An Analytical Model for Oceanic Whitecap Coverage

NORDEN E. HUANG

NASA Goddard Space Flight Center, Greenbelt, MD 20771

LARRY F. BLIVEN AND STEVEN R. LONG

NASA Goddard Space Flight Center, Wallops Flight Facility, Wallops Island, VA 23337

CHI-CHAO TUNG

North Carolina State University, Raleigh, NC 27695

(Manuscript received 12 November 1985, in final form 11 February 1986)

ABSTRACT

Using a threshold criterion governing the onset of wave breaking, we derived an analytical expression for the whitecap coverage of the ocean. This expression is a function of the wave steepness in terms of the significant slope and the ratio of the frictional velocity of the wind to the phase velocity of the energy-containing waves. Theoretically, this analytical expression works only for the narrowband wave field. However, the comparison with the field data of Snyder et al. suggests that the present model could be applied to fresh wind-wave fields. Since the present approach is based on the probability density function of wave breaking with or without wind stress, it is believed that this analytical expression will offer a more reliable answer than the traditional empirical formula.

1. Introduction

Oceanic whitecapping is a consequence of wave breaking and occurs when a patch of white water, which is the turbulent air–water mixture at the crest, runs down the forward face of the wave (e.g., see Phillips, 1977; Cokelet, 1977). This event occurs only at the crest of the larger waves, where higher local energy density is available to produce such a violent motion. Soon after breaking and releasing the excessive energy, the wave crest will become rounded, and the local slope will reduce. This change of the wave crest geometry will allow part of the white foamy water to be swept by the crest and degenerate into streaks of foam. Because the formation of whitecaps is a consequence of the breaking, the existence or absence of the whitecaps is a good indicator of the sea state. For example, whitecaps appear mostly in choppy fresh sea rather than in swell-dominated conditions. The extent of whitecap coverage has also been traditionally used by mariners as a measure of the local wind strength, because fresh waves are all generated by the local wind. This tradition has even been carried over into the space age, for one of the designated means of oceanic surface wind measurement is by the passive microwave radiometer, a device that senses the brightness temperature change of the ocean surface due to whitecap and foam coverage (e.g., see Lipes, 1982).

The relationship between whitecap coverage and the local wind has been derived empirically through hundreds of years of shipboard observations. Most of these observations, however, cannot be classified as scientifically rigorous. Recently, due to the need for remote sensing, there have been renewed efforts to establish such a relationship on a firmer scientific ground. Numerous trials have been reported in various articles, with the most complete one being represented by the result of Monahan and O'Muircheartaigh (1980), who concluded that

$$W = 2.95 \times 10^{-6} U^{3.52} \tag{1.1}$$

or

$$W = 3.84 \times 10^{-6} U^{3.41}, \tag{1.2}$$

in which U is the wind speed in meters per second, and W is the simple fraction of the sea surface covered by whitecaps. The expression given in (1.1) was derived using the ordinary least-square fitting, while (1.2) was derived using the robust biweighted fitting of the same dataset. While there is no denying that (1.1), (1.2), or any other similar empirical formula could probably give a reasonable approximate extent of the whitecap coverage under a given local wind condition, difficulties still exist.

Strictly speaking, the occurrence of whitecaps is a consequence of wave breaking, and any attempt to describe it should at least be sea-state dependent. To put this statement in another way, we should ask: Is the local wind the only mechanism to cause wave breaking? If it is, then what is the condition of wind to produce a local breaking in a deterministic case? The answers to both these questions can be shown to be negative. This implies that the empirical formulae relating whitecap and wind lack generality and can only be indirect. This is true because even in the open ocean, wave breaking will be influenced by other mechanisms that affect the local wave energy density, such as wave current interactions at the strong current boundaries (Kenyon, 1971; Phillips, 1977), sea surface boundary stability, and the preexisting sea state (Phillips and Banner, 1974). Under special conditions, the wind can not even be considered as the primary cause of whitecapping (e.g., see Valenzuela, et al., 1983)

Since whitecapping is produced by wave breaking, it is only logical to seek the relationship between the whitecap coverage and the criteria of wave breaking. In fact, recent studies by Kennedy and Snyder (1983) and Ochi and Tsai (1983) have clearly established the relationship between wave breaking and the wave field probability characteristics. Kennedy and Snyder (1983) used an acceleration threshold criteria for wave breaking to relate the observed whitecap coverage in the field to wave parameters with a high degree of success. Ochi and Tsai (1983) adopted an amplitude criteria that was applied to the amplitude period joint-probability distribution. When the method is applied to some laboratory data, the results are also highly successful. Indeed, it can be shown that the methods adopted by Kennedy and Snyder, and Ochi and Tsai are equivalent; they produced similar results, but also suffered the same drawback in that both models of breaking are based on the Gaussian statistics of the wave field.

Since whitecaps are produced by wave breaking at the crest of the larger waves in fresh choppy seas, these waves are likely to be steep and nonlinear, hence their statistical properties are also more likely to be non-Gaussian (Huang and Long, 1980; Huang et al., 1983, 1984). In this paper, we report an analytical model for the whitecap coverage. The approach is based on a threshold criterion applied to a statistical distribution of the wave field originally proposed by Longuet-Higgins (1969). The breaking criterion used here is based on the one modified by Banner and Phillips (1974) to include the effect of the surface drift current. More importantly, the statistical model of the wave field is non-Gaussian. The final result is compared with the field data of Kennedy and Snyder (1983), and the agreement is encouraging.

2. The statistical model

The statistical model of breaking waves proposed by Longuet-Higgins (1969) was based on the classical theory of waves. For a single wave train, breaking will occur at the crest when the local acceleration at the peak reaches $-\frac{1}{2}g$. At this point, the limiting amplitude, $a_{\max}$, is given by

$$a_{\max}\omega^2 = \frac{1}{2}g, \tag{2.1a}$$

or

$$a_{\max} = g/2\omega^2, \tag{2.1b}$$

in which ω is the wave frequency, and g the gravitational acceleration. Equations (2.1a) and (2.1b) provided a link between the amplitude criterion and the acceleration criterion for a narrowband wave field. For a random wave field, Longuet-Higgins (1969) proposed a characteristic maximum amplitude, a_c, as

$$a_c = g/2\overline{\omega^2} \tag{2.2}$$

in which $\overline{\omega^2}$ is the characteristic frequency defined as

$$\overline{\omega^2}\int_\omega \phi(\omega)d\omega = \int_\omega \omega^2\phi(\omega)d\omega, \tag{2.3}$$

where $\phi(\omega)$ is the wave energy spectrum in terms of frequency. For any wave in the wave field with an amplitude greater than a_c, breaking will occur; therefore, the probability of breaking is given by

$$\int_{a_c}^{\infty} P(a)da, \tag{2.4}$$

where $P(a)$ is the probability density function for the amplitude which was taken to be a Rayleigh distribution by Longuet-Higgins (1969).

The Rayleigh distribution has been shown to be a very accurate model for a linear wave field by Longuet-Higgins (1952) and Cartwright and Longuet-Higgins (1956). Even for a strongly nonlinear wave field, it still governs the amplitude distribution to the second order of approximation as shown by Tayfun (1980). However, the justification for using a Rayleigh distribution in the wave breaking computation is weak. In a strongly nonlinear wave field, the distributions of the crest and trough are different due to the harmonic distortion. The fact that breaking occurs only at the crest of large waves offers a perfect illustration of the asymmetric nature of the breaking events along the wave profile. Based on this observation, we propose a slight modification to the existing model by applying the breaking criterion to Eq. (2.4) with the probability density function substituted by one that is for the crest amplitude only.

In this section, we will first derive the probability density function of the crest and trough amplitudes. In doing so, we will only consider the strongly nonlinear effect in the sense as discussed by Schwartz and Fenton (1982). Under this consideration, for a narrowband wave field, the ocean surface can be represented by

$$\zeta = a\cos\chi + a^2k\cos^2\chi, \tag{2.5}$$

where the amplitude, a, is Rayleigh distributed; the phase function, χ, uniformly distributed; and the wavenumber indicated by k. This model has been used successfully by Tayfun (1980) and Huang et al. (1983, 1984) to study various non-Gaussian statistical properties of the nonlinear wave field.

According to this wave model, the crest and trough amplitudes are given by

$$\eta = a \pm a^2 k. \tag{2.6}$$

Even though the amplitude a is still Rayleigh distributed, η is not Rayleigh anymore. If we normalize η with respect to $(\overline{a^2})^{1/2}$, we have

$$y = \frac{\eta}{(\overline{a^2})^{1/2}} = \frac{a}{(\overline{a^2})^{1/2}} \pm \frac{a^2}{(\overline{a^2})^{1/2}} k. \tag{2.7}$$

If we define

$$a/(\overline{a^2})^{1/2} = x, \tag{2.8}$$

then the density for x is

$$P(x) = 2xe^{-x^2}, \tag{2.9}$$

according to Longuet-Higgins (1952).

In terms of this variable, the crest amplitude can be written as

$$y = x + x^2\sqrt{2}\sigma k, \tag{2.10}$$

where $\sigma = (\overline{a^2})^{1/2}/\sqrt{2}$ is the standard deviation of the surface displacement to the third order. Therefore, k is a measure of the surface steepness of the wave field, which can be related to the significant slope, §, defined by Huang and Long (1980), and Huang et al. (1981), so that

$$2\pi\S = \sigma k.$$

From (2.10), we can solve x in terms of y to get

$$x = \frac{-1 + (1 + 4\sqrt{2}\sigma k y)^{1/2}}{2\sqrt{2}\sigma k} \tag{2.11}$$

for the crest amplitude. Then by the mapping theorem (e.g., see Papoulis, 1984), we have

$$P(y) = \frac{(1 + 4\sqrt{2}\sigma k y)^{1/2} - 1}{\sqrt{2}\sigma k(1 + 4\sqrt{2}\sigma k y)} \times \exp\left[-\left(\frac{(1 + 4\sqrt{2}\sigma k y)^{1/2} - 1}{2\sqrt{2}\sigma k}\right)^2\right] \tag{2.12}$$

for the probability density function of the crest amplitude. The corresponding density function for the trough is

$$P(y) = \frac{(1 - 4\sqrt{2}\sigma k \tilde{y})^{1/2} - 1}{\sqrt{2}\sigma k(1 - 4\sqrt{2}\sigma k \tilde{y})^{1/2}} \times \exp\left[-\left(\frac{(1 - 4\sqrt{2}\sigma k \tilde{y}) - 1}{2\sqrt{2}\sigma k}\right)^2\right], \tag{2.13}$$

where $\tilde{y} = x - x^2\sqrt{2}\sigma k$ is the trough amplitude.

In using this statistical model, there is one difficulty in the trough density function as given in Eq. (2.13), i.e., when

$$1 - 4\sqrt{2}\sigma k \tilde{y} < 0,$$

we may encounter imaginary numbers for the solution of x. This limits the validity of the equation to a very small range of values. Higher expansion will alleviate this restriction somewhat, but the algebra will become much more involved. A simple way out is to expand the square-root terms to get the asymptotic expression of $P(\tilde{y})$ as

$$P(\tilde{y}) = 2(\tilde{y} - 3\sqrt{2}\sigma k \tilde{y}^2) \exp[-(\tilde{y} - \sqrt{2}\sigma k \tilde{y}^2)^2]. \tag{2.14}$$

The use of (2.14) should not offer better accuracy but simply circumvent the difficulty of having to deal with the imaginary numbers in the density function. Since we will not need the probability density function of the trough in our computations, we will not discuss it any further.

Having derived the analytic forms of the statistical model, we can make comparisons with some experimental data. The data used here are the same set as reported in Huang et al. (1984) but processed differently to satisfy the specific needs in the present cases. Distributions of crest and trough amplitudes are based on the distance from the mean water level to the highest and lowest points between two adjacent upgoing zero-crossing points, and the wave amplitudes are based on the mean of crest and trough amplitudes. Two sets of typical density curves are presented in Figs. 1 and 2. In each set of figures, (a) represents the wave amplitude, (b) the crest amplitude, and (c) the trough amplitude. As we can see from the figures, the Rayleigh distribution fits the wave amplitude distribution very well, irrespective of the variations of the σk values. This is in agreement with the conclusion of Longuet-Higgins (1952, 1980). The distributions for the crest and trough amplitudes, however, are quite different. The crest (trough) amplitude becomes increasingly broad (narrow) as the σk value increases. This is clearly the consequence of harmonic distortion, which produces sharp crests and rounded-off troughs in the wave profile. The nonzero density value near the zero amplitude is caused by high-frequency noise in the data. But even with the noise, the overall agreement between the data and the model is remarkably good. This is especially true for the tails of the crest amplitude, which are also the crucial point in the breaking probability computation.

Since wave breaking occurs only at the crest, the breaking probability should be defined as

$$Q = \int_{y_c}^{\infty} P(y)dy \tag{2.15}$$

where $P(y)$ is given in Eq. (2.12), and the critical value y_c is to be defined in the next section.

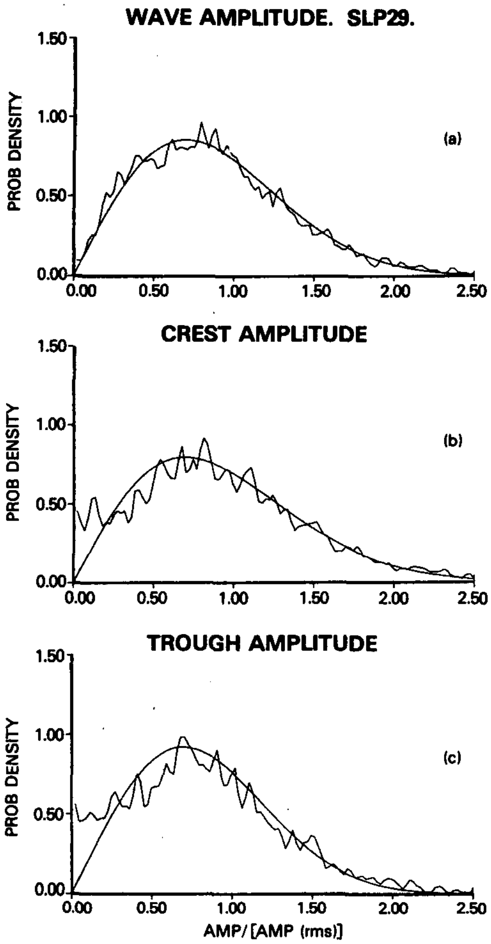

FIG. 1. Probability density functions of (a) wave amplitude, (b) crest amplitude, and (c) trough amplitude for $\sigma k = 0.0377$.

3. The breaking criterion

In order to establish the breaking criterion, there are several issues to be settled.

First, we have to determine the maximum amplitude for a free gravity wave. According to the classical theory, a surface gravity wave will break when the acceleration reaches $-\frac{1}{2}g$. Laboratory experimental studies have consistently shown that the upper limit is somewhat lower than this theoretically established value. For example, Danel (1952) and Ramberg et al. (1985) gave the same equivalent critical value of $-0.41g$, while Ochi and Tsai (1983) found the critical value to be $-0.39g$. Furthermore, Ramberg et al. concluded from their laboratory experiment that the onset of deep-water wave breaking is determined by the criterion of local wave steepness, and it is independent of the way the waves reached such a state. If we convert the steepness criterion to acceleration, then the statement by Ramberg et al. seems to be confirmed by the field results, too. For the field study, Snyder et al. (1983) found the critical value to lie between $-0.3g$ and $-0.5g$ statistically, even for wind waves. Based on these data, we decided to adopt a critical value of $-0.4g$ as the breaking criterion. To accommodate the possible variation and to preserve generality, we decided to introduce a coefficient, α, for the time being. Using this assumption, one can easily show that the characteristic maximum amplitude, a_c, given in Eq. (2.2) can be expressed as

$$a_c = g/2\alpha\overline{\omega^2}. \tag{3.1}$$

When $\alpha = 1$, we have the classical result; when $\alpha = 1.25$, we have the result adopted for this study, which is in the closest agreement with the recent observations.

Second, we must realize that the breaking of wind waves is a little different from the breaking of free waves. According to laboratory observations (e.g., see Stoker, 1957), waves will break at a gentler steepness under the influence of the wind. Banner and Phillips (1974) have successfully modeled this effect and found the wind-induced drift current to have a strong influence on the incipient breaking. According to their model, the maximum amplitude will be modified by a factor

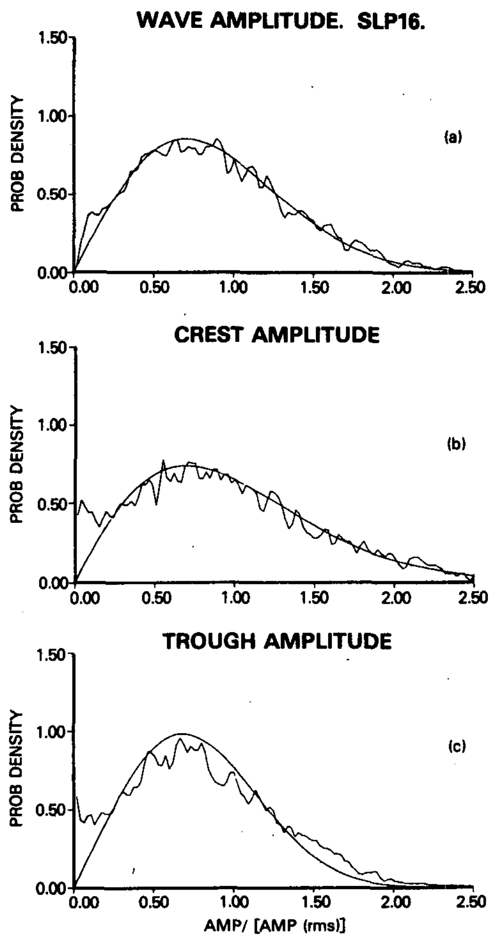

FIG. 2. As in Fig. 1 but with $\sigma k = 0.0829$.

$$\left(1-\frac{q_0}{c_0}\right)^2 \tag{3.2}$$

where c_0 is the phase velocity of the wave, and q_0 is the drift current, which is usually fixed at one half of the frictional velocity of the wind, u_*. Consequently, for a random wave field, the characteristic maximum crest amplitude will be

$$a_c=\frac{g}{2\alpha\overline{\omega}^2}\left(1-\frac{u_*}{2\overline{c_0}}\right), \tag{3.3}$$

in which $\overline{c_0}$ is the characteristic phase velocity of the wave having the characteristic frequency. The specific expression for $\overline{c_0}$ will be defined after we have defined the characteristic frequency.

Finally, we have to determine the characteristic frequency, which depends on the spectral function adopted. For this study, we have used the simplified Wallops spectrum (Huang and Long, 1980) for its flexibility and simplicity. The spectrum is given by

$$\left.\begin{aligned}\phi(\omega)&=\frac{\beta g^2}{\omega^m\omega_0^{5-m}}, & \omega&\geqslant\omega_0\\ &=0, & \omega&<\omega_0\end{aligned}\right\} \tag{3.4}$$

where $\beta = (m-1)(\sigma k)^2$, and

$$m=\left|\frac{\log(k/\sqrt{2})^2}{\log 2}\right|.$$

After some simple computation, we get

$$\overline{\omega^2}=\left(\frac{m-1}{m-3}\right)\omega_0^2$$

$$\overline{c_0}=\left(\frac{m-3}{m-1}\right)^{1/2}c_0. \tag{3.5}$$

Combining (3.3) and (3.5), we have

$$a_c=\frac{g}{2\alpha\omega_0^2}\left(\frac{m-3}{m-1}\right)\left[1-\frac{u_*}{2c_0}\left(\frac{m-1}{m-3}\right)^{1/2}\right]^2. \tag{3.6}$$

In the normalized variable,

$$y_c=\frac{1}{2\alpha\sqrt{2}\sigma k}\left(\frac{m-3}{m-1}\right)\left[1-\frac{u_*}{2c_0}\left(\frac{m-1}{m-3}\right)^{1/2}\right]^2. \tag{3.7}$$

This is the breaking criterion for the crest amplitude that we will use in the subsequent computations. Although the criterion is in terms of crest amplitude, it is equivalent to the acceleration criterion adopted by Snyder et al. (1983); for, under the narrowband assumption, (3.6) gives exactly the acceleration criterion

$$a_c\omega_0^2=\frac{g}{2\alpha}\left(\frac{m-3}{m-1}\right)\left[1-\frac{u_*}{2c_0}\left(\frac{m-1}{m-3}\right)^{1/2}\right]^2. \tag{3.8}$$

Having arrived at the breaking criterion, we can carry out the integration in Eq. (2.15), with $P(y)$ given in Eq. (2.12), and y_c given in Eq. (3.7). The integration is involved algebraically but is straightforward in principle. The final result of the breaking probability can be expressed simply as

$$Q=\frac{1}{2}e^{-s_c}, \tag{3.9}$$

where

$$s_c=\frac{1}{8\sigma^2k^2}\left\langle\left\{1-\frac{2}{\alpha}\left(\frac{m-3}{m-1}\right)\times\left[1-\frac{u_*}{2c_0}\left(\frac{m-1}{m-3}\right)^{1/2}\right]^2\right\}^{1/2}-1\right\rangle^2.$$

The quantity obtained in Eq. (3.9) is equivalent to the probability of breaking at a point given by Snyder and Kennedy (1983). However, Snyder and Kennedy used the Gaussian statistical model; as a result, there was a considerable difference between the analytic result and the observed data. A comparison between the present result and the data reported by Snyder et al. (1983) is given in Fig. 3. Note that the independent variable is the significant slope with the frictional wind velocity entered as part of a parameter. Three lines of Q values with c_0/u_* as parameter are given in the figure. Because their experimental site was in a semienclosed bay, the waves cannot grow to large amplitude and length. We estimated the c_0/u_* values to be somewhere between 5 and 20. An additional line of $c_0/u_* = \infty$ is also given as a limiting case for the no wind condition. Data from Snyder et al. (1983, Tables 1 and 2) are plotted in the figure. Although there is some scattering of the data, it is no worse than the empirical formula based on the wind speed alone. In this aspect, the agreement is encouraging.

4. Discussion

Wave breaking is an important phenomenon in almost all studies of the dynamics of the ocean surface and the upper layer. Limited by available analytic tools, most studies of breaking waves in a random wave field have been empirical. Essentially, the breaking or the whitecap coverage is related to the local wind speed. Such results may fit special sets of observational data, but they shed no light on the mechanism of breaking and the generation of the whitecaps. The results of the two recent studies of Snyder et al. (1983) and Ochi and Tsai (1983) have pushed forward our understanding of the breaking of random waves considerably. Both of these studies showed the intimate relationship between wave breaking and sea state. Although the study of Ochi and Tsai was confined to laboratory cases free of the surface wind stress, the approach adopted by them was new and seems more reasonable than the simple height criterion for a narrowband sea. They considered the finite bandwidth case and used the joint distribution of wave elevation and period defined by

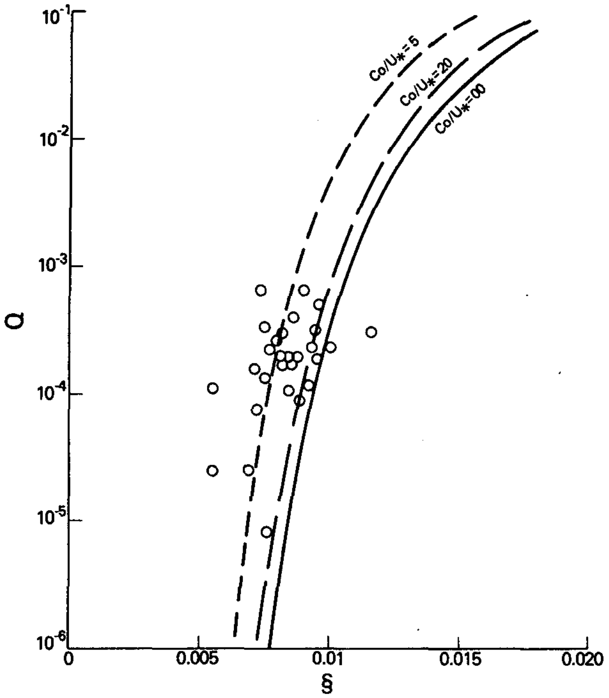

FIG. 3. Comparison between the observed whitecap coverage as reported by Snyder et al. (1983) and the computed values according to Eq. (3.9).

the zero-crossing time. However, the statistical model was still based on the Gaussian assumption. Since breaking occurs only at the crest of the highly nonlinear waves, we feel the nonlinear effect should be emphasized. In the present model, we have incorporated such effects but left out the influences of finite bandwidth and the joint distribution. Our justification is that whitecaps are generated only by the violent breaking of the larger waves; therefore, we can concentrate on the energy-containing range of the wave spectrum, where the narrowband assumption is usually acceptable.

In the study by Snyder et al., field data of wind-generated waves were reported. The importance of their study lies not only on their high quality data collection and analysis but, we feel, more on the fact that they succeeded in establishing a definite relationship between whitecap coverage and the sea state by leaving the wind stress out completely. As such, their study contributed greatly toward the understanding of the mechanism of whitecap formation. Limited by the available analytic tools, they also adopted the Gaussian statistical model for the surface acceleration. Since acceleration is the second derivative of the surface elevation, its spectrum will put an undue emphasis on the high frequency range. Undoubtedly, small waves are more likely to break, but it is the breaking at or near the crests of the "larger" waves that generates the whitecaps. For whitecap coverage computation, the breaking of the smaller waves is of no consequence and its effects are negligible. Because of these considerations, we feel our strategy of adopting the crest amplitude distribution is justifiable. In this amplitude approach, we are dealing only with the energy-containing waves. That part of the spectrum can be adequately approximated by the Wallops spectrum model. We also included the effect of the surface wind drift, which can be significant for low c_0/u_* cases. Such cases are usually represented by the freshly developed or developing stage when whitecap formation is most likely.

Taking everything into consideration, we feel that the present result offers a workable model for whitecap coverage computation. The result is at least as good as the empirical formulae based on wind speed alone. While we do not doubt the correlation between wind speed and whitecap coverage, the premise that whitecap coverage is a function of wind alone is unacceptable. As we stated before, whitecaps are generated by breaking waves, but there are many causes that contribute to wave breaking; effects such as current–wave interaction along the Gulf Stream or at the vicinity of a coastal inlet are equally efficient in causing wave breaking. Even if the wind waves are the only motions present, the nonlinear nature of the wind–wave interaction seems to indicate that the initial conditions or the state of the background waves play a critical role. We do not think wind speed alone can fully parameterize the whole event of wave breaking either explicitly or implicitly. Further studies along the present line of investigation are of course needed. Most importantly, the future efforts should include the finite bandwidth effect, and the joint amplitude–frequency distribution. Additionally, rigorous tests should be carried out to establish whether breaking in the field is correctly modeled by classical theory with the modification by Banner and Phillips (1974). For example, it is known that high wind can shear off the crest of a wave. The significance of such phenomena has not been included, but it should be investigated.

Finally, a brief discussion on the Banner–Phillips breaking mechanism is in order. Shortly after publication of the paper by Banner and Phillips (1974), it was pointed out by Wright (1976) that although the augmentation of wave breaking by surface drift was true, its effect was considerably smaller. Data from a wave tank measured at a fetch of 2.7 m was used by Wright to prove that the wave steepness indeed exceeded the limit set by the Banner–Phillips theory. In order to shed some additional light on this controversy, we used all the data collected in the Wallops Wind–Wave Research Facility to check Wright's claim by plotting in Fig. 4 the theoretical upper limit on the wave slope as a function of the surface drift speed as

$$\frac{2a_c\omega^2}{g} = 2\sqrt{2}k_0\zeta_{\rm rms} = \left(1 - \frac{q_0}{c_0}\right)^2,$$

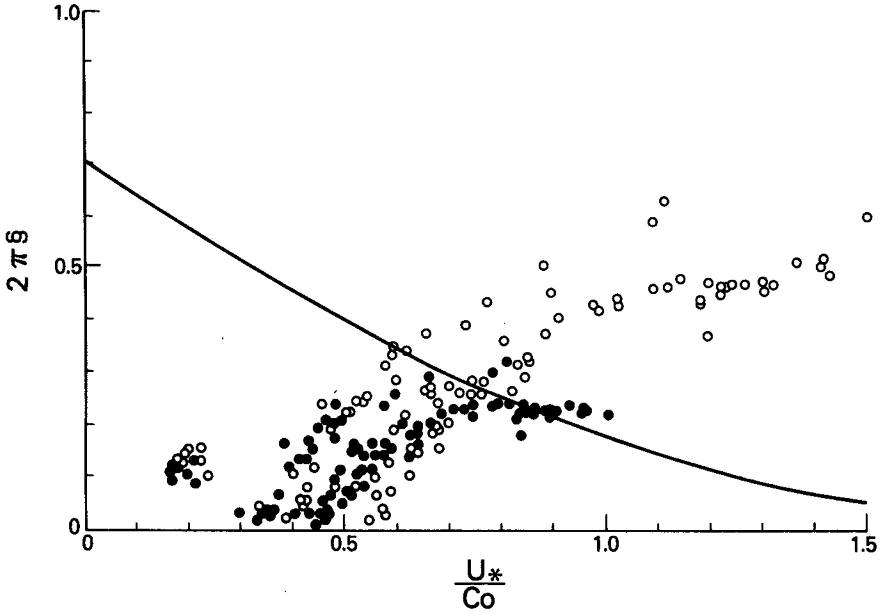

FIG. 4. Testing of the Phillips–Banner breaking limit (solid line) with laboratory data, with (●) and without (○) the Doppler frequency shift due to the surface drift current.

which can be reduced to

$$2\pi\S = \frac{1}{2\sqrt{2}}\left(1 - \frac{u_*}{2c_0}\right)^2. \tag{4.1}$$

A total of 93 cases were used, which included pure wind-generated waves and wind over mechanically generated random waves (Huang et al., 1984). When we plotted the observed values, the result was identical to that of Wright; the experimental points went right over the theoretical limit as given by (4.1).

In a close examination, however, we concluded that using the observed values was not strictly correct. The reason being that if surface drift does exist and cause premature breaking, it can also cause a Doppler shift of frequency. Consequently, the apparent quantities such as the observed frequency and phase velocity need corrections.

If we let the observed apparent frequency be $\tilde{n}$, then

$$\tilde{n} = n + k \cdot u_d \tag{4.2}$$

in which n is the intrinsic frequency satisfying the dispersion relationship, and u_d is the depth-averaged drift current. Its value should depend upon the length of the specific wave considered and be somewhat smaller than q_0 used in (3.2). Here, for simplicity, we adopt

$$u_d = u_*/r$$

with $r > 2$. With this assumption, it can be easily shown that the true values are

$$\left.\begin{aligned} \frac{u_*}{c_0} &= \frac{r}{2}\left[\left(1 + \frac{4u_*\tilde{n}}{gr}\right)^{1/2} - 1\right] \\ n &= \frac{rg}{2u_*}\left[\left(1 + \frac{4u_*\tilde{n}}{gr}\right)^{1/2} - 1\right] \\ k_0 &= \tilde{k}_0(n_0/\tilde{n}_0)^2 \end{aligned}\right\}. \tag{4.3}$$

For $r = 2$, the corrected values of u_*/c_0 and k_0 were plotted again for the experimental data in the solid symbols in Fig. 4. A few points still go over the theoretical limit, but they represent a small percentage of the points, and the general trend of the data suggests that the theoretical limit indeed offers a good first-order approximation. Several explanations can be advanced for the overshoot of the theory. First, at wave breaking, the amplitude of the wave can be momentarily higher than the theoretical limit (e.g., see Cokelet, 1977). Second, there is some scattering in the data.

This Doppler shift should be more important for Wright's data because his fetch was extremely short, but such correction was not included. It should be pointed out that the Banner–Phillips mechanism is only effective for short waves. For longer waves, the value of

$$u_*\tilde{n}/g = u_*/\tilde{c}_0$$

will be too small to show its influence in Eqs. (3.6) and (4.3). Under that condition, the actual limitation on the mean steepness of the wave field will be limited by other considerations as discussed by Huang (1985).

This paper offers an alternative to the present models for whitecap coverage computation; it incorporates some improvements but also contains some deficiencies. Further refinements are still needed. The most urgent ones are obviously those involving the breaking criterion and the determination of the proper Doppler shift in the dispersion relationship correction. Only extremely simple solutions of both these problems are employed here to show that the Banner–Phillips breaking mechanism and the non-Gaussian statistical model are both relevant modifications; they should be properly accounted for in the wave breaking studies.

5. Conclusions

Based on the results presented here, we believe that we have derived an analytic expression for oceanic whitecap coverage as a function of both sea state and surface wind stress. Although limitations exist, such as the narrowband assumption, idealized breaking criterion, and simple Doppler shift correction, etc., we think this new model offers a better solution in parameterizing the whitecap coverage as a function of the environmental variables. More field tests with complete and accurate measurements of the whitecap coverage statistics, together with the sea state and the wind stress, are urgently needed to firmly test this and other models.

REFERENCES

Banner, M. L., and O. M. Phillips, 1974: On the incipient breaking of small scale waves. *J. Fluid Mech.*, 77, 825–842.

Cartwright, D. E., and M. S. Longuet-Higgins, 1956: The statistical distribution of the maxima of a random function. *Proc. Roy. Soc. London,* **A 237,** 212–232.

Cokelet, E. D., 1977: Breaking waves. *Nature,* **267,** 769–774.

Danel, P., 1952: On the limiting clapotis. *Gravity Waves,* National Bureau of Standards Circular 521, 35–38.

Huang, N. E., 1985: An estimate of the influence of breaking waves on the dynamics of the upper ocean. *Proc. Symp. on Wave Dynamics and Radio Probing of the Ocean Surface.* Plenum, (in press).

——, and S. R. Long, 1980: An experimental study of the surface elevation probability distribution and statistics of wind generated waves. *J. Fluid Mech.,* **101,** 179–200.

——, ——, C. C. Tung, Y. Yuen and L. F. Bliven, 1981: A unified two-parameter wave spectral model for a general sea state. *J. Fluid Mech.,* **112,** 203–224.

——, ——, ——, —— and ——, 1983: A non-Gaussian statistical model for surface elevation of nonlinear random wave field. *J. Geophys. Res.,* **88,** 7597–7606.

——, ——, L. F. Bliven and C. C. Tung, 1984: The non-Gaussian joint probability density function of slope and elevation for a nonlinear gravity wave field. *J. Geophys. Res.,* **89,** 1961–1972.

Kennedy, R. M., and R. L. Snyder, 1983: On the formation of whitecaps by a threshold mechanism. Part II: Monte Carlo experiments. *J. Phys. Oceanogr.,* **13,** 1493–1504.

Kenyon, K. E., 1971: Wave refraction in ocean currents. *Deep Sea Res.,* **18,** 1023–1034.

Lipes, R. G., 1982: Description of SEASAT radiometer status and results. *J. Geophys. Res.,* **87,** 3385–3395.

Longuet-Higgins, M. S., 1952: On the statistical distribution of the heights of sea waves. *J. Mar. Res.,* **11,** 245–266.

——, 1969: On wave breaking and the equilibrium spectrum of wind-generated waves. *Proc. Roy. Soc. London,* **A310,** 151–159.

——, 1980: On the distribution of the heights of the sea waves: Some effects of nonlinearity and finite band width. *J. Geophys. Res.,* **85,** 1519–1523.

Monahan, E. C., and I. O'Muircheartaigh, 1980: Optimal power law description of oceanic whitecap coverage dependence on wind speed. *J. Phys. Oceanogr.,* **10,** 2094–2099.

Ochi, M. K., and C. H. Tsai, 1983: Prediction of occurrence of breaking waves in deep water. *J. Phys. Oceanogr.,* **13,** 2008–2019.

Papoulis, A., 1984: *Probability, Random Variables, and Stochastic Processes,* 2nd ed., McGraw Hill, 576 pp.

Phillips, O. M., 1977: *The Dynamics of the Upper Ocean,* 2nd ed., Cambridge University Press, 336 pp.

——, and M. L. Banner, 1974: Wave breaking in the presence of wind drift and swell. *J. Fluid Mech.,* **66,** 625–640.

Ramberg, S. E., M. E. Barber and O. M. Griffin, 1985: Laboratory studies of steep and breaking deep water waves in a convergent channel. NRL Memo. Rep. 5610, Naval Research Laboratory, Washington, DC, 42 pp.

Schwartz, L. W., and J. D. Fenton, 1982: Strongly nonlinear waves. *Ann. Rev. Fluid Mech.,* **14,** 39–60.

Snyder, R. L., and R. M. Kennedy, 1983: On the formation of whitecaps by a threshold mechanism. Part I: Basic formalism. *J. Phys. Oceanogr.,* **13,** 1482–1492.

——, L. Smith and R. M. Kennedy, 1983: On the formation of whitecaps by a threshold mechanism. Part III: Field experiment and comparison with theory. *J. Phys. Oceanogr.,* **13,** 1505–1518.

Stoker, J. J., 1957: *Water Waves.* Interscience, 567 pp.

Tayfun, M. A., 1980: Narrow-band nonlinear sea waves. *J. Geophys. Res.,* **85,** 1548–1552.

Wright, J. W., 1976: The wind drift and wave breaking. *J. Phys. Oceanogr.,* **6,** 402–405.

Valenzula, G. R., D. T. Chen, W. D. Garrett and J. A. Kaiser, 1983: Shallow water bottom topography from radar imagery. *Nature,* **303,** 687–689.