller radii occurs with a constant speed in radius space.] The observed spectra actually show an increase in spectral density toward smaller radii even at small radii, suggesting that some small bubbles are, in fact, produced during the injection process.

The differences in the size spectra between depths may be associated with the range of injection depths, with a depth-dependent dissolution rate and the effects of surface contamination, and with other neglected processes such as Langmuir circulation and turbulent diffusion. These last two processes are, in fact, likely to have been responsible for the presence of bubbles, albeit at low concentration, at 3.5 m which is below the expected initial injection depth. Langmuir circulation, in particular, could advect bubbles to much greater depths than the initial injection, as discussed by Thorpe (1984a).

6. Conclusions

There is much to be learned about the connection between bubble size spectra and turbulence levels in the upper ocean. A simple formula involving the surface tension and dissipation rate gives the bubble radius, the ''Hinze scale,'' such that 95% of the air is contained in smaller bubbles. It implies dissipation rates in breaking waves in the surf zone of the order of 0.1 W kg^{-1}. It is difficult, however, to account for the decade or more of smaller bubbles unless surface tension limitation of breakup actually occurs at these smaller scales rather than at the Hinze scale. Alternatively, using the results of Martínez-Bazán et al. (1999b) to establish a one-to-one relationship between the local dissipation rate and bubble size, the average dissipation rate is approximately 40 W kg^{-1}, which is more in line with expectations for a rapidly damped wave. It is possible that some of the small bubbles are a lingering effect of partially dissolved bubbles from previous events, though a rapid rise in the number of small bubbles in the vicinity of a breaking event suggests the dominance of the creation of new bubbles.

A similarity argument based on the average dissipation rate predicts a power law for the initial bubble size spectrum, comparable with that observed. The cascade to small scales, however, should lead to an accumulation of a large number of bubbles at a scale determined by surface tension. An alternative scenario is that the size spectrum is associated with intermittency of the dissipation rate; this is not ruled out by the observations as the preponderance of small bubbles does not indicate an unphysical preponderance of high dissipation rates. Perhaps it is possible to argue for both results: a spectral shape determined by our dimensional result and the prevention, by intermittency, of a pileup at one scale.

Given an initial size spectrum from a wave breaking and bubble injection event, very simple arguments show how dissolution will tend to flatten the size spectrum for small bubbles and buoyancy will tend to steepen the time-averaged spectrum for large bubbles as they rise to the surface. The actual slope change predicted for an initial power-law spectrum, and the transition radius, are in reasonable accord with observations. This lends some support to the conceptual model in which there is a highly turbulent injection event followed by a rather quiescent evolution of the bubbles, and suggests that this scenario is worth pursuing as a complement to models, such as that of Thorpe (1984b), with a statistically steady source function and constant turbulent mixing. The estimates and simple models presented in this paper attempt only to explore the basic physics and build intuition. Clearly more data are required, along with more elaborate theoretical models allowing for factors such as a depth-dependent dissolution rate and Langmuir circulation.

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