NOTES AND CORRESPONDENCE

On the Reflection of Internal Wave Groups from Sloping Topography

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24 February 2000 and 6 April 2001

ABSTRACT

Strong interactions, often leading to wave breaking and mixing, are known to occur between a train of internal waves, incident on a plane sloping ocean boundary, and its reflection propagating away from the slope. Intermittent processes of internal-wave generation at the sea surface or ocean floor, however produce packets or *groups* of internal waves. The size of the volume within which an internal wave group of narrow bandwidth overlaps with its reflection from a plane boundary depends on the finite dimensions of the group. The height above a slope within which interactions can occur is calculated as a function of the direction of approach of the incident group to the plane. Wave groups traveling with positive downslope group velocity components, such as internal tidal rays generated at the shelf break, produce relatively thicker interaction regions than do groups of equal dimensions and frequency approaching a slope from deep water. In some regions, the thickness of layers of enhanced mixing over continental slopes or on the flanks of deep ocean ridges may be determined by the scale of reflecting internal wave groups rather than by, for example, the straining of the ambient wave field by radiating internal tidal waves.

1. Introduction

Internal wave generation is an intermittent process occurring mainly at the sea surface or near rough bottom topography, leading to wave packets or ''groups'' (Thorpe 2001a, manuscript submitted to *J. Phys. Oceanogr.*; Thorpe 2001b) which can impress a scale and pattern to the patches of mixing within the deep ocean (Thorpe 1999a). Here we describe the effect that the propagation of waves in groups has on the volume within which instability and mixing may occur as a result of their reflection from sloping boundaries. This is partly motivated by the Polzin et al. (1997) and Ledwell et al. (2000) observations of regions of greatly enhanced mixing up to scales of 0.5–1 km above the rough topography of the Mid-Atlantic Ridge east of the Brazil Basin. The thickness of mixing regions has consequences for deep ocean circulation, which is sensitive to horizontal variations in diapycnal mixing (Samelson 1998). Hasumi and Suginohara's (1999) model of the global ocean demonstrates that, when vertical diffusion is greater over rough topography than elsewhere, upwelling is confined to the regions of enhanced diffusion and a three-dimensional pattern of circulation develops different from that predicted when the diffusion is horizontally homogeneous. A major uncertainty in their numerical model remains the vertical extent of mixing over regions of rough topography.

Much interest has already been given to the reflection of internal waves from sloping topography. Several authors (e.g., Eriksen 1982, 1985; Ivey and Nokes 1989; Slinn and Riley 1998; Taylor 1993; Thorpe 1999a) have discussed the reflection of trains of waves incident on a plane boundary inclined at angle, γ, to the horizontal in a uniformly stratified fluid of constant buoyancy frequency, N, and particularly the wave steepening, formation of fronts and mixing when slopes are near critical, that is, when the slope angle, γ, is near the angle at which the group vector makes with the horizontal, β. The angle β is related to the wave frequency, σ, by the dispersion relation, $\sigma^2 = N^2 s_\beta^2 + f^2 c_\beta^2$ [Gill 1982, Eq. (8.4.13)], where $s_\beta = \sin\beta$ and $c_\beta = \cos\beta$. Thorpe (1987, 1997, 2001) has described the resonant interaction that occurs between the incident and reflected waves, leading to significant wave steepening and reduced stability. De Silva et al. (1997) have made laboratory experiments in which an internal wave ray reflects from a plane sloping boundary at zero azimuth. These two-dimensional experiments are reproduced numerically by Javam et al. (1999). Both studies show that there are signs of wave instability, notably overturning or convective instability with the production of statically unstable regions within, but not outside, the reflection region on

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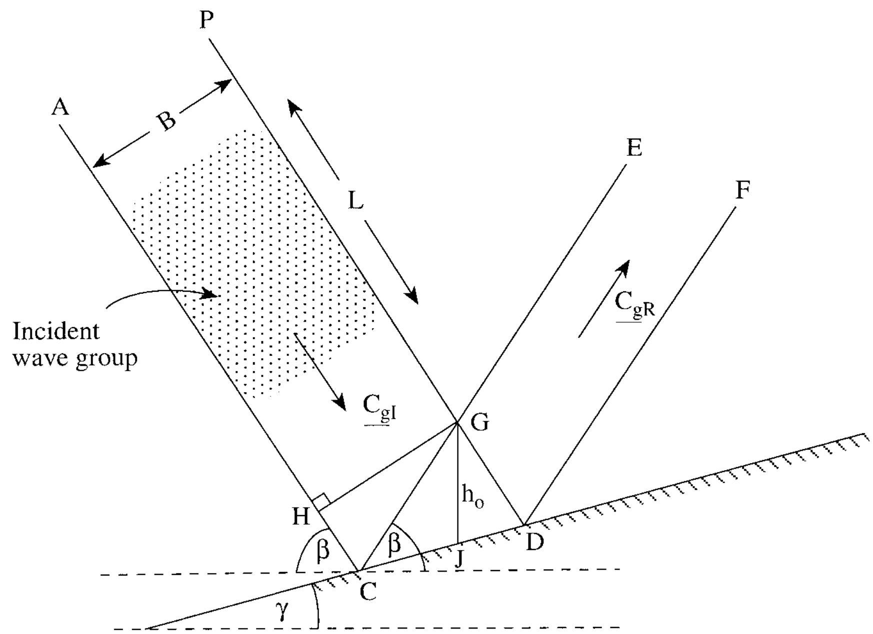

FIG. 1. Sketch showing a wave group, stippled, approaching a plane slope inclined at angle γ to the horizontal. Lines PD and AC mark the boundaries of the group parallel to the group velocity $\mathbf{c}_{gI}$ as it approaches the slope, and DE, CE are the corresponding boundaries of the (modified) group reflected away from the slope with group velocity $\mathbf{c}_{gR}$. The point G at height h_0 above the plane determines the maximum height to which the incident and reflected wave groups may occupy the same space and possibly interact.

the slope where the incident and reflected wave coexist. Overturn is particularly evident when the slope is near critical.

In all these studies, it is implicit that the incident wave train is periodic and of great duration and extent. Because of the transience of their generation and nonlinear interactions in midwater, internal waves in the ocean may reach its sloping bottom boundaries as short packets or groups of finite dimensions. This has an important consequence, which is considered here: the domain within which the incident and reflected waves composing a group overlap and can interact is limited in its extent above the topography by the size of the incident wave groups.

2. Wave groups reflecting from sloping boundaries

We suppose that wave groups or packets impinge on the sloping topography of the floor of an ocean with constant buoyancy frequency, N, where the Coriolis frequency is f. For simplicity, a group of very narrow frequency bandwidth containing waves of wavenumber K is considered. The group has length, L, in the direction of its propagation, $\mathbf{c}_{gI}$, and breadth, B, and it approaches a sloping boundary at zero azimuth (i.e., so that $\mathbf{c}_{gI}$ lies in a plane normal to the slope) as shown in Fig. 1 for $\gamma < \beta$. (The extent in the third direction is immaterial, but supposed large.) The slope angle is γ, and $\mathbf{c}_{gI}$ is inclined to the vertical at angle β. The incident group lies between the lines AC and PD and, preserving frequency (and hence β because of the dispersion relation), reflects between CE and DF at angle β to the horizontal; CE and PD intersect at G. Interaction between the incident and reflected groups will occur to the height of G above the plane, provided that the rear of the incident group has not passed G before the front of the reflected group arrives. This is so if the time taken for the incident group to pass through G ($L/\mathbf{c}_{gI}$) exceeds the total time taken for its advance from H to C (at speed $|\mathbf{c}_{gI}| = c_{gI}$) and C to G (at speed c_{gR}), where H is the foot of the perpendicular from G to AC. Since B = HG, this condition can be written $L/B >$ HC/HG + (CG/HG) (c_{gI}/c_{gR}), = $(L/B)_0$, say. Angle HCG = 180° − 2β, so HC/HG = $-\cot(2\beta)$, and CG/HG = cosec2β. Using the expression for the group velocity $\mathbf{c}_g = (c_{gx}, c_{gz})$, where

$$c_{gx} = s_\beta c_\beta^2(N^2 - f^2)/[K(N^2 s_\beta^2 + f^2 c_\beta^2)^{1/2}], \quad \text{and}$$

$$c_{gz} = -c_\beta s_\beta^2(N^2 - f^2)/[K(N^2 s_\beta^2 + f^2 c_\beta^2)^{1/2}],$$

with

$$c_g = |\mathbf{c}_g| = s_\beta c_\beta (N^2 - f^2)/K\sigma \qquad (1)$$

(Thorpe 2001b), the ratio c_{gI}/c_{gR} is equal to the ratio of wavelengths of the reflected and incident waves. This equals $\sin(\beta + \gamma)/\sin(\beta - \gamma)$ (Eriksen 1985), so $(L/B)_0$ becomes

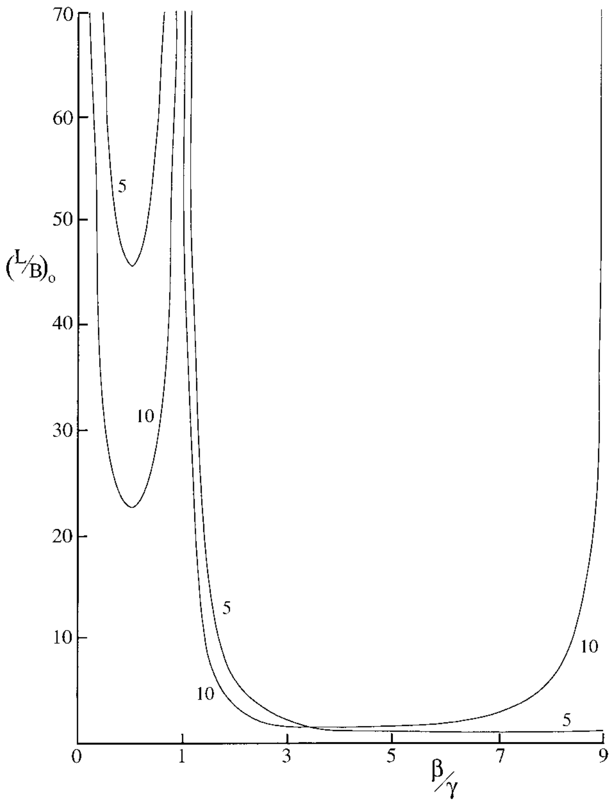

FIG. 2. The condition for the incident and reflected groups to coincide at G in Fig. 1 is $L/B > (L/B)_0$. The curves show $(L/B)_0$ as a function of β/γ for $\gamma = 5°$ and 10° (labeled). The horizontal scale is stretched between $\beta/\gamma = 0$ and 1 to aid definition of the curves in this range. Note that $\beta/\gamma = 9$ when $\beta = 90°$ and $\gamma = 10°$.

$$\left(\frac{L}{B}\right)_0 = \frac{\sin(\beta + \gamma)}{[\sin(\beta - \gamma)\,\sin 2\beta]} - \cot 2\beta,$$

$$= \frac{[3\,\sin(\beta + \gamma) - \sin(3\beta - \gamma)]}{[2\,\sin 2\beta\,\sin(\beta - \gamma)]}. \qquad (2)$$

(It may be shown that the sign is reversed when $\beta < \gamma$.) Figure 2 shows $(L/B)_0$ as a function of β/γ for $\gamma = 5°$ and 10°. Here $(L/B)_0 = \sin(45° + \gamma)/\sin(45° - \gamma)$ when $\beta = 45°$, which is close to unity when γ is small. However the ratio tends to infinity as β tends to zero or to γ, when the ratio of the incident to reflected wavelengths also tends to infinity and the width of the reflected group, $B\,\sin(\beta - \gamma)/\sin(\beta + \gamma)$, tends to zero. For small γ, a minimum of $(L/B)_0$ is found to be $(7/2)\gamma$ when $\beta = \gamma/2$ [i.e., when the minimum $(L/B)_0$ is $(7/4)\beta$; see later]. The length of the incident group must be much greater than its breadth if the incident and reflected groups are to coexist at point G for groups of near-inertial waves or when the bottom slope is near critical (i.e., when $\gamma = \beta$.) The time taken for the group to reflect at the point C is L/c_{gI}, which is equal to the length of the reflected group divided by c_{gR}, so the length of the reflected group is $L(c_{gI}R/c_{gI}) = L\,\sin(\beta - \gamma)/\sin(\beta + \gamma)$. The area of the reflected group is therefore approximately $LB[\sin(\beta - \gamma)/\sin(\beta + \gamma)]^2$ and will generally be smaller than that of the incident group if $\beta > \gamma$; the structure of wave groups will change on reflection, even from a uniform slope.

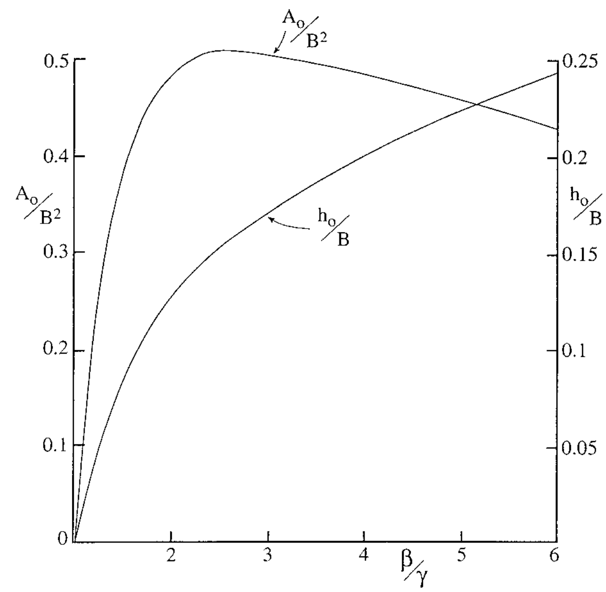

FIG. 3. The maximum height h_0 and area of interaction A_0 between an incident group and its reflection, both normalized using the breadth B of the incident group, as functions of β/γ. The slope angle γ is 5° and the incident group is at zero azimuth.

Suppose now that the incident and reflected groups overlap fully so that, for zero azimuth, (2) is satisfied. To what height, $h_0 = $ GJ, above the boundary and over what area, GCD, will interaction occur? Since CG $= B$ cosec2β, and the sine rule in triangle CJG gives $h_0 = $ CG $\sin(\beta - \gamma)/\cos\gamma$, or

$$h_0/B = \sin(\beta - \gamma)/(\cos\gamma\,\sin 2\beta), \qquad (3)$$

which tends to zero as β tends to γ. The area, A_0, of CGD within which interactions may occur is found to be

$$A_0/B^2 = \sin(\beta - \gamma)/[2\,\sin 2\beta\,\sin(\beta + \gamma)]. \qquad (4)$$

Figure 3 shows relations (3) and (4) expressed in terms of β/γ when $\gamma = 5°$. The ratio h_0/B increases steadily as β/γ increases, but A_0/B^2 has a maximum of 0.52 near $\beta = 13°$, or approximately $0.43/\sin\gamma$ at $\beta = \sin^{-1}[(3 + 2\sqrt{2})^{1/2}\,\sin\gamma]$ when $\gamma\ (<\beta)$ is small.

These results may be extended to include variations in the azimuth, θ_I, of the incident group

$$\frac{h(\theta_I)}{h_0} = \frac{\sin\phi_R\,\sin 2\beta}{[\sin(\phi_I - \phi_R)\,\sin(\beta - \gamma)]}, \quad \text{and} \qquad (5)$$

$$\frac{A(\theta_I)}{A_0} = \frac{\sin\phi_R\,\sin(\beta + \gamma)\,\sin 2\beta}{[\sin\theta_I\,\sin(\phi_I - \phi_R)\,\sin(\beta - \gamma)]}, \qquad (6)$$

where $\cos\phi_I = s_\gamma s_\beta\cos\theta_I - c_\gamma c_\beta$ and $\cos\phi_R = s_\gamma s_\beta\cos\theta_R + c_\gamma c_\beta$ with $\tan(\theta_R/2) = \tan(\theta_I/2)\,\sin(\beta - \gamma)/\sin(\beta +$

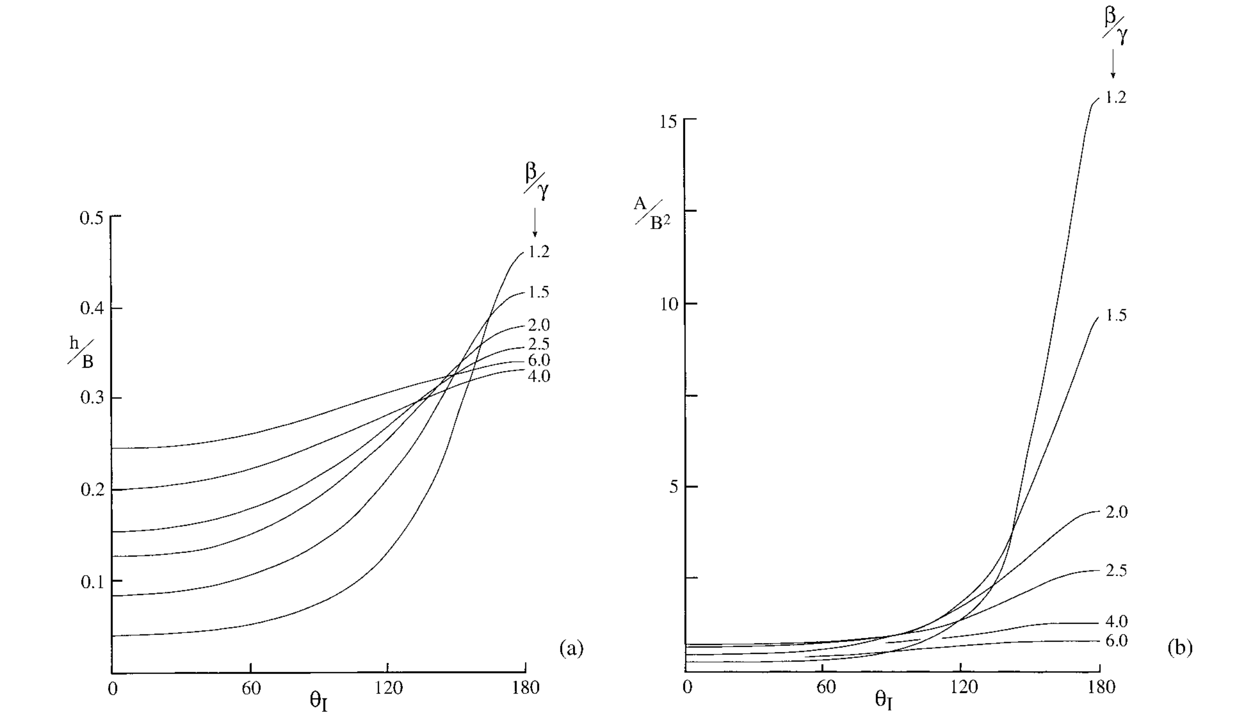

FIG. 4. (a) The maximum height h and (b) the area of interaction A between an incident group and its reflection, both normalized using the breadth B of the incident group, as functions of the azimuth angle of the incident group, θ_I. The slope angle γ is 5° and the curves show various values of β/γ (>1) as labeled.

γ). The angle θ_R is the azimuth of the reflected wave group. Figure 4 shows the variations of h/B and A/B^2 as functions of θ_I for various values of β/γ at $\gamma = 5°$. Each increases with θ_I, but only slightly so if $\theta_I < 60°$. If the direction of approach shown in Fig. 1 is reversed so that the wave azimuth is 180° and the incident group moves in direction EC with breadth, B, then (3) and (4) become $h_0/B = \sin(\beta + \gamma)/(\cos\gamma \sin 2\beta)$ and $A_0/B^2 = \sin(\beta + \gamma)/[2 \sin 2\beta \sin(\beta - \gamma)]$, which now tends to infinity as γ tends to β. This is consistent with the increasingly large values of h/h_0 and A/A_0 at $\theta_I = 180°$ as β/γ tends to unity.

Consider the case in which an internal wave group containing waves of frequencies ranging from σ to $\sigma + \Delta\sigma$ (with $\Delta\sigma/\sigma \ll 1$) and wavenumber K is generated by processes acting for a period T over a finite horizontal length of the sea surface, l. The group will become broader and longer as it propagates. This is because the β is related to wave frequency through the dispersion relation and the speed at which waves of different frequency propagate also depends on frequency via (1). At a time, t, after the onset of generation the breadth of the group, B, normal to a mean $\mathbf{c}_g$, is approximately $B = ls_\beta + c_g t\Delta\beta$, where $\Delta\beta$ is the small angle between the directions of the σ and the $\sigma + \Delta\sigma$ wave directions and ls_β is the initial extent of the group normal to $\mathbf{c}_g$. The length of the group is $L = lc_\beta + Tc_g + t\Delta\beta\partial c_g/\partial\beta$, where $\partial c_g/\partial\beta = c_g(f^2c_\beta^4 - N^2s_\beta^4)/(N^2s_\beta^2 + f^2c_\beta^2)$, so

$$L/B = [c_\beta + Tc_g/l + (\chi/c_g)\partial c_g/\partial\beta]/(s_\beta + \chi), \quad (7)$$

where $\chi = c_g t\Delta\beta/l$. (The wave group reaches the seafloor at depth, D, at time D/c_{gz} when $\chi = D\Delta\beta/ls_\beta$.) Differentiation of (7) with respect to χ shows that L/B decreases as χ increases, provided $\beta < \pi/4$ (where c_g reaches its maximum value); the maximum value of L/B is then $(c_\beta + Tc_g/l)/s_\beta$. For near-inertial waves, when β is small, L/B does not exceed the minimum value of $(L/B)_0$ of $(7/4)\beta$ found below (2) to be required for interaction between the incident and reflected group to the full height of G in Fig. 1, unless $Tc_g/l > (3/4)\beta$. Since $c_g \approx \beta N^2/Kf$ when $\beta \ll f/N$ and $(f/N)^2 \ll 1$, this condition becomes $NT > (3f/4N)(lK)$. While (7) is at best a rough estimate, being based on a crude model of group generation, it indicates that the vertical extent of the near-bottom region of strong interaction can be smaller than the values given in Figs. 3 and 4 (particularly if the generation time, T, is small), which are themselves significantly less than the breadth of the incident group.

3. Discussion

Earlier studies of resonant or near-resonant interactions between incident and reflected waves on a slope identified regions within the wave field in which static instability may occur (e.g., Thorpe 1987). No physical limit was found on the thickness of the layer adjacent to the slope within which such resonant instability, and potentially mixing, might occur. Similarly, in the study of the generation of near-boundary alongslope currents generated as a consequence of convective instability resulting from an interaction between an incident wave train and its reflection, it was necessary to impose an arbitrary length scale proportional to the wavelength of the incident wave to characterize the thickness of the interaction zone (Thorpe 1999c). An important conclusion of the present study is that the thickness of the region within which interactions between incident and reflected waves may lead to instability, with consequent mixing and enhanced diapycnal transfers, has an upper bound determined by the dimensions of wave groups incident on the boundary and is therefore determined by the processes (e.g., those at the sea surface) leading to the formation of wave groups.

Diffusivities of (2–4) $\times$ 10^{-4} m^2 s^{-1} at 500 m above abyssal hills on the flank of the Mid-Atlantic Ridge and approximately 10^{-3} m^2 s^{-1} near the boundary are found by Ledwell et al. (2000). Over the one month period of existing observations, the diffusivities have a fortnightly modulation, suggesting an association with the spring–neap tidal cycle. The enhanced near-bottom mixing is ascribed to breaking internal waves generated by tidal currents flowing over the rough topography.

Munk and Wunsch (1998) find that about 2 TW of power is required to support diapycnal transfer in the abyssal ocean. About half, (1 $\pm$ 0.25) TW, may be supplied by tidal dissipation, probably through the scattering of barotropic tidal energy into internal waves (Egbert and Ray 2000). Internal waves generated by the scattering of the tidal flow over rough bathymetry may appear as topographic lee waves of intrinsic frequency that depends on the length scales of the topography and the flow speed (Thorpe 1996, 2001b) or, when radiated from slopes close to critical, may take the form of internal wave rays with tidal frequency, that is, internal tidal waves (Baines 1974). Some of the internal waves of both types generated on the shallower topography will propagate downward, to be reflected from slopes at greater depths.

The other 1 TW needed to support diapycnal mixing is contributed by the wind at the water surface, a dominant part of which is that transmitted to internal waves that subsequently lose their energy by breaking (Munk and Wunsch 1998; Wunsch 1998). Both the surface and topographic sources of internal waves are intermittent, those at the surface because of the variations that occur in, for example, the wind stress, mixed layer turbulence, pressure patterns, or interacting surface wave fields that generate the internal waves (Thorpe 1975) and those at the bottom because the conditions for generation of internal lee waves by the flow of the fluctuating tidal currents over topography are satisfied only during certain parts of the tidal period (Thorpe 1996) or simply because their generation has a spring–neap tidal cycle. Even accounting for the possible interactions in midwater, which may impose further modulations on the structure of the wave packets or sustain the integrity of wave groups (Thorpe 2001a,b), much of the internal wave field propagating toward abyssal slopes will consequently have an intermittent nature or ''grouplike'' structure.

We have discussed the consequences of this group structure on boundary mixing. The approach to explaining enhanced near-bottom mixing over topography differs somewhat from that envisaged by Ledwell et al. (2000). Here mixing is caused mainly by the interactions within groups of waves that approach and reflect from sloping boundaries, groups coming from all sources, surface and topographic (the latter perhaps dominated by tidal forcing). Ledwell et al. propose that scattering of tidal energy from topography into internal waves accounts for the enhanced mixing, presumably as a consequence of wave breaking caused by processes such as the straining of the ambient wave field by the tidal internal wave rays radiating from the scattering regions. Not only the intensity of mixing, but the thickness scale of the mixing region in the two cases may differ. Neglecting the presence of other, relatively thin, sublayers, for example, that induced by bottom shear of scale u_*/N, where u_* is the friction velocity, typically of order 10^{-4} m s^{-1}, it is possible that in some regions a double boundary layer structure exists, internal tidal wave rays scattered and radiating directly from the topography locally dictating one scale and wave groups another.

The finding by Polzin et al. (1997) and Ledwell et al. (2000) of enhanced mixing to heights of 0.5–1 km over topography does however imply very broad groups of incident groups if incident–reflected wave group interaction is the primary cause of the observed mixing. The boundaries of the ocean are however not plane, as assumed in the simple calculations made here, but irregular, leading to scattering of part of the incident wave groups in azimuth and wavenumber (Thorpe 2001b), possibly thickening the region of interaction. Establishing the causes and extent of mixing near topography is hindered by a lack of information about the patchy nature, propagation characteristics, or group structure of internal waves approaching or radiating from the lateral boundaries—the continental slopes and sides of islands, seamounts, and ridges—of the ocean.

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