Resistance to Barotropic Tidal Flow in Straits by Baroclinic Wave Drag

ANDERS STIGEBRANDT

Department of Oceanography, Earth Sciences Centre, Göteborg University, Göteborg, Sweden

(Manuscript received 18 August 1997, in final form 16 March 1998)

ABSTRACT

Energy transfer from barotropic tides to baroclinic motions may take place at the ends of straits connecting stratified basins, implying generation of internal waves propagating into the basins. Different aspects of this have been described in the literature, including quantification of the energy transfer, studies of the resulting internal tides, and the relationship to diapycnal mixing in the basins. However, the accompanying resistance to oscillatory barotropic strait flows, by so-called baroclinic wave drag, has not been implemented earlier in models for strait flow. In the present paper a formulation for the instantaneous baroclinic wave drag force is suggested. This leads to a linear relationship between the strait flow and the synoptic sea level difference between the adjacent basins. The model is used to study tidal response in land-locked basins. The response is computed for three fjords where the resistance to the barotropic flow comes essentially from baroclinic wave drag. It is concluded that the strait-flow model handles baroclinic wave drag in a satisfactory way since the observed responses are well predicted by the model. Baroclinic wave drag should provide the major resistance to surface tides in the deep ocean, which indicates its importance in a wider context.

1. Introduction

Several mechanisms may cause resistance to barotropic flow in straits and channels. Friction against the sea bed is certainly the most commonly considered type of flow resistance. It is often modeled with a quadratic relationship between the bottom stress and the barotropic velocity using a drag coefficient to characterize the frictional properties of the sea bed. Another type of flow resistance is due to large-scale longitudinal variations of the vertical cross-sectional area of the strait causing contraction followed by expansion of the flow. This class of flow resistance, appropriately termed barotropic form drag, includes the resistance caused by contraction–expansion induced by the ends of straits having smaller vertical cross-sectional areas than the adjacent basins. In the cases of frictional drag and large-scale form drag, barotropic energy is transferred to turbulence in the bottom boundary layer in the strait and to large-scale eddies in the expanding current downstream of the strait, respectively. The present paper, however, draws attention to a third class of resistance, operating on oscillating barotropic strait flow. It is due to generation of baroclinic (internal) waves in the adjacent stratified basins by the ends of the strait and is appropriately termed baroclinic wave drag. For this class of resistance barotropic energy is thus transferred to baroclinic waves in the adjacent basins.

An oscillatory barotropic transport through the mouth of a land-locked basin causes sea level variations in the basin. This opens for a convenient way to investigate barotropic strait flows using observations of the sea level response in land-locked basins to external sea level variations. Co-oscillating tides in land-locked basins with quite narrow mouths are known to be smaller and lag those outside the mouth. The Baltic Sea, for instance, is an extreme case where externally forced sea-level oscillations of periods shorter than a few weeks are strongly damped or choked, for example, Samuelsson and Stigebrandt (1996). Other examples from Norwegian and Canadian fjords and tropical coastal lagoons have been described in, for example, McClimans (1978), Tinis (1995), and Rydberg and Wickhom (1996). Tidal choking has been modeled for cases where the barotropic transport capacity of the mouth is restricted due to flow resistance by one or both of the following two mechanisms: frictional bottom drag in the mouth (Glenne and Simensen 1963) and barotropic form drag due to acceleration–deceleration at the ends of the mouth (McClimans 1978). A unified model including both mechanisms was given in Stigebrandt (1980).

If the basins connected by a strait are stratified, the reduction of depth and vertical cross-sectional area introduced by the strait may cause flow resistance due to baroclinic wave drag. This mechanism of flow resistance to barotropic strait flows may be looked upon as another

Corresponding author address: Dr. Anders Stigebrandt, Department of Oceanography, Earth Sciences Centre, Göteborg University, Box 460, S-40530 Göteborg, Sweden.
E-mail: anst@oce.gu.se

kind of form drag, alternate to the barotropic form drag under unstratified conditions. For well-behaved topography and stratification the baroclinic basin response to a fluctuating barotropic current over steep topography can be computed using simple analytical models, for example, Stigebrandt (1976, 1980). The models rely on the boundary condition of vanishing normal velocity at nonhorizontal topography, which in the present case occurs at the basin wall at the basin–strait junction. A verification of the model for barotropic to baroclinic tidal energy conversion at fjord sills in Stigebrandt (1976) is presented by Parsmar and Stigebrandt (1997), who show that this model explains the observed damping of the fundamental barotropic seiche, of period slightly less than 2 h, quite well in the Gullmar Fjord.

If there is no dissipation of tidal energy in a fjord, co-oscillating barotropic tides should take the shape of standing waves. However, if tidal energy is dissipated, progressive barotropic tidal wave components will provide the energy required by dissipation. The presence of progressive components means that the phase of the tide increases up-fjord. There is a diagnostic method to estimate dissipation using observed up-fjord changes of the phase of tidal constituents, for example, McClimans (1978) and Farmer and Freeland (1983). The method, based on an assumption of linear fjordic response, was improved and reviewed by Tinis (1995). However, as further discussed later in the present paper, the diagnostic method to estimate tidal energy dissipation does not reveal the nature of the energy loss and cannot be used to predict fjordic tidal response.

Stacey (1984) found that dissipation in some Canadian fjords, estimated using the diagnostic method mentioned above, is approximately equal to the energy transfer from barotropic to baroclinic tides computed by a modified version of the models for barotropic to baroclinic energy conversion in Stigebrandt (1976, 1980). From this he concludes that the tidal energy losses in these fjords are essentially of baroclinic nature and due to the generation of internal tides.

Internal waves in fjord basins generated by oscillating barotropic currents over sills seem to provide most of the power required by turbulence and diapycnal mixing in the deepwater of many fjords, for example, Stigebrandt (1976), Stigebrandt and Aure (1989), Simpson and Rippeth (1993), and Tinis (1995). Estimates in these and other papers show that regularly about 5% of the computed energy transfer from barotropic to baroclinic tides is used for irreversible work against the buoyancy forces (i.e., diapycnal mixing) in the basin water of fjords. The paths taken by the energy flux from the barotropic to baroclinic energy conversion at sills to turbulent dissipation and diapycnal mixing at small scales are not well understood. However, some pieces of the puzzle are available. De Young and Pond (1989) found in Canadian fjords that the radiation of baroclinic energy away from the generation area (sill) is only 50%–80% of the expected radiation, while Stigebrandt (1979) found ~80% for the Oslo Fjord. Thus, it seems that there is some transfer of baroclinic energy to small-scale turbulence already in, or close to, the area where the internal waves are generated.

Although the barotropic to baroclinic energy transfer at the ends of straits connecting stratified basins now must be regarded as well understood and documented, the resistance to barotropic strait flows induced by this process has not yet been implemented in strait-flow models. In the present paper an expression for the instantaneous baroclinic wave drag force is derived and included in a new, general model for barotropic flow through relatively narrow and shallow straits, that is, straits where the flow speed is appreciably greater than the speed of the barotropic flow in the adjacent basins. The model computes the instantaneous energy transfer due to 1) baroclinic wave drag for stratified conditions in the adjacent basins, 2) frictional bottom drag in the strait, and 3) barotropic form drag under unstratified conditions in the adjacent basins. The latter two drag forces were implemented already in Stigebrandt (1980). The new model may predict the phase lag of barotropic tides in stratified land-locked basins, and the results are easily checked using sea level data only as demonstrated later in this paper.

The outline of the paper is as follows. The next section describes a general model for barotropic strait flows including the baroclinic wave drag force. Together with the volume budget for a land-locked basin, this model constitutes a general model for tidal response. Thereafter, in section 3, the theory is tested using published data from some land-locked basins (fjords). Finally, the paper is concluded in section 4 by some remarks.

2. Theory

In this section a model for barotropic strait flow is discussed. The new contribution can be found in section 2c where baroclinic wave drag is treated. However, for completeness all three mechanisms causing resistance to barotropic strait flow, discussed in the previous section, are presented. It is assumed that the strait is relatively narrow and of constant (rectangular) cross-sectional area. These assumptions are introduced for clarity and simplicity of the model and no essential dynamics are lost by this. One particular goal of this section is to put the diagnostic method to estimate tidal dissipation, mentioned in the previous section, into its proper context.

a. Barotropic energy loss in narrow straits

From basic principles it follows that the momentary rate of energy loss for barotropic flow between two basins, separated by a relatively narrow strait, is

$$\varepsilon = Qg\rho\Delta\eta. \tag{1}$$

The flow rate Q has the same sign as $\Delta\eta = h_o - h_i$,

h_o (h_i) is the sea level in the outer (inner) basin, g the acceleration of gravity, and ρ the density of water. For Eq. (1) to be valid, the velocities both far upstream and far downstream of the strait have to be small, implying that the vertical cross-sectional area should be appreciably smaller in the strait than in the adjacent basins. This limitation is introduced only for clarity and simplicity; it should be easy to account for nonvanishing velocities in the adjacent basins. Equation (1) does not reveal, or depend on, the nature of the energy loss and should apply independently of the details of the flow in and around the strait. To study the mechanics of the energy loss a dynamical model for the flow in and around the strait is needed as discussed later in this paper.

In a so-called tidal response (choking) model, used later in this paper, the following volume budget is applied to the land-locked basin inside the strait:

$$A_i \frac{dh_i}{dt} = Q + Q_f. \quad (2)$$

Here A_i is the horizontal surface area, Q_f the freshwater supply, and h_i the time-dependent horizontal mean sea level of the land-locked basin. From observed time series of h_i and Q_f one may compute the barotropic transport Q from Eq. (2). Thus, it is extremely easy to estimate barotropic transports through straits connecting land-locked basins and the sea, and it should be hard to obtain better transport estimates even from extensive current measurement programs. This should be true for relatively small basins with small internally forced sea-level variations so that volume changes may be estimated with good accuracy from sea level measurements in only one location.

The rate of dissipation of a tidal component may be estimated in a diagnostic way from Eq. (1) if measurements of h_o and h_i are available, thus without any knowledge of the dynamics of the strait flow. By guessing the functional form h_i of the tidal response in a land-locked basin, an analytical expression for the mean rate of dissipation may be derived for example, McClimans (1978), Farmer and Freeland (1983), Stacey (1984), and Tinis (1995). Putting $h_o = a_o \sin(\omega t)$, one thus assumes that the response in the land-locked basin is $h_\mathrm{i} = a_\mathrm{i} \sin(\omega t - \varphi)$. The transport Q can then be expressed as a function of h_i using Eq. (2) with $Q_f = 0$ and the relationship between a_o, a_i and the phase angle φ is readily found to be $a_i = a_o \cos(\varphi)$. Inserting in the expression for momentary dissipation, Eq. (1), and averaging over one tidal cycle, the following expression for the mean dissipation ε_av may be obtained:

$$\varepsilon_\mathrm{av} = \frac{1}{4}\rho g A_i \omega a_o^2 \sin(2\varphi). \quad (3)$$

This is identical to the expression derived by Tinis (1995), who also pointed out that some earlier published expressions for ε_av based on the assumption $a_i = a_o$ overestimate the mean dissipation by a factor of 2. Of course, a_i and φ cannot be predicted with a guessed tidal response in the fjord, but Eq. (3) can be used to estimate tidal dissipation if a_o and φ are known from observations. To really predict a_i and φ for a given land-locked basin, one evidently needs a dynamical model for the strait flow. Such a model is derived below.

b. Energy losses due to bed friction and barotropic form drag

For completeness, energy losses due to bed friction and barotropic form drag are first briefly discussed. Due to friction against the sea bed in the strait, energy is lost to turbulence in the boundary layer at the sea bed. For the purpose of the present paper it is sufficient to consider a topographically simple strait of constant vertical cross-sectional area giving a constant velocity U_s in the strait. If C_D is the drag coefficient and A_s the surface area of the strait bed, the frictional energy loss in the strait may be written

$$\varepsilon_f = \rho C_D U_s^2 A_s |U_s|. \quad (4)$$

The second mechanism of flow resistance is due to (nonviscous) large-scale barotropic form drag under unstratified conditions causing flow acceleration into the strait, implying a drop of the sea level and the pressure at a given level. In homogeneous water this pressure drop is usually not recovered at the lee side of the strait where the flow separates from the walls and takes the form of a dissipating jet. Possibly, the pressure may be partially recovered if the transitions between the strait and the basins are smooth and gradual but, for simplicity, this is not accounted for here. Thus, in the case of downstream flow separation, the rate of energy loss due to the barotropic form drag force is

$$\varepsilon_\mathrm{a} = |Q|\frac{\rho}{2}U_s^2. \quad (5)$$

Here we have used the already mentioned assumption that the strait is relatively narrow, so the far upstream and downstream velocities may be neglected. For later reference it is noted that for a rectangular strait of width B and depth h the barotropic form drag force can be written $F_{fD} = (\rho_0/2)BhU_s^2$. As discussed later, this flow force is replaced by the baroclinic wave drag force if the amplitude U_t of the barotropic strait flow is less than the group velocity of internal waves c_g in the stratified adjacent basins. Thus, the barotropic form drag is expected to act when $U_t/c_g > 1$, that is, under practically unstratified conditions.

If only energy losses due to bed friction and barotropic form drag under practically unstratified conditions apply, the energy balance for the strait flow becomes

$$\varepsilon = \varepsilon_a + \varepsilon_f. \quad (6)$$

From Eqs. (1), (4), (5), and (6) the following expression for the flow $Q(t)$ through a rectangular strait may be derived:

$$Q^2 = \frac{2g\Delta\eta B^2h^2}{1 + 2C_D(L/h)}. \quad (7)$$

Here the area A_s of the sea bed of the strait is approximated by L times B where L is the length of the strait. The sign of Q is equal to that of $\Delta\eta$. Equations (2) and (7) together constitute a so-called tidal choking model for unstratified land-locked basins identical to that earlier presented in Stigebrandt (1980).

c. Energy losses due to baroclinic wave drag

The relationship between Q and $\Delta\eta$, Eq. (7), is thus fairly well known for unstratified conditions, but what does it look like when the flow resistance is due to baroclinic wave drag? This not earlier considered question is the main interest of the present paper. Internal waves are generated in the adjacent basins if these are stratified and deeper than the strait and the barotropic strait flow varies in time with an amplitude U_t less than the group velocity c_g of the internal waves. The latter condition can be expressed in terms of a particular densimetric Froude number $F_t = U_t/c_g$, already used in section 2b, that thus must be less than one for generation of internal waves, for example, Stigebrandt (1976). The terms ‘‘jet basin’’ and ‘‘wave basin’’ were used in Stigebrandt and Aure (1989) for basins characterized by $F_t > 1$ and $F_t < 1$, respectively. The physical reason for internal wave generation is that internal waves and the barotropic flow together may satisfy the boundary condition of vanishing normal velocity at the basin walls at the strait–basin junctions (sills). For continuous vertical stratification in the adjacent basins several vertical modes of internal waves are needed to satisfy the boundary condition at the sills, for example Stigebrandt (1980).

The mean energy flux by a specified progressive internal wave mode is

$$\overline{\varepsilon_w} = BEc_g, \quad (8)$$

where B is the width of the strait (the wave front), c_g the group velocity, and the energy density E of the wave mode in the adjacent basins can be computed from

$$E = \frac{1}{T}\int_0^T\int_0^H \rho u_i^2 \, dz \, dt. \quad (9)$$

Here $u_i(z)$ is the wave orbital speed and H the water depth in the adjacent basins. The property that kinetic and potential energy densities are in phase and of equal magnitude in long progressive internal waves has been used. This is true for narrow channels where rotational effects may be neglected. Note that for long internal waves the group velocity equals the phase speed.

For a two-layer approximation of the stratification in a basin with the interface at the crest of the sill, the boundary condition at the end of the strait is satisfied by the barotropic plus the only baroclinic mode provided that $F_t < 1$. The momentary transport of baroclinic energy away from the end of the strait is then estimated by

$$\varepsilon_w = (h\rho_1u_1^2 + d\rho_2u_2^2)Bc_g. \quad (10)$$

Here h, ρ_1, u_1 (d, ρ_2, u_2) are thickness, density, and orbital velocity of the upper (lower) layer, and the internal wave has the property that $hu_1 = du_2$. This expression for the momentary transport of baroclinic energy away from the end of a strait (or a sill) apparently gives the correct mean energy flux, as given by Eq. (8).

When internal waves are generated in a two-layer stratification with the interface at sill depth, one readily finds that $u_1 = U_sd/(h + d)$, for example, Stigebrandt (1976). The momentary transport of baroclinic energy from the end of a strait in Eq. (10) can be rewritten in terms of the internal wave drag force F_{wD}, thus

$$\varepsilon_w = F_{wD}c_g. \quad (11)$$

If it is assumed that $\rho_1 \approx \rho_2 \approx \rho_0$, F_{wD} is defined by

$$F_{wD} = \rho_0\frac{hd}{h + d}U_s^2B. \quad (12)$$

It should be noted that the internal-wave drag force derived here for well-behaved stratification and topography does not contain any adjustable parameter, and it should therefore be known exactly. This may be compared to the drag force due to bottom friction that depends on the empirical drag coefficient C_D. This was also pointed out in Sjöberg and Stigebrandt (1992), who presented an expression for the baroclinic wave drag force equivalent to that in Eq. (12). These authors also show that for continuous vertical stratification, the total wave drag force is independent of the modal structure of the response. The energy transfer away from a step in the bottom, like in the case of a fjord sill, however, depends on the actual modal response because the internal wave modes have different group velocities.

Comparing the expressions for barotropic and baroclinic form drag forces, F_{fD} and F_{wD}, one finds that for cases with deep lower layers, that is, $d/(h + d) \approx 1$, $F_{wD} \approx 2F_{fD}$. Thus, the effect of stratification can be thought of as increasing the ‘‘effective’’ cross-sectional area of the strait and, by that, increasing the drag force. A comparison of the energy losses gives that $\varepsilon_w \approx 2(c_g/U_s)\varepsilon_a$ showing that compared to barotropic form drag, baroclinic wave drag is quite efficient in removing barotropic energy. There may be an additional factor of 2 in favor of the baroclinic wave drag since baroclinic wave drag losses may occur at both ends of a strait.

For the implementation of baroclinic wave drag in a strait-flow model, consider a strait with stratified adjacent basins where $U_t < c_g$ (i.e., $F_t < 1$). In such a strait, there is energy loss due to baroclinic wave drag. Often there are contributions from wave drag from both ends of the strait (ε_{wo}, ε_{wi}), so the total rate of energy loss for the strait flow equals

$$\varepsilon = \varepsilon_f + \varepsilon_{wi} + \varepsilon_{wo}. \tag{13}$$

Putting $\varepsilon_f = 0$, that is, consider a short strait with negligible effects of bottom friction, and assuming $\varepsilon_{wo} = \varepsilon_{wi}$ for simplicity, one obtains from Eqs. (1), (11), (12), and (13) for the two-layer approximation

$$Q = \frac{g\Delta\eta}{2c_g}\frac{Bh(d + h)}{d}. \tag{14}$$

This equation, the major theoretical result of this paper, suggests that transports through straits are proportional to $\Delta\eta$ if the only drag force is that due to baroclinic wave drag. In straits with frictional and barotropic form drag forces, however, the transport is proportional to the square root of $\Delta\eta$ [cf. Eq. (7)]. From simultaneous measurements of Q and $\Delta\eta$ it should be possible to verify Eq. (14) and by that the expression for the baroclinic wave drag force in Eq. (12). As discussed previously, the easiest way to test Eq. (14) should be to apply it to the strait of a land-locked basin. Equations (2) and (14) together constitute a variant of a tidal response model for land-locked stratified basins applicable to cases where $F_t < 1$ and bottom friction in the strait is negligible compared to baroclinic wave drag.

For a periodic barotropic current in a strait the mean importance of bed friction compared to baroclinic wave drag from both ends of the strait can be estimated as $C_D L U_{sm}/(4hc_g)$, where U_{sm} is the mean of the absolute value of the barotropic current velocity during the period.

The simple model presented here should predict the time-dependent flow through relatively narrow straits when internal tides are generated in the adjacent basins (i.e., $F_t < 1$) for the case where the stratification in the adjacent basins can be approximated by two homogeneous layers with the interface at the sill crest. The model does not require the computation of the details of the sea level going from one basin to the other. The barotropic strait flow for cases with continuous stratification in the adjacent basins may be somewhat different. It may, however, be computed in a similar way using the proper condition at the boundary between the strait and the basin to determine the amplitudes of the different vertical modes of internal waves, for example, Stigebrandt (1980) and Sjöberg and Stigebrandt (1992).

For the application to the Idefjord below, one is reminded that it has been observed that the mixing in the basin water of wave fjords may be explained if about 5% of the energy transferred from the surface tide to internal tides at the sill in the fjord mouth is used for irreversible (diapycnal) work against the buoyancy forces. This holds also for wave fjords that are much shorter than the wavelengths of internal tides, for example, Stigebrandt and Aure (1989).

3. Model results

First, some general results of the model applied to land-locked basins are presented and discussed. The model is thereafter used to compute the tidal response in some land-locked fjords and the predictions are compared with the known responses.

a. Characteristics of tidal choking in land-locked basins with linear and quadratic strait flow resistance: Some general results

Tidal response in land-locked basins and dissipation of barotropic energy by strait flow have been computed numerically using the strait model derived in section 2 above together with volume conservation in the land-locked basin [Eq. (2)]. The sea level in the outer basin is assumed to vary harmonically with the amplitude a_o. For a linear relationship between the strait flow Q and the (forcing) sea level difference $\Delta\eta$, as in Eq. (14) for the case of baroclinic wave drag ($F_t < 1$) in a short strait ($\varepsilon_f = 0$), the numerical solution gives, as expected, exactly the same relationship between the mean dissipation ε_{av} and the phase lag φ as given by Eq. (3). This shows that the assumed fjordic response h_i used for the derivation of Eq. (3) actually is the correct response for the case of linear drag. The phase delay in this linear case is independent of the external amplitude a_o of the tide. Maximum dissipation occurs when the phase lag in the land-locked basin equals 45° and the amplitude a_i in the land-locked basin then equals $2^{-1/2}a_o$, which also can be seen from Eq. (3). These general results for the case of linear response were discussed by McClimans (1978).

For the case of quadratic drag, as in Eq. (7) (valid for $F_t > 1$ and $\varepsilon_f \geq 0$), the numerical solution of the present model shows that maximum dissipation is slightly less than for the case of linear drag (97.4%) and occurs for a slightly larger amplitude in the land-locked basin, $a_i = 0.741a_o$. However, for a given value of the phase lag φ, the difference in dissipation between the linear and nonlinear cases is rather small. Equation (3) can therefore be used for all types of straits to obtain an approximate estimate of tidal dissipation if phase information is available. Effects of local freshwater supply Q_f on the tidal response of land-locked basins for the case of quadratic drag in the strait are discussed in Stigebrandt (1980).

b. The Oslo Fjord

The choking model for the case of baroclinic wave drag, that is, using Eq. (14) for the strait flow, has been applied to the inner Oslo Fjord for which $A_f = 200$ km^2, $B = 600$ m, $h = 15$ m, $d = 65$ m, and $c_g = 0.8$ m s^{-1} (two-layer stratification). The mouth (the Drøbak Strait) is extremely short and sea bed friction may be neglected (cf. Stigebrandt 1976). A semidiurnal tide of amplitude

0.15 m is assumed to exist outside the fjord. Using these figures the model predicts that the dissipation is about 2 × 590 kW (internal waves emitted from both sides of the sill). The phase lag of the semidiurnal tide in the fjord is predicted to be 22°. As discussed above this is independent of the amplitude as long as $F_t < 1$ (wave basin) and Eq. (14) applies. The phase lag should thus be approximately the same for all semidiurnal components of the tide. For the computation it is assumed that the internal wave speed outside the sill equals that inside, which probably is an overestimate since the stratification is weaker outside than inside the sill most of the time. Thus, the predicted phase shift of M_2 across the sill is probably overestimated by some degrees. The observed phase shift is about 15° (Anonymous 1997).

c. Knight Inlet

The inner sill of Knight Inlet has the width $B = 1250$ m and depth $h = 60$ m, while $d = 300$ m and $A_f = 220$ km^2. The M_2 tidal component has an amplitude 1.5 m and for summertime stratification $c_g = 0.83$ (0.50) m s^{-1} for internal wave vertical mode 1 (2) (Webb and Pond 1986). These authors report that the phase shift of M_2 across the inner sill in Knight Inlet is 2°–3° and from time series of velocity and density at different depths in the fjord they find that the internal wave response is a mixture of vertical modes 1 and 2. Stacey (1984) estimated the dissipation inside the sill to about 7 MW under similar conditions of stratification. Assuming wave generation at both sides of the inner sill and no resistance due to sea bed friction, Eq. (14) should describe the flow over the sill. The present model then predicts that the phase shift across the inner sill is 3° (2°) and the dissipation inside the sill is predicted to be 10.1 (5.8) MW for mode 1 (2) internal wave response. With a mixed mode 1 and 2 response, the model thus seems to predict the conditions in Knight Inlet rather well.

d. The Idefjord

The Idefjord, denoted basin 2 in the computations below, is situated at the Skagerrak coast and constitutes part of the border between Sweden and Norway. The mouth of the Idefjord has two narrow (~70 m) and shallow (~9 m) sills separated by a wider and deeper small basin (basin 1). As pointed out in section 2 above, it is expected that baroclinic wave drag acts even in quite small basins, and thus at both sides of both sills as long as $F_t < 1$. Baroclinic wave drag should then be much greater than frictional drag. From Munthe Kaas (1970) one may obtain the following numbers: $A_{f1} = 0.27$ km^2, $d_1 = 12$ m, $h_1 = 9$ m, $B_1 = 58$ m, $c_{g1} = 0.5$ m s^{-1}, $A_{f2} = 20.4$ km^2, $d_2 = 19$ m, $h_2 = 9.5$ m, $B_2 = 78$ m, and $c_{g2} = 0.6$ m s^{-1}, where subscript 1 (2) refers to basin 1 (2). By simultaneously solving Eqs. (2) and (14) for basins 1 and 2 coupled in series, one obtains for a semidiurnal tidal component a phase shift of 33° between the coastal water and the Idefjord, and the amplitude in the fjord is 83% of that in the coastal water. These results are very close to the observed phase shift (36°) and amplitude reduction in the fjord (R. Parsmar 1997, personal communication).

4. Concluding remarks

The most convenient way to check models for barotropic flows in straits is to study the sea level response in a land-locked basin to externally forced time-dependent sea level variations due to, for example, tides outside the mouth of the land-locked basin. The model for barotropic strait flows developed in the present paper is used in a so-called tidal response model that is applied to land-locked basins where the resistance to the barotropic strait flow is dominated by baroclinic wave drag. The model seems to predict the conditions in the land-locked basins (fjords) it has been tested upon quite well. This means that it well computes the barotropic strait flow, including the baroclinic wave drag. It should be noted that in cases with well-behaved stratification and topography the expression for the baroclinic wave drag force contains no adjustable parameter why it actually should be known exactly in such cases.

Baroclinic wave drag should act on fluctuating barotropic currents across nonhorizontal bottoms everywhere in the ocean where $F_t < 1$. Sjöberg and Stigebrandt (1992) computed the barotropic to baroclinic tidal energy transfer for most of the deep World Ocean (below 1000-m depth) using a model based on the same boundary condition as used in the present paper, that is, vanishing normal velocity at sloping bottoms. Using the coupling between the barotropic to baroclinic energy transfer and turbulent dissipation established for fjord basins (Stigebrandt and Aure 1989), they predict strongly enhanced tidal dissipation and mixing along mid-ocean ridges and continental rises, similar to the patterns of turbulent dissipation recently observed by Polzin et al. (1997).

Since baroclinic wave drag is a singular process—at least at vertical walls—it may be difficult to implement in ocean general circulation models. However, Sjöberg and Stigebrandt (1992) pointed out a possible way to do this using the baroclinic wave drag force, which should be proportional to the slope of the bottom.

The flow resistance in the Öresund Strait (between Sweden and Denmark) is mainly quadratic and due to bottom drag on the vast (about 12 × 15 km^2) and shallow (3–8 m) sill at the border to the Baltic Sea. From field measurements Mattsson (1995) found that there also is a component of linear flow resistance. He explained this as due to rotational effects as outlined by Toulaney and Garrett (1984). However, north of the sill the strait is deeper and strongly stratified ($c_g \sim 1$ m s^{-1}), so baroclinic wave drag should contribute to the resistance to fluctuating barotropic flow. According to the theory in the present paper even the flow resistance due to baroclinic wave drag is linear [cf. Eq. (14)]. Putting in figures for Öresund ($d \sim 10$ m, $h \sim 5$ m) one finds that baroclinic wave drag may be of the same order of magnitude as rotational resistance. From observations of the baroclinic response in the stratified basin north of the sill it should be possible to make an independent estimate of the baroclinic wave drag.

Acknowledgments. This work was supported by the Swedish Natural Science Research Council (NFR).

REFERENCES

Anonymous, 1997: Tidevannstabeller (Tide Tables) for den norske kyst. Statens Kartverk, Sjøkartverket, Norway, 80 pp. [Available from Sjøkarteverket, Box 36, N-4001 Stavanger, Norway.]

de Young, B., and S. Pond, 1989: Partition of energy loss from the barotropic tide in fjords. *J. Phys. Oceanogr.,* **19,** 246–252.

Farmer, D. M., and H. J. Freeland, 1983: The physical oceanography of fjords. *Progress in Oceanography,* Vol. 12 (2), Pergamon, 147–220.

Glenne, B., and T. Simensen, 1963: Tidal current choking in the landlocked fjord of Nordåsvatnet. *Sarsia,* **11,** 43–73.

Mattsson, J., 1995: Observed linear flow resistance in the Öresund due to rotation. *J. Geophys. Res.,* **100,** 20 779–20 791.

McClimans, T., 1978: On the energetics of tidal inlets to land-locked fjords. *Mar. Sci. Commun.,* **4,** 121–137.

Munthe Kaas, H., 1970: Iddefjorden og dens forurensningsproblemer. Rapp. nr 2: Situasjonsrapport pr. 1 desember 1969. NIVA, Oslo, Rep. O-113/64, 33 pp. [Available from NIVA, Box 173 Kjelsas, N-0411 Oslo, Norway.]

Parsmar, R., and A. Stigebrandt, 1997: Observed damping of barotropic seiches through baroclinic wave drag in the Gullmar Fjord. *J. Phys. Oceanogr.,* **27,** 849–857.

Polzin, K. L., J. M. Toole, J. R. Ledwell, and R. W. Schmitt, 1997: Spatial variability of turbulent mixing in the abyssal ocean. *Science,* **276,** 93–96.

Rydberg, L., and L. Wickbom, 1996: Tidal choking and bed friction in Negombo Lagoon, Sri Lanka. *Estuaries,* **19,** 540–547.

Samuelsson, M., and A. Stigebrandt, 1996: Main characteristics of the long-term sea level variability in the Baltic Sea. *Tellus,* **48A,** 672–683.

Simpson, J. H., and T. Rippeth, 1993: The Clyde Sea: A model of the seasonal cycle of stratification and mixing. *Estuarine Coastal Shelf Sci.,* **37,** 129–144.

Sjöberg, B., and A. Stigebrandt, 1992: Computations of the geographical distribution of the energy flux to mixing processes via internal tides: Its horizontal distribution and the associated vertical circulation in the ocean. *Deep-Sea Res.,* **39,** 269–291.

Stacey, M. W., 1984: The interaction of tides with the sill of a tidally energetic inlet. *J. Phys. Oceanogr.,* **14,** 1105–1117.

Stigebrandt, A., 1976: Vertical diffusion driven by internal waves in a sill fjord. *J. Phys. Oceanogr.,* **6,** 486–495.

——, 1979: Observational evidence for vertical diffusion driven by internal waves of tidal origin in the Oslofjord. *J. Phys. Oceanogr.,* **9,** 435–441.

——, 1980: Some aspects of tidal interactions with fjord constrictions. *Estuarine Coastal Mar. Sci.,* **11,** 151–166.

——, and J. Aure, 1989: Vertical mixing in the basin waters of fjords. *J. Phys. Oceanogr.,* **19,** 917–926.

Tinis, S. W., 1995: The circulation and energetics of the Sechelt Inlet System, British Columbia. Ph.D. thesis, University of British Columbia, Vancouver, Canada, 173 pp. [Available from Dept. of Earth and Ocean Sciences, University of British Columbia, 6339 Stores Road, Vancouver, BC V6T 1Z4, Canada.]

Toulaney, B., and C. Garrett, 1984: Geostrophic control of fluctuating barotropic flow through straits. *J. Phys. Oceanogr.,* **14,** 649–655.

Webb, A. J., and S. Pond, 1986: A modal decomposition of the internal tide in a deep, strongly stratified inlet: Knight Inlet, British Columbia. *J. Geophys. Res.,* **91,** 9721–9738.