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LIMITED ENGLISH PROFICIENCY AND SPATIAL VISUALIZATION
IN MIDDLE SCHOOL STUDENTS' CONSTRUCTION
OF THE CONCEPTS OF REFLECTION AND ROTATION

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Abstract

This study attempted to show that there can exist an environment in which English proficient (EP) and limited English proficient (LEP) students can work together using computers to construct the concepts of reflection and rotation. Under investigation were the effects of a dynamic instructional environment and visualization level independently and interactively on students' construction of the concepts of reflection and rotation. Also examined was the effect of a dynamic instructional environment on students' two- and three-dimensional visualization ability. Of particular interest was the relationship between performance of EP students and their LEP peers on concept construction and visualization ability based on the instructional environment the students experienced. After controlling for initial differences, it was concluded that students experiencing the dynamic instructional environment significantly outperformed students experiencing a traditional instructional environment on content measures of the concepts of reflection and rotation as well as on measures of two-dimensional visualization ability. LEP students did not perform statistically significantly differently than their EP peers on any of the dependent variables when experiencing the same instructional environments. If this research is supported by replicated studies, a reassessment of educational policy may be required.

Introduction

Effective instructional strategies are needed for use with language minority students. According to Mathematics for Language Minority Students, the National Council of Teachers of Mathematics' (NCTM) position statement, "cultural background or difficulties with the English language must not exclude any student from full participation in the school's mathematics program" (NCTM, 1987). The *Curriculum and Evaluation Standards for School Mathematics* states, "students whose primary language is not standard

English may require support to facilitate their learning of mathematics" (NCTM, 1989, p. 80). Judging by the low percentage of language minority students receiving post-secondary degrees in mathematics-related fields, language minority students are not receiving adequate preparation in school mathematics. These students tend to spend most of their time on the prerequisite basic skills and rarely have exposure to higher-order mathematics skills (Schwartz, 1991; Secada & Carey, 1990; Stoloff, 1989).

The *Curriculum and Evaluation Standards for School Mathematics* include the following as objectives in the standard for middle-level geometry: "in grades 5 - 8, the mathematics curriculum should include the study of the geometry of one, two, and three dimensions in a variety of situations so that students can visualize and represent geometric figures with special attention to developing spatial sense; [and] explore transformations of geometric figures" (NCTM, 1989, p. 112). This standard is not being implemented in many middle schools in the United States. *The Second International Mathematics Study: Report for the United States* (Crosswhite, 1985) identified topics taught in other countries which were not covered by the majority of eighth grade classes in the United States. "For example, topics in transformational geometry, taught in some countries, were reported taught by only 12% of United States eighth grade teachers" (p. 20). Furthermore, the results of the study indicated that teachers primarily taught according to their textbook with very little use of manipulatives or other materials that may not be included with the text. This finding was corroborated by data collected in a comparative study with seventh and eighth grade students in the United States and Japan (Iben, 1988). Lack of emphasis on transformation geometry may be due, in part, to the need for an effective and accessible instructional strategy.

In order for students to develop spatial sense, students need many and varying experiences with drawing, measuring, transforming, visualizing, comparing, and classifying geometric shapes (NCTM, 1989). "Spatial sense is an intuitive feel for one's surroundings and the objects in them. To develop spatial sense, children must have many experiences that focus on geometric relationships; the direction, orientation, and perspectives of objects in space; the relative shapes and sizes of figures and objects; and how a change in shape relates to a change in size" (NCTM, 1989, p. 49). The term, spatial sense, identifies what has also been labeled spatial visualization, spatial perception, visual imagery, spatial ability, visual skill, spatial reasoning, mental rotations, and visual processes (Bishop, 1983; Davey & Holliday, 1992; Stanic &

Owens, 1990; Wheatley, 1990). Curriculum and instruction must reflect appropriate experiences for students in this area as it reflects an important aspect of mathematics (National Council of Supervisors of Mathematics [NCSM], 1989; NCTM, 1989; National Research Council (NRC] 1989). The recent availability of computers in classrooms has provided the tool for such instruction to take place. The dynamic graphic capabilities of the microcomputer allows for geometry to be introduced to students through transformations (Kantowski, 1987). NCSM (1989) supports the use of computers throughout the mathematics curriculum as well as instruction in visualization and transformations.

The *Standards* (NCTM, 1989, 1991) stress communication as an important part of teaching and learning mathematics. Mathematics can be taught and learned visually; communication does not necessarily refer exclusively to verbal exchanges. However, visual communication in mathematics education is especially important to language minority students or students having limited proficiency in English (Cummins, 1984; Dawe, 1983; Presmeg, 1989).

Use of the *Geometer's Sketchpad* (Jackiw, 1991) for the dynamic presentation of rotations and reflections is an appropriate strategy in order to facilitate full participation of limited English proficient (LEP) students while encouraging the development of all students' visualization abilities in the middle school mathematics program. *The Geometer's Sketchpad* is a dynamic program for the Macintosh computer. *The Sketchpad* "has great potential for teachers and students to use in investigations of ideas in geometry" (Wilson, 1992, p. 157).

Achievement in mathematics in a given language seems to be related to the degree of proficiency in that language (Secada, 1992a). Contrary to what many people seem to believe, the study of mathematics does not transcend language barriers. The changing demographics of the United States dictates that appropriate mathematical content, teaching strategies, learning tools, and classroom environments must be incorporated in all schools to close the gap of achievement in mathematics between English speaking and LEP students. Current programs designed with the pretense of closing this gap are "...evaluated based on their ability to curtail student drop out and to improve student scores on standardized tests. As a consequence, compensatory programs mimic the tests by which they are evaluated and which are focused on lower level and computational skills" (Secada, 1989, p. 38). This focus on lower level skills is not consistent with the Standards (NCTM, 1989) and is

not an acceptable solution to the problem of the achievement gap. LEP students can make progress leaning mathematics when the teacher recognizeS their special needs and uses manipulatives, hands-on experiences, first language (L1) of the students when possible, and systematically teaches the necessary vocabulary (Kober, 1991).

This study investigated the effects of a dynamic instructional environment and students' visualization level on LEP students' identification and performance of reflections and rotations and the effects of the dynamic instructional environment on the relationship between the students' level of visualization and identification and performance of reflections and rotations. A second purpose of this study was to examine the effects of the dynamic instructional environment on performance of LEP students on measures involving transformation geometry and visualization compared to their English proficient (EP) peers. Third, the study was designed to ascertain the effects of a dynamic instructional environment on LEP students' visualization. Students were introduced to and explored the transformations through a dynamic instructional environment incorporating activities using *The Geometer's Sketchpad* (Jackiw, 1991) within an English-dominated classroom. This research attempted to show that there can exist an environment in which EP and LEP students can work together with computers to construct knowledge of reflections and rotations through inquiry and collaboration.

The Constructivist theory of teaching and learning was used to explain how teachers can create environments in which students can construct knowledge. Vygotsky's (1978, 1986) zone of proximal development (ZPD) explained how students working together, with adult guidance as needed, and using proper tools, could construct this knowledge socially. And, finally, Cummins' (1984) theory of context-embedded language versus context-reduced language explained how EP and LEP students could work together in the same instructional setting.

Literature Review

Although the Constructivist theory of teaching and learning is given a variety of expressions, there seems to be general agreement on the following basic principles (Confrey, 1990; Davis, Maher, & Noddings, 1990; Goldin, 1990; Kamii & Lewis, 1990; Noddings, 1990; von Glasersfeld, 1990; Wheatley, 1991):

1. Students build their own knowledge, they do not receive knowledge prepackaged from others.
2. Knowledge is not built passively but through physical and mental action.
3. Truths are not found, rather, interpretations are built to explain experiences.
4. Learning takes place through social interaction.

Inherent in these principles is the need for an environment conducive to student exploration and interaction. "Constructivists in mathematics education contend that cognitive constructivism implies pedagogical constructivism; that is, acceptance of constructivist premises about knowledge and knowers implies a way of teaching that acknowledges learners as active knowers" (Noddings, 1990, p. 10).

The Constructivist view of learning mathematics is consistent with the belief that students come to the classroom with differing understandings and ways of conceptualizing mathematics (Kamii & Lewis, 1990; Steffe, 1990; Wheatley, 1991). Different students construct concepts in different ways depending on many variables, including their native and learned abilities, aptitudes, dispositions, learning styles, native language, and past experiences. The Constructivist view of teaching allows for these differences by providing experiences for children to make sense of mathematics through varying instructional strategies. These strategies include meaningful mathematical exploration and experiences, use of multiple representations, multiple modes of instruction, and, in the case of this research, a dynamic medium for learning (Kamii & Lewis, 1990; Noddings, 1990; Wheatley, 1991).

Vygotsky (1978) posited that the pedagogical and social needs of the student could be met in an environment that promoted collaborative inquiry about concepts using appropriate tools. In so doing, the students' ZPD would be formed and learning would be legitimized. According to Vygotsky (1978, 1986), the ZPD is the distance in developmental level between problems the student can solve independently and problems the student can solve in collaboration with more capable peers or under guidance from an adult. Students' potential to learn is then judged by that which they cannot yet do alone but can accomplish in collaboration with others rather than that which they can already perform. Furthermore, Vygotsky believed that learning within this zone was the only worthwhile learning. He proposed "...that an

essential feature of learning is that it creates the ZPD; that is, learning awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers. Once these processes are internalized, they become part of the child's independent development achievement" (Vygotsky, 1978, p. 90).

Vygotsky suggested that the student was working within his ZPD when he was able to solve given problems only with assistance from more capable peers or adult guidance and was not able to solve the problems independently. Students working in collaboration are able to help each other within their zones of proximal development because, according to the Constructivist view of learning, each student brings different experiences to the learning situation and hence may be more capable on different aspects of the same problem. In this situation, the "more capable peer" may alternate from one student to the other while working on the same problem. And, in the event that neither student can provide the proper catalyst to solve the problem, the teacher, acting as facilitator, can supply the necessary guidance. This role of the teacher is in accordance with both the Constructivist view of teaching and learning mathematics and Vygotsky's ZPD (Vygotsky, 1978, 1986; Wright, 1990).

For LEP students in particular and possibly all students in general, this collaborative inquiry should be placed in a context-embedded situation. A context-embedded situation is one which provides comprehensible input so that the students do not need to guess about the teacher's intentions but may gather an understanding of the intentions based on linguistic as well as situational cues. This context-embedded language may include pictures, interpersonal interactions, and nonverbal information (Fradd, 1987). According to Cummins (1984) and Fradd (1987), this environment is more compatible to LEP students than a context-reduced situation (the traditional classroom lecture style of environment) where students must rely solely on linguistic cues in a language with which the LEP student is not yet proficient.

The degree of cognitive involvement in activities where communication is used can be explained through a continuum where at one end exists "...communicative tasks and activities in which the linguistic tools have become largely automatized (mastered) and thus require little active involvement for appropriate performance" (Cummins, 1984, p. 139). At the opposite "...end of the continuum are tasks and activities in which the communicative tools have not become automatized and thus require active cognitive involvement" (Cummins, 1984, p. 139). Contextual support is most

important when the degree of cognitive involvement is the greatest (Cummins, 1981, 1984; Fradd, 1987).

When students are exploring new mathematical concepts with new mathematical vocabulary, clearly, the students, especially the LEP students, will not have automatized the associated communicative tools. Students may be experiencing just this kind of situation when working within their ZPDs. And, according to Fradd, "comprehensible input is the foundation of effective instruction" (1987, p. 143).

Bilingual-education-program research and evaluation have been driven by concerns for the development of English and of academics among LEP students. These studies have taken for granted the school mathematics curriculum that LEP students are exposed to and, even when problems in instruction are noted, those concerns get cast in terms of language development (Secada, 1992b, p. 218).

This same dilemma does not exist in school science (Sutman & Guzman, 1992). Cheche Konnen (Rosebery, Warren, & Conant, 1992; Warren & Rosebery, 1992; Warren & Rosebery, 1993) is a program conducted by the Teacher Education Research Center in Cambridge. Through working with bilingual teachers and LEP students, Warren and Rosebery "...are attempting to elaborate an approach to science teaching and learning that supports the development of scientific sense-making communities in the classroom" (1992).

Through pilot projects, the researchers concluded that the training of the teachers must include both science and science pedagogy so that the teachers could become effective facilitators, co-investigators, and mentors to the students. This was not successful when the teachers did not experience scientific sense-making in a setting conducive to collaborative inquiry themselves. Sutman and Guzman (1992) agree that even though a discovery/inquiry lesson has been planned, it is up to the teacher to maintain the environment in which students are free to discover and inquire about the concept.

As in the current research, the emphasis on collaborative inquiry in the training of the teachers and the experiences of the students build on Vygotsky (1978), in that "robust knowledge and understandings are socially constructed through talk, activity, and interaction around meaningful problems and tools" (Rosebery, personal conversation, 2/17/94; Warren & Rosebery, 1992, p.

280). The collaborative inquiry approach allows for shared responsibility in learning; this is especially helpful for the LEP students as they are contending with learning the language as well as the science. However, Rosebery and Warren might argue that the two, language and science, cannot be separated.

A major goal of Cheche Konnen is to forge links between learning science and doing science, and among science, mathematics, and language. This is in large part what makes it a powerful model for language minority students, in particular, and perhaps for all students (Rosebery, Warren, & Conant, 1992, p. 3).

Sutman and Guzman (1992) report a theme-based program similar to Cheche Konnen for teaching science to LEP students. The primary emphasis of this program is illustrated through its assumption that instruction in science and the English language can coexist effectively without placing excessive emphasis on the students' first language. However, unlike Cheche Konnen, the curricular content of this program is predetermined by the instructors. Thematic lessons are used in this approach to scientific inquiry and discovery to give students the opportunity to become accustomed to the vocabulary and syntax associated with one concept. These have been developed for both the elementary and secondary curriculum and include preparation and materials for teachers, objectives, hands-on activities for students, and questions that students may ask.

The study of transformations provided an excellent opportunity for a successful yet challenging instructional environment in mathematics for all students regardless of their proficiency in the English language. The idea of including transformations in the school curriculum is not new. Throughout the recent history of mathematics education, influential reform groups have devoted chapters and entire textbooks to transformations. Organizations and individuals having promoted transformations include the University of Illinois Committee on School Mathematics in 1952, the School Mathematics Study Group in 1958, the Commission on Mathematics of the College Entrance Examination Board in 1959, the Cambridge Conference on School Mathematics in 1963, Arthur Coxford and Zalman Usiskin in 1971, and the research program Concepts in Secondary Mathematics and Science in 1981. Dissertations have also been published that look at the teaching and learning of the rigid motion transformations. These dissertations include publication dates of 1972 through 1991.

Several studies have been conducted involving geometric transformations over the past twenty years. Some studies have involved extensive instruction in performing transformations (Edwards, 1991; Ernest, 1986; Johnson-Gentile, 1990; Pleet, 1990; Williford, 1972). However, most studies have been characterized by a brief introduction to transformations, if any at all, before testing or interviewing the students on their ability to perform transformations (Edwards & Zazkis, 1993; Hart, 1981; Kidder, 1976; Law, 1991; Moyer, 1978; Schultz & Austin, 1983; Soon, 1989). Five studies have involved computers in the presentation of transformations (Edwards, 1991; Edwards & Zazkis, 1993; Ernest, 1986; Johnson-Gentile, 1990; Pleet, 1990).

As it was this researcher's belief that students' visualization could be improved by working with reflections and rotations in a dynamic environment, studies involving extensive instruction in performing transformations were of particular interest. Additionally, the research focused on the mathematics and instruction appropriate for middle school students; the following studies reflect this focus. Edwards' (1990) dissertation most closely matched the current research as to target age, instructional environment and experience of the students. However, the students in Edwards' study worked in a computer microworld based on Logo, a computer language the students had previously learned. As with the current research the students worked in pairs to construct a working knowledge of transformations. Edwards reported that the twelve students in grades 6 through 8 who made up this study experienced success in this construction.

Ernest (1986) conducted an experiment to determine the effects of computer gaming on the performance of fifteen-year-old students in transformational geometry. His study differed from Edwards' in that his students did not need to learn a computer language in order to work with the computers; in addition, he used comparative statistics rather than qualitative methods to evaluate students' learning. The treatment group played transformation computer games whereas the control group played computer games having no relation to transformations. While the experimental group performed significantly better on the transformations specifically related to the game, there was no significant difference in performance on the general test.

Pleet (1990) compared the use of the Motions computer program to the use of the Mira hands-on manipulative on eighth grade students' ability to perform transformations and mental rotations. There was no significant difference between the Motions computer program group and the Mira hands-on manipulative group on either acquisition of transformation geometry

concepts or mental rotation ability. Johnson-Gentile (1990) also investigated the effects of computer and non-computer environments on students' achievement with transformation geometry. The researcher examined the effects of a Logo-computer version and a non-computer version of a "motions" unit. The non-computer groups worked with paper and pencil, transparencies, and the Mira while the Logo groups worked with Logo on the computer. The control group did not participate in any "motions" activities. A pretest of achievement in geometry was given to all groups in the Fall of 1988; the study was conducted during the spring of 1989. The motions unit lasted two weeks. Both treatment groups scored significantly higher than the control group on the posttest and the retention test. There was no significant difference between the Logo and non-computer groups on the posttest but there was a significant difference between the groups on the retention test. The Logo group scored higher on the retention test compared to the posttest and the non-computer group scored lower. There was no significant difference between the Logo and non-computer groups on interview measures but both treatment groups scored significantly higher than the control group on the same measures.

There is lack of agreement in the results of the studies previously reviewed. This lack of agreement suggests a need for more research. According to the Constructivist view of teaching and learning mathematics, a thorough investigation of students' ability to perform transformations must allow for an environment including student opportunities for communication with one another and the teacher as well as time to explore the mathematical concepts so knowledge may be built. The appropriate technology and a well organized instructional strategy will help to create this environment. Edwards' (1991) study, which was conducted at a small private school, included the components for the necessary environment; however, her microworld necessitated facility with Logo. It is unrealistic to assume that the already overburdened and under-funded public schools will develop curriculum that will be altered to include a computer programming course in the middle school before students can learn to perform transformations.

Ernest (1986) incorporated technology into his instruction through computer games related to transformations. The use of the computer games did not require knowledge or experience with programming. While the students had access to computers, they were not allowed sufficient time to explore transformations through the use of this technology. The students were exposed to the software for two half-hour sessions. Perhaps this brief duration

of time explains the successful performance of the experimental group only on transformations directly related to those experienced in the computer game.

Pleet (1990) did not combine the hands-on manipulative with the computer. He did, however, employ explanation, discussion, and activity (with either Motions or Mira) in both treatment groups. Pleet recommended that teachers who would be teaching transformations using either a computer graphics program or hands-on manipulatives receive sufficient training in both use of and teaching strategies for the appropriate program or manipulative. Johnson-Gentile (1990) combined hands-on manipulatives with the Logo computer program for one group and hands-on manipulatives without Logo for a second group. The results of the study showed no difference between the two groups for the short term but a significant difference favoring the Logo group for long-term retention. It should be noted, however, that the Logo group had been using a different curriculum (Clements/Battista curriculum) previous to the beginning of the study and after the administration of the pretest. The pretest was given in the Fall of 1988 and the study began in the Spring of 1989. The groups may not have been equivalent with respect to geometry ability at the start of the study.

The Geometer's Sketchpad (Jackiw, 1991), first published in 1991 by Key Curriculum Press, is a relatively new and not yet sufficiently exploited Geometry drawing program for the Macintosh computer. The *Sketchpad* is a highly visual and dynamic tool for exploring and discovering geometric properties. Based on the lack of research using the *Sketchpad*, it may be that its potential is not being realized in most middle schools. This study attempted to establish the *Sketchpad's* effectiveness and accessibility as a basis for a strategy to teach LEP students within an English dominated classroom to successfully recognize and perform rotations and reflections while simultaneously improving students' visualization.

Researchers have experienced success, illustrating that middle school students' visualization can actually be improved; however, controversy has surrounded the discussion of whether or not visualization is related to achievement in mathematics. Visualization studies tend to be involved with either the interaction between students' spatial ability and performance in specified areas of mathematics (Battista, Wheatley, & Talsma, 1982; Connor & Serbin, 1985; Ferrini-Mundy, 1987; Kiser, 1990; Moses, 1977; Perunko, 1982; Tillotson, 1984) or students' trainability with visualization (Baker, 1990; Battista, Wheatley, & Talsma, 1982; Ben-Chaim, Lappan, & Houang, 1988, 1989; Brinkmann, 1966; Connor, Schackman, & Serbin, 1978; Connor

& Serbin, 1985; Ferrini-Mundy, 1987; Lord, 1985; Mendicino, 1958; Miller & Miller, 1977; Moses, 1977; Pallrand & Seeber, 1984; Smith & Schroeder, 1979; Tillotson, 1984).

The relationship between spatial ability and mathematical problem-solving performance, as well as the effect of instruction in perceptual tasks on spatial ability, was explored by Moses (1977) using tests including those which were incorporated into the current research. Moses found that the Card Rotation Test and the Punched Holes Test were good measures of visualization ability for fifth grade students. Moses found a positive correlation between spatial ability and mathematical problem-solving performance. Training in perceptual tasks positively affected spatial ability but not mathematical problem-solving performance. It is interesting to note, however, that the experimental group experienced gains in spatial mathematical problems whereas the control group did not experience such gains.

As with the current research, Tillotson (1984) and Connor and Serbin (1985) were interested in spatial visualization ability as both a predictor of success in mathematics and as a trainable skill. Tillotson worked with sixth-grade students and Connor and Serbin worked with seventh- and tenth-grade students in a series of three studies. All used the Card Rotation Test and the Punched Holes Test among other instruments to measure spatial ability. The specific purpose of Tillotson's research was to determine if spatial visualization ability was a predictor of problem-solving performance, and if training in spatial visualization would improve spatial visualization ability as well as problem-solving performance.

Based on the collected data, Tillotson concluded that spatial visualization is a trainable skill and a predictor of problem-solving performance. However, instruction in visualization skills did not affect high visualization students differently than low visualization students.

After testing seventh- and tenth-grade students with a wide variety of both spatial orientation and visualization instruments and measures of mathematical achievement in two separate studies, Connor and Serbin (1985) concluded that spatial ability is a predictor of mathematical achievement for both boys and girls.

Ben-Chaim, Lappan, and Houang (1988, 1989) conducted a study with sixth-, seventh-, and eighth-grade students that was motivated by the researchers observation that middle school students tended to have difficulty communicating information about three-dimensional solids. Building

description tasks were created "...to determine whether students' preference for representation mode and rate of success on the task would be affected by instruction in spatial visualization activities" (Ben-Chaim, Lappan, & Houang, 1989, p. 123). This study supported the claim that training can improve spatial visualization skills (Ben-Chaim, Lappan, & Houang, 1988). "It should be noted that a significant positive training effect was also evident in students' performance on the Middle Grades Mathematics Project Spatial Visualization Test. In addition, a persistence over time of the effects was demonstrated" (Ben-Chaim, Lappan, & Houang, 1989, p. 142).

As with the previous study, Brinkmann (1966) improved eighth-grade students' spatial visualization through training. The training in Brinkmann's study was in the form of programmed instruction in elementary geometry including topics about points, lines, line segments, rays, angles, planes, plane figures, and solids. The treatment group followed programmed instruction during mathematics class for three weeks. The control group received no training. The treatment group performed significantly better than the control group on a geometry inventory based on the instruction as well as on spatial visualization.

Further proof of students' trainability with respect to visualization skills was also exemplified in Iben's (1988) comparative study between seventh- and eighth-grade students in Japan and the United States. Then investigated students' development of space relations and found that Japanese students had significantly more developed space relations when compared to U.S. students of the same grade. Four explanations were offered for this observed difference. Japanese students have spent more time in school when they are in seventh or eighth grade than U.S. students of the same grade, approximately 2.4 school years in U.S. equivalence. The Socratic method is used to teach Japanese students whereas students in the U. S. are primarily taught through direct instruction. Students in the U.S. experience computational space relations activities while Japanese students experience hands-on space relations activities. And, space relations is a large part of the Japanese first-grade curriculum through paper folding activities. Students in the United States do not tend to be exposed to space relations activities until seventh or eighth grade.

Based on Iben's observations, Japanese students' superior performance on space relations activities compared to the performance on the same activities by U. S. students may be due in part to the substantially greater amount of time spent in such training as well as the type of training received.

As with the Second International Mathematics Study (Crosswhite, 1985), students in the United States tend to experience much less hands-on activity in the classroom than students in other countries.

Methodology

The study consisted of nine classes of eighth-grade students divided into treatment and control groups. A three-factor, nonequivalent control-group design was used for the study. This quasi-experimental research design involved a 2X2X3 matrix to examine three factors. The three factors were the level of English proficiency, the level of computer use, and the level of visualization of the students. There were two groups, one Treatment Group and one Control Group, with four classes in the Treatment Group and five classes in the Control Group.

The measures for the study consisted of three covariates and four posttests. The covariates consisted of the Paper Folding Test and the Card Rotation Test of the Kit of Factor-Referenced Cognitive Tests (Ekstrom, *et al.*, 1976) as well as the Language Assessment Battery (LAB). Two posttests designed by the researcher were versions of a Rotation/Reflection Instrument. The Card Rotation and Paper Folding Tests were also used as posttests. The Card Rotation Test, S-1 of the Spatial Orientation Factor and the Paper Folding Test, VZ-2 of the Visualization Factor were used to measure students' visualization. The Card Rotation Test required that the student differentiate between figures that are equivalent in every way but orientation from cards that have been "flipped." The Paper Folding Test required that the student imagine a piece of paper being folded and having holes punched through all thicknesses according to drawings. The student then must choose the appropriate result once the imagined paper has been unfolded. Both tests contain two parts and each part has a time limit of three minutes. Each of the tests call for spatial orientation and visualization skills required to perform rotations and reflections. The Card Rotation Test represents two-dimensional motion and the Paper Folding Test represents three-dimensional motion.

The Reflection/Rotation Instruments (a paper version and a computer version) were designed by the researcher, administered during a pilot study, and content-validated by four experts, two in the field of mathematics education, one in the field of mathematics, and one in the field of educational technology. The instruments were adapted from the exam designed for a large

scale British study which was part of Concepts in Secondary Mathematics and Science (Hart, 1981).

The computer and paper versions of the Reflection/Rotation Instrument are similar in content and difficulty; however the computer version was designed to be taken on the computer and was dynamic in nature. The dynamic aspect of the computer version involved motion having to do with that which is inherent in the rigid motion transformations. The objectives of both versions of the Reflection/Rotation Instruments are given in Table 1. A description of the motion for the computer version of the Reflection/Rotation Instrument is given in Table 2.

Table 1 <i>Objectives of Reflection/Rotation Instruments</i>	
Items	Objective: Test the student's ability to...
1-4	recognize examples and nonexamples of reflections.
5-8	perform reflections given a figure, a mirror line, and background grid.
9-12	perform reflections given a figure and a mirror line.
13-16	recognize examples and nonexamples of reflections when the mirror line is hidden and draw mirror lines where appropriate.
17-20	recognize examples and nonexamples of rotations.
21-24	perform rotations given a figure, a center point, and a reference circle.
25-32	perform rotations given a figure and a center point.
33-36	draw the center of rotation given a figure and its image.
37-40	determine the correct composition of transformations given the figures and their images without mirror lines or center points.

A pilot study was conducted in order to establish the reliability of the previously described instruments and verify the logistics of the treatment.

All classes were taught how to use the *Geometer's Sketchpad* at the beginning of the study prior to the collection of the data. The training lessons were based on angle measure and distance and taught by the researcher. At no time during the training did the students witness or perform reflections or rotations using *The Geometer's Sketchpad*.

Items	Description of motion.
1-4	The figure moves from the pre-image to the image following the properties of a reflection when the example portrays a reflection.
5-12	The figure moves from the pre-image to the mirror line then disappears. The movement follows the properties of a reflection.
13-16	The figure moves from the pre-image to the image following the properties of a reflection when the example portrays a reflection.
17-20	The figure moves from the pre-image to the image following the properties of a rotation. The center of rotation may or may not be correctly drawn each example.
21-32	The figure moves from the pre-image approximately 200 in the direction of the given reflection then disappears. The movement follows the properties of a rotation.
33-36	The figure moves from the pre-image to the image following the properties of a rotation. The center point is hidden in each example.
37-40	The figure moves from pre-image to image to image after composition following the properties of rotations and reflections as appropriate. All mirror lines and center points are hidden.

Following the training of the students in the treatment and control groups on use of *The Geometer's Sketchpad*, all classes were given the Card Rotation Test and the Paper Folding Test. Upon completion of the above measures, the control group was presented content on reflections and rotations using the traditional teacher directed, textbook approach. The classroom teachers for the control group taught their respective classes. All treatment lessons and problems were pre-planned so that any eighth-grade mathematics teacher with substantial knowledge of and experience with use of *The Geometer's Sketchpad* could teach the lessons. The researcher taught the lessons due to her experience teaching in computer labs and her experience using *The Geometer's Sketchpad*. The researcher assumed the role of facilitator and problem poser for the treatment group. Students in the Treatment Group worked in pairs at computers using *The Geometer's Sketchpad*.

The reflection and rotation units for the treatment and control groups each lasted approximately two weeks. The paper and computer versions of the Reflection/Rotation Instrument and the Card Rotation and Paper Folding Tests were administered at the end of the unit.

Students in the treatment classes spent approximately two-and-one-half weeks learning how to use *The Geometer's Sketchpad* and using the computer software to conjecture about and construct knowledge of reflections and rotations. Since the students' middle school followed block scheduling, the eighth- grade students spent approximately two consecutive hours in mathematics class every other school day. Mathematics class was held in the computer lab throughout the unit on reflections and rotations.

As there were fifteen computers and from twenty-eight to thirty-two students in each mathematics class, the students mainly worked in pairs at the computers. For the most part, students chose their own partners. Several pairs consisted of students whose first language was Spanish. These students tended to converse in their first language when discussing problems posed by the instructor as well as the properties of the geometric transformations.

The reflection/rotation unit consisted of lessons that required students to conjecture about the location and orientation of the images of objects after the objects had been reflected over a given mirror line or rotated a specified angle about a given center of rotation. The students received immediate feedback about their conjectures by testing their hypotheses using *The Geometer's Sketchpad*. Originally all problems were posed by the instructor; eventually the students were directed to pose problems to each other and ultimately to themselves.

After one week elapsed, the students were assigned a project that was to become part of the unit grade. A major goal of the unit was to have the students explore and discover the properties of reflections and rotations. The students had been exploring the properties through the aforementioned activities.

The students, working in pairs, spent approximately four hours (two blocks) in two days on the project. They had at most four screens (on *The Geometer's Sketchpad*) to demonstrate properties of reflections and rotations. The students were required to use drawings, color, at least one circle, measurement, and written explanation (in Spanish or English) to illustrate the properties they had discovered. The researcher acted as facilitator, answering questions generated by the students dealing with use of the software as well as

the geometric transformations. The researcher did not, however, supply the properties.

The students presented their projects to the class via a Power Macintosh linked to a classroom-size television monitor. Students asked questions about the demonstrations when they were unclear about or in disagreement with the illustrated properties.

An analysis of covariance (ANCOVA) was used to control for initial differences between groups. Two tests from the Kit of Factor-Referenced Cognitive Tests Ekstrom *et al.*, 1976) and the Language Assessment Battery (LAB) served as covariates. The Paper Folding Test was the discontinuous measure used to assign students to visualization level. The Card Rotation Test was the continuous covariate used to control for initial differences between groups. The objective of the design was to determine the effects of the independent variables (English proficiency, computer use, and visualization level) individually and interactively, on the dependent variables posttest scores). Of interest was performance on two dependent variables, the computer version of the posttest and the paper version of the posttest as well as the performance of the sample and a subsample consisting of the LEP students in each group. Two separate ANCOVAs were run to determine the effects of the independent variables on each of the dependent variables.

The first ANCOVA described the effect of the independent variables individually and interactively on the performance of the sample on the paper version of the posttest. The second ANCOVA described the effect of the independent variables individually and interactively on the performance of the sample on the computer version of the posttest. An additional hypothesis was tested using a 2)(2 matrix to examine the effects of English proficiency and a dynamic instructional environment on students' visualization. The last two ANCOVAs were run with the purpose of describing the effect of the two independent variables on the performance of the sample on the Card Rotation Test and the Paper Folding Test.

Results

This study investigated the effects of a dynamic instructional environment and students' visualization level on LEP students' identification and performance of reflections and rotations and the effects of the dynamic instructional environment on the relationship between the students' level of

visualization and identification and performance of reflections and rotations. A second purpose of this study was to examine the effects of the dynamic instructional environment on performance of LEP students on measures involving transformation geometry and visualization compared to their English proficient (EP) peers. Third, the study was designed to ascertain the effects of a dynamic instructional environment on LEP students' visualization. This research attempted to show that there can exist an environment in which EP and LEP students can work together with computers to construct knowledge of reflections and rotations through inquiry and collaboration.

After controlling for initial differences through the use of the Card Rotation Test and the Paper Folding Test (determiner of visualization level) and distinguishing between LEP and EP students by way of the Language Assessment Battery (LAB), the researcher was able to deduce from the data that there were no statistically significant interactions between the students' English proficiency and the treatment (Paper Instrument: $F=.23$, $df=1,218$, $p=.63$; Computer Instrument: $F=.9$, $df=1,212$, $p=.34$) or between the students' level of visualization and the treatment (Paper Instrument: $F=1.88$, $df=2,218$, $p=.16$; Computer Instrument: $F=.55$, $df=2,218$, $p=.58$) on performance on either the paper or computer instruments. The relationship between English proficiency and performance on the instruments was the same regardless of the students' instructional environment. Students experiencing the dynamic instructional environment did not benefit more than expected whether they were EP or LEP. Finding no significant interaction, the interaction terms were removed from the model. After removal of the interaction terms, it was determined that LEP students did not perform statistically significantly differently on either the Paper Reflection/Rotation Instrument ($F=1.09$, $df=1,221$, $p=.3$) or the Computer Reflection/Rotation Instrument ($F=.88$, $df=1,215$, $p=.35$) when compared to their EP peers. Furthermore both the LEP and EP students performed significantly better on the dependent measures after experiencing the dynamic instructional environment than their LEP and EP peers who experienced the traditional textbook approach Paper Instrument: $F=67.55$, $df=1,221$, $p=.00$; Computer Instrument: $F=47.28$, $df=2,215$, $p=.00$). Visualization level was also a statistically significant predictor of LEP and EP students' performance on the dependent measures (Paper Instrument: $F=10.16$, $df=2,221$, $p=.00$; Computer Instrument: $F=14.05$, $df=2,215$, $p=.00$) with students having high visualization levels performing better than students having medium

visualization levels. Students having medium visualization levels performed better than students having low visualization levels.

After controlling for initial differences through pretest scores, it was determined that, once again, LEP students did not perform statistically significantly differently than their EP peers on the dependent measures. The dependent measures under investigation were the Card Rotation Test ($F=.7$, $df=1,227$, $p=.4$) and the Paper Folding Test ($F=.02$, $df=1,224$, $p=.88$). Furthermore, LEP and EP students receiving the dynamic treatment performed significantly better on the Card Rotation Test than their LEP and EP peers who did not receive the dynamic treatment ($F=5.81$, $df=1,227$, $p=.02$). There was not a significant difference between the treatment and control groups on the Paper Folding Test ($F=2.4$, $df=1,224$, $p=.12$).

Discussion and Implications of the Results

This study has demonstrated that limited English proficient students can succeed alongside their English proficient peers at learning important mathematics as described in the *Standards* (NCTM, 1989, 1991). Furthermore, it has shown that LEP students can learn to conjecture and test those conjectures in a Constructivist teaching and learning environment while simultaneously learning new mathematical vocabulary through context-embedded situations. The answers to the questions addressed in this study lead this researcher to more questions. Since students are able to increase their visualization through experiences such as those described in this study and since visualization level is a significant predictor of LEP students' performance on measures of transformation geometry, would increasing LEP students' level of visualization help narrow the achievement gap between LEP students and their EP peers in mathematics? Since visualization level is positively correlated with success in certain types of mathematics and two-dimensional visualization skills can be improved, are there educational experiences that may enhance LEP students' visualization involving three-dimensional manipulation? Since there was not a significant difference between LEP and EP students on measures of performance with reflections and rotations or visualization when experiencing the same instructional environment, does it follow that LEP students should always coexist in mathematics classes with their EP peers?

Mathematics educators need to ask themselves whether or not they are meeting the needs of LEP students in their classes by providing environments in which students are furnished with comprehensible input to situations involving important mathematics. This research has demonstrated that a computer-based, dynamic instructional environment can provide for successful outcomes in LEP students' learning of reflections and rotations, a small part of geometry and an even smaller portion of middle school mathematics. It remains to be shown that similar instructional environments can afford LEP students the opportunity to construct other areas of mathematics described in the *Standards*.

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