

# Research on Moving Force Estimation of the Bridge Structure using the Adaptive Input Estimation Method

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**ABSTRACT:** An on-line adaptive input estimation method that estimates the moving force inputs of the bridge structure is presented in this research. By using the inverse method, input forces acting on bridge structural system can be estimated from the measured dynamic responses. The algorithm includes the Kalman filter (KF) and the recursive least squares estimator (RLSE). This work presents an efficient weighting factor  $r$  of the RLSE, which is capable of improving the estimation results. The capability of the proposed algorithm is demonstrated through several examples of the bridge structure system with different types of the time-varying moving forces as the unknown inputs.

**KEYWORDS:** Adaptive input estimation method, Moving force, Kalman filter.

## 1. INTRODUCTION

The vehicle/bridge interaction moving forces are important for the bridge structure design and the reliability evaluation. Therefore, the dynamic moving forces produced by the vehicles on the bridge structure must be determined by adopting the estimation method or measurement techniques. Direct measurement of the vehicle moving forces using sensors with higher precision is expensive and is subject to bias, while results from the modeling errors [1~4] produced during the computation. The Weight-in-Motion (WIM) systems have been developed to solve above problems by Davis et al [5~7], but these techniques can only measure the static vehicle axle loads. However, the vehicle dynamic moving forces usually cause the damage of the road surfaces more easily than the static loads [8] do. Therefore, it is important to obtain the histories of the vehicle/bridge interaction dynamic moving forces.

In recent years, many researchers have studied the histories of the dynamic responses of the continuous beams due to moving vehicle wheel loads. Henchi et al. [9] used the finite element method to obtain the exact mode shapes and frequencies. Zhang et al. [10] used the assumed mode shape function to solve the vibration behavior of a non-uniform Bernoulli–Euler beam. In addition,

Dugush and Eisenberger [11] used the finite element method to determine the natural frequencies and mode shapes, and the solution is obtained by using the application of modal analysis and the direct integration method. All the above methods are mainly concentrated on the forward resolution, i.e. the detection of the bridge dynamic responses due to moving vehicle loads. However, the inverse problem of the vehicle moving force determination from the bridge dynamic responses is also significant and needed to be studied.

Some researchers addressed the determination method for the above problem. Doyle [12] developed a method for determining the location and magnitude of an impact force by using the phase difference of the signals measured at two different locations straddling the impact point. Busby and Trujillo [13] reconstructed the force history using the standing wave approach. Hollandsworth and Bushy [14] verified this experiment by applying a force at a known location and adopting the accelerometers as sensors. Druz et al.[15] formulated a non-linear inverse problem and tried to find the location and magnitude of the external force.

Other researchers studied the determination of the vehicle moving forces a few years ago. The time domain approach by Law et al. [16] models the structure and forces with a set of second order differential equations. The forces are modeled as the

step functions in a small time interval. These equations of motion are then expressed in the modal co-ordinates, and they are solved by using convolution in the time domain. The forces are then determined by using the modal superposition principle. The frequency and time domain approach by Law et al. [17] performs Fourier transformation with respect to the equations of motion, which are expressed in modal co-ordinates, and the histories of the forces are obtained directly by using the least-squares method. The modal approach identifies the forces completely in the modal co-ordinates by Chan et al. [18]. Measured displacements are converted into modal displacements with an assumed shape function. The forces are then determined by solving the uncoupled equations of motion in the modal co-ordinates. The above-mentioned approaches require the resolution of inverse matrix problems, which are computationally consuming and not numerically well-functioned in dealing with the on-line real-time signal problem.

To resolve the about-mentioned problems, an on-line recursive inverse method to estimate the input forces of the beam structures is presented. The inverse method is based on the Kalman filter and the recursive least square algorithm. Tuan et al. [19, 20] presented the input estimation algorithm to cope with the 1-D and 2-D inverse heat conduction problems. Ma et al.[21] presented an inverse method to estimate the impulsive loads on lumped-mass structural systems. The capability of the proposed algorithm is demonstrated in these examples. The algorithm is an efficient on-line recursive inverse method to estimate the input forces. The method is computationally more economical than the batch process when estimating complex structure system input forces. In the presented work, the input force estimation method is applied to the bridge structure systems to cope with the moving forces. The precision of the presented method is verified through several examples with different types of the time-varying moving forces as unknown inputs. The simulation results show that the method is effective in determining the moving forces.

## 2. PROBLEM FORMULATION

To illustrate the practicability and precision of the presented approach in estimating unknown input moving forces, numerical simulations of a bridge structure are investigated here. As shown in Figure 1,

the bridge structure is modeled as a simple beam with the total span  $L$ , the flexible stiffness constant  $EI$ , the mass per unit length  $\rho$  and the damping coefficient  $C$ . The beam is assumed to be a Bernoulli-Euler beam, in which the effects of shear deformation and rotary inertia are not taken into account.

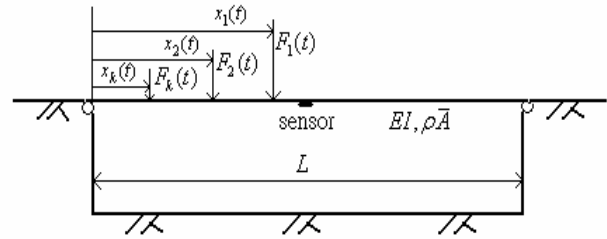


Figure 1. The bridge structure model under the multi-vehicle moving force inputs.

Considering the group of vehicle forces moving from left to right at a constant speed, the equation of motion [22] can be expressed as:

$$\rho \bar{A} \frac{\partial^2 u(x,t)}{\partial t^2} + C \frac{\partial u(x,t)}{\partial t} + EI \frac{\partial^4 u(x,t)}{\partial x^4} = \sum_{k=1}^N F_k(t) \delta(x - x_k(t)) \quad (1)$$

where  $\bar{A}$  is the cross section of beam,  $u(x,t)$  is the displacement of beam,  $F_k(t)$  is the vehicle force matrix,  $\delta(t)$  is the Dirac delta function,  $x_k(t) = v_k t$  is the position of the  $k$ th vehicle force and  $v_k$  is the speed of the  $k$ th vehicle. Based on modal superposition, the solution of Equation 1 can then be expressed as:

$$u(x,t) = \sum_n \Phi_n(x) Y_n(t) \quad (2)$$

where  $\Phi_n(x)$  is the  $n$ th modal shape function and  $Y_n(t)$  is the  $n$ th modal amplitude. By substituting Equation 2 in Equation 1, multiplying each term by  $\Phi_r(x)$ , integrating it over the length of the beam and applying orthogonal conditions, the equation of motion in terms of the modal amplitude can be rewritten as:

$$M_n \ddot{Y}_n(t) + C_n \dot{Y}_n(t) + K_n Y_n(t) = F_n(t) \quad (3)$$

where

$$M_n = \int_0^L \rho \bar{A} [\Phi_n(x)]^2 dx \quad (4)$$

$$K_n = \int_0^L EI [\Phi_n''(x)]^2 dx \quad (5)$$

$$F_n(t) = \int_0^L F_k(t) \delta(x - x_k(t)) [\Phi_n(x)] dx \quad (6)$$

$M_n$ ,  $K_n$ , and  $F_n(t)$  are the modal mass, the modal stiffness and the modal force of the  $n$ th mode, respectively. The modal damping coefficient,  $C_n = \alpha M_n + \beta K_n$ , where  $\alpha$  and  $\beta$  are constants with proper units.

Input estimation is a analysis method based on the state space. The state-space model of a beam structure system needs to be constructed before applying the input estimation method. After converting to the state-space model, the state variables of the second order dynamic system with  $n$  degrees of freedom are represented by a  $2n \times 1$  state vector, i.e.  $X = [Y(t) \dot{Y}(t)]^T$ . From Equation 1, the continuous-time state equation and measurement equation of the structure system can be formulated as:

$$\dot{X}(t) = AX(t) + BF_n(t) \quad (7)$$

$$Z(t) = HX(t) \quad (8)$$

where

$$A = \begin{bmatrix} 0_{n \times n} & I_{n \times n} \\ -M_n^{-1}K_n & -M_n^{-1}C_n \end{bmatrix}$$

$$B = \begin{bmatrix} 0_{n \times n} \\ M_n^{-1} \end{bmatrix}$$

$$H = [I_{2n \times 2n}]$$

$$X(t) = [X_1(t) \ X_2(t) \ \dots \ X_{2n-1}(t) \ X_{2n}(t)]^T$$

$A$  and  $B$  are constant matrices composed of the mass, damping ratio and stiffness of the beam structure system.  $X(t)$  is the state vector.  $Z(t)$  is the observation vector and  $H$  is the measurement matrix.

The noise interference exists in the practical circumstances. The noise interference was not considered in Equations 7 and 8. In order to approximate the noise to simulate the practical condition, the noise interference with statistical characteristics was added in the state equation and measurement equation of structure system. This random noise interference was represented by the Gaussian white noise. The statistical characteristic of a random variable was described in detail by means of the probability distribution and the density function, and it can be represented by using the mean and variance values of the random process [23]. On account of the above reason, by sampling Equation 7 with the sampling interval,  $\Delta t$ , the discrete-time statistical model of the system with

processing noise input [24] becomes:

$$X(k+1) = \Phi X(k) + \Gamma[F(k) + w(k)] \quad (9)$$

where

$$X(k) = [X_1(k) \ X_2(k) \ \dots \ X_{2n-1}(k) \ X_{2n}(k)]^T$$

$$\Phi = \exp(A\Delta t)$$

$$\Gamma = \int_{k\Delta t}^{(k+1)\Delta t} \exp\{A[(k+1)\Delta t - \tau]\} B d\tau$$

$$F(k) = [F_1(k) \ F_2(k) \ \dots \ F_{n-1}(k) \ F_n(k)]^T$$

$$w(k) = [w_1(k) \ w_2(k) \ \dots \ w_{n-1}(k) \ w_n(k)]^T$$

$X(k)$  represents the state vector.  $\Phi$  is the state transition matrix.  $\Gamma$  is the input matrix.  $\Delta t$  is the sampling interval.  $w(k)$  is the processing noise vector, which is assumed to be the Gaussian white noise with zero mean and variance,  $E\{w(k)w^T(k)\} = Q\delta_{kj}$ , where  $Q = Q_w \times I_{2n \times 2n}$ .  $Q$  is the discrete-time processing noise covariance matrix.  $\delta_{kj}$  is the Kronecker Delta function. To consider the measurement noise, the discrete-time measure equation is expressed as:

$$Z(k) = HX(k) + v(k) \quad (10)$$

where

$$Z(k) = [Z_1(k) \ Z_2(k) \ \dots \ Z_{2n}(k)]^T$$

$$v(k) = [v_1(k) \ v_2(k) \ \dots \ v_{2n}(k)]^T$$

$Z(k)$  is the observation vector.  $v(k)$  represents the measurement noise vector and is assumed to be the Gaussian white noise with zero mean and variance,  $E\{v(k)v^T(k)\} = R\delta_{kj}$ , where  $R = R_v \times I_{2n \times 2n}$ .  $R$  is the discrete-time measurement noise covariance matrix.  $H$  is the measurement matrix.

### 3. ADAPTIVE WEIGHTED RECURSIVE INPUT ESTIMATION METHOD

Adaptive weighted input force estimation is a process of determining the applied loads from the system measurements. The presented adaptive weighted input estimation method consists of two portions, the Kalman filter and the estimator. The Kalman filter is used to generate the residual innovation sequence. The residual innovation sequence connotes bias or systematic error of the unknown time-varying input item and the variance or random error of the measurement. The estimator is then adopted to compute the forces over time by applying the residual innovation sequence to the adaptive weighted recursive least square algorithm.

The detailed formulation of this technique can be found in the paper by Tuan et al [25]. The equations formulating the Kalman filter are as follows.

$$\bar{X}(k/k-1) = \Phi \bar{X}(k-1/k-1) \quad (11)$$

$$P(k/k-1) = \Phi P(k-1/k-1) \Phi^T + \Gamma Q \Gamma^T \quad (12)$$

$$\bar{Z}(k) = Z(k) - H \bar{X}(k/k-1) \quad (13)$$

$$S(k) = H P(k/k-1) H^T + R \quad (14)$$

$$K_a(k) = P(k/k-1) H^T S^{-1}(k) \quad (15)$$

$$\bar{X}(k/k) = \bar{X}(k/k-1) + K_a(k) \bar{Z}(k) \quad (16)$$

$$P(k/k) = [I - K_a(k) H] P(k/k-1) \quad (17)$$

In Equations 11 to 17, superscript ‘-’ represents the estimation value.  $\bar{X}(k/k-1)$  denotes the state estimation.  $P(k/k-1)$  is the state estimation error covariance.  $\bar{Z}(k)$  is the bias innovation caused by the measurement noise and the input disturbance.  $S(k)$  represents the innovation covariance.  $K_a(k)$  is the Kalman Gain.  $\bar{X}(k/k)$  is the state filter.  $P(k/k)$  represents state filter error covariance. The state transition matrix  $\Phi$ , the measure matrix  $H$ , the discrete-time process noise covariance matrix  $Q$ , and the discrete-time measurement noise covariance matrix  $R$  of the Kalman filter must be obtained to implement the filtering process. After the initial value,  $X_0$  and  $P_0$ , are adopted, as the observation vector is being inputted continuously, the output of Kalman filter can be obtained in real-time. The estimation value,  $\bar{X}(k/k-1)$ , and the state estimation error covariance,  $P(k/k-1)$ , of the structure system can be determined immediately. The formulation of the adaptive weighted recursive least square algorithm is as follows:

$$B_s(k) = H [\Phi M_s(k-1) + I] \Gamma \quad (18)$$

$$M_s(k) = [I - K_a(k) H] [\Phi M_s(k-1) + I] \quad (19)$$

$$K_b(k) = r^{-1} P_b(k-1) B_s^T(k) [B_s(k) r^{-1} P_b(k-1) B_s^T(k) + S(k)]^{-1} \quad (20)$$

$$P_b(k) = [I - K_b(k) B_s(k)] r^{-1} P_b(k-1) \quad (21)$$

$$\hat{F}(k) = \hat{F}(k-1) + K_b(k) [\bar{Z}(k) - B_s(k) \hat{F}(k-1)] \quad (22)$$

where  $\bar{Z}(k)$  denotes innovation value,  $K_b(k)$  is the correction gain,  $B_s(k)$  and  $M_s(k)$  are the sensitivity matrices,  $P_b$  represents the error covariance of the estimation, and  $\hat{F}(k)$  is the estimated input vector. The weighting factor  $r$  is used to compromise between the tracking capability and the degradation of estimation precision. In this study, the adaptive weighting function is presented. The detailed derivation of this function can be found in the paper of Tuan et al [20]. The adaptive

weighting function is as the following equation.

$$r(k) = \begin{cases} 1 & |\bar{z}(k)| \leq \sigma \\ \frac{\sigma}{|\bar{z}(k)|} & |\bar{z}(k)| > \sigma \end{cases} \quad (23)$$

In Equations 18 to 23, the estimator computes the Kalman Gain  $K_a(k)$  by applying the innovation covariance  $S(k)$  and innovation matrix  $\bar{Z}(k)$  produced by Kalman filter. By substituting Equation 23 in Equations 20 and 21 for weighting factor  $r$ , the adaptive weighted recursive least square estimator can be constructed.

The procedure to estimate the unknown moving force inputs using the inverse method is summarized as follows:

- Step1: Construct the discrete-time state-space model of the system, i.e. Equations 7 and 9, and measure the system responses  $X(k)$ .
- Step2: Use the Kalman filter, i.e. Equations 11 to 17, to obtain the innovation matrix  $\bar{Z}(k)$ , the innovation covariance  $S(k)$  and the Kalman gain  $K_a(k)$ .
- Step3: Use the adaptive weighted recursive least square algorithm, i.e. Equations 18 to 22, to estimate the unknown moving force  $\hat{F}(k)$ .

#### 4. RESULTS AND DISCUSSION

To verify the practicability and precision of the presented approach in estimating the unknown moving input forces, the bridge structure is modeled as a simple beam with the total span  $L = 30m$ , the flexible stiffness constant  $EI = 1.27914 \times 10^{11} Nm^2$ , the mass per unit length  $\rho = 1.2 \times 10^4 kg/m$ , the damping coefficient  $C_n = \alpha M_n + \beta K_n$ , where  $\alpha = 0.01$ , and  $\beta = 0.001$ , and the modal shape function  $\Phi_n = \sin(\pi x_k / L)$ . The initial conditions of the error covariance are given as  $p(0/0) = diag[10^4]$  for the KF and  $p_b(0) = 10^4$  for the adaptive weighted recursive least square estimator. The simulation conditions are set as follows. The sampling interval,  $\Delta t = 0.01$  s. The sensitivity matrix,  $M(0)$ , is null. The weighting factor is an adaptive weighting function. The error used to quantify the deviations between the estimated and actual input moving forces is defined as percent root mean square difference (PRD) [26]:

$$Error(\%) = \frac{\sqrt{\sum_{i=1}^n [F_{ex}(t_i) - F_{es}(t_i)]^2}}{\sqrt{\sum_{i=1}^n [F_{ex}(t_i)]^2}} \times 100\% \quad (24)$$

where  $n$  is total number of estimation time steps,  $F_{ex}(t_i)$  and  $F_{es}(t_i)$  are the actual and estimated forces at time  $t_i$ , respectively.

#### 4.1 Singular-vehicle moving force input estimation

The singular-vehicle moving force input is simulated by adopting a vehicle with the static weight,  $F_k = 200KN$ , acting on the bridge structure, and the constant velocity,  $v_k = 10m/sec$ , over the bridge. According to Equation 6, the time-varying moving force input is simulated as below:

$$F_n(t) = \begin{cases} F_k \sin(\pi v_k t / L) & t_i \leq t \leq t_d \\ 0 & 0 \leq t \leq t_i, t \geq t_d \end{cases} \quad (25)$$

where  $t_i$  represents the initial time when the vehicle enters the bridge. There is time delay for 0.3s in order to obtain a well-determined result of simulation. The terminal time when the vehicle leaves the bridge,  $t_d = L/v$ . The dynamic response of the bridge is obtained by adopting a numerical method with the system noise and the measurement noise. The Kalman estimation parameters used in the numerical model are given as follows. The covariance matrix of process noise,  $Q = Q_w \times I_{2n \times 2n}$ . Set  $Q_w = 10^7$ . The covariance matrix of measurement noise,  $R = R_w \times I_{2n \times 2n}$ . Set  $R_w = \sigma^2 = 10^{-10}$ . Figure 2 shows the tracking capability of the estimator with different weighting factors. The tracking capability of the estimator is superior with a smaller value of weighting factor  $r$ , the error between the estimated and exact moving forces input is smaller. On the contrary, the opposite effect with larger value of weighting factor  $r$  is presented.

Figure 3 shows the singular-vehicle moving force input estimation result. The displacement was measured at the middle of the bridge. The result reveals a very good estimating ability, that is, the estimation values converge to the actual values rapidly. Since  $p(0/0)$  and  $P_b(0)$  are normally unknown, the estimator was initialized with large values of  $p(0/0)$  and  $P_b(0)$ , such as 104, which introduces the effect of treating the initial error as a large value, so that the estimator will ignore the first few estimates.

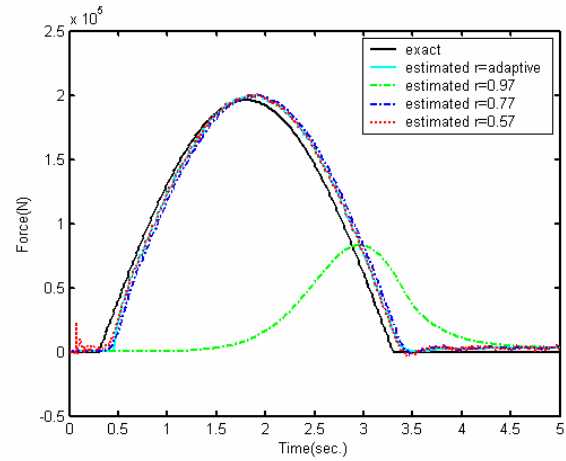


Figure 2. History of the estimated and actual singular-vehicle moving force inputs with different weighting factors. ( $F = 200(KN), v = 10m/sec, Q_w = 10^7$ , and  $\sigma = 10^{-5}$ )

The error (PRD) of the estimated singular-vehicle moving force input is approximately 8.08%. The influence of the process and measurement noises on the estimation results has been considered. The estimation results have demonstrated the availability of the presented inverse estimation algorithm in coping with the singular-vehicle moving force input.

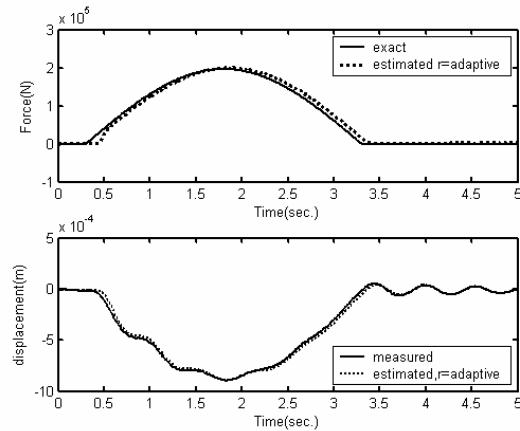


Figure 3. History of the estimated and actual singular-vehicle moving force inputs and the displacements at the middle of the bridge(  $F = 200(KN), v = 10m/sec, Q_w = 10^7, \sigma = 10^{-5}$ , and error = 8.08%)

Figure 4 depicts the corresponding history of the singular-vehicle moving force input estimation result and the displacements. The singular-vehicle moving force input is simulated by adopting a mid-size vehicle with the static weight,  $F_k = 150KN$ , acting on the bridge structure, and the constant velocity,  $v = 20m/sec$ . The error (PRD) of the estimated singular-vehicle moving force input is approximately 16.74%.

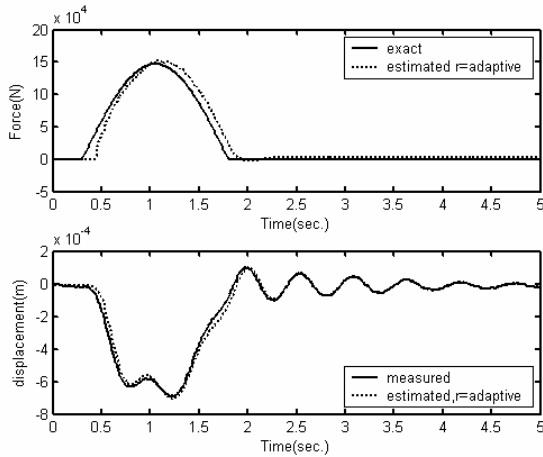


Figure 4. History of the estimated and actual singular-vehicle moving force inputs and the displacements at the middle of the bridge. ( $F = 150(KN)$ ,  $v = 20m/sec$ ,  $Q_w = 10^7$ ,  $\sigma = 10^{-5}$ , and error = 16.74%)

Figure 5 shows the history of the singular-vehicle moving force input estimation result and the displacements. The moving force input is simulated by adopting a compact vehicle with the static weight,  $F_k = 100KN$ , acting on the bridge structure, and the constant velocity,  $v = 30m/sec$ . The error (PRD) of the estimated singular-vehicle moving force input is approximately 27.42%.

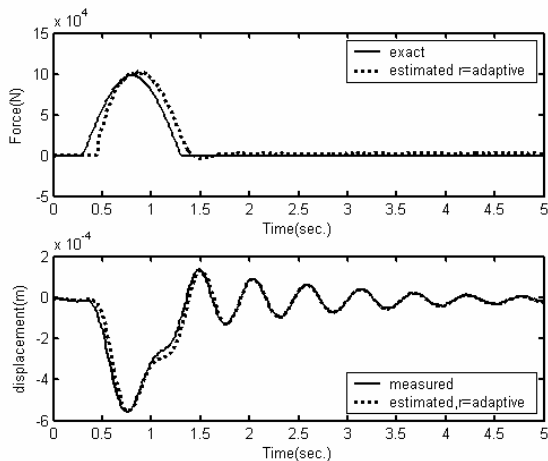


Figure 5. History of the estimated and actual singular-vehicle moving force inputs and the displacements at the middle of the bridge. ( $F = 100(KN)$ ,  $v = 30m/sec$ ,  $Q_w = 10^7$ ,  $\sigma = 10^{-5}$ , and error = 27.42%)

According to Figures 3 to 6, the performance of the estimator is not satisfying when adopting the static light-weight and the high-velocity vehicle, the error (PRD) is relatively higher. However, the presented inverse method still has the tracking capability to identify the moving force inputs. To obtain better estimation results, the values of

Kalman estimation parameters are adjusted that  $Q_w = 10^8$ , and  $R_v = \sigma^2 = 10^{-14}$ . Figure 6 shows the history of the singular-vehicle moving force input estimation result and the displacements. The result reveals a very good estimating ability, that is, the error (PRD) of the estimated singular-vehicle moving force input is apparently reduced (17.97%).

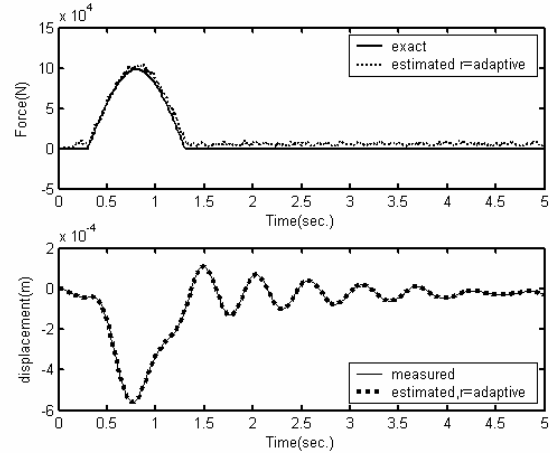


Figure 6. History of the estimated and actual singular-vehicle moving force inputs and the displacements at the middle of the bridge. ( $F = 100(KN)$ ,  $v = 30m/sec$ ,  $Q_w = 10^8$ ,  $\sigma = 10^{-7}$ , and error = 17.97%)

#### 4.2 Multi-vehicle moving force input estimation

Three moving force inputs are simulated by adopting multiple vehicles with the static weights,  $F_{k=1-3} = 100KN$ , acting on the bridge structure, and the constant velocities,  $v_{k=1-3} = 10m/sec$ . The initial time  $t_i$  as the first vehicle enters the bridge is delayed for 0.3s to obtained a more well-determined result of simulation. The time interval between any two vehicles' entries is 0.5s. According to Equation 6, the time-varying moving force inputs are simulated as follows:

$$F_n(t) = \begin{cases} F_k \sin(\pi v_k t / L) & t_i \leq t \leq t_d \\ 0 & 0 \leq t \leq t_i, t \geq t_d \end{cases} \quad (26)$$

where  $t_d = L/v_{k=1-3}$ , which represents the terminal time as the vehicles leave the bridge. The Kalman estimation parameters used in the numerical model are adopted as follows: The covariance matrix of the processing noise,  $Q_w = 10^7$ . The covariance matrix of the measurement noise,  $R_v = \sigma^2 = 10^{-10}$ . The dynamic response of the bridge is obtained by using a numerical method with system noise and measurement noise taken into account. The history of the multi-vehicle moving force input estimation result and the displacements at the middle of the

bridge are shown in Figure 7. The error (PRD) of the estimated multi-vehicle moving force input is approximately 10.66%. The estimation result demonstrates the availability of the presented inverse estimation algorithm in use of estimating the multi-vehicle moving force inputs.

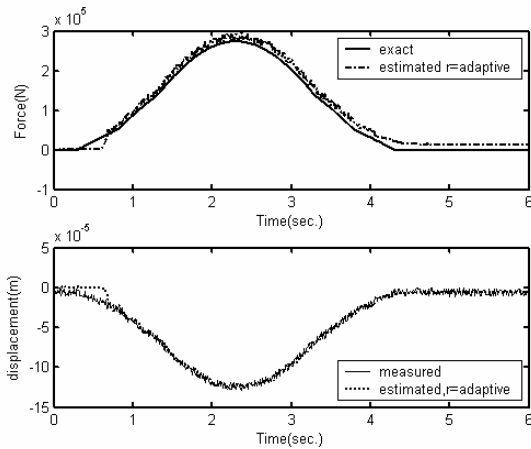


Figure 7. History of the estimated and actual multi-vehicle moving force inputs and the displacements at the middle of the bridge. ( $F_1 = F_2 = F_3 = 100(KN)$ ,  $v_1 = v_2 = v_3 = 10m/sec$ ).

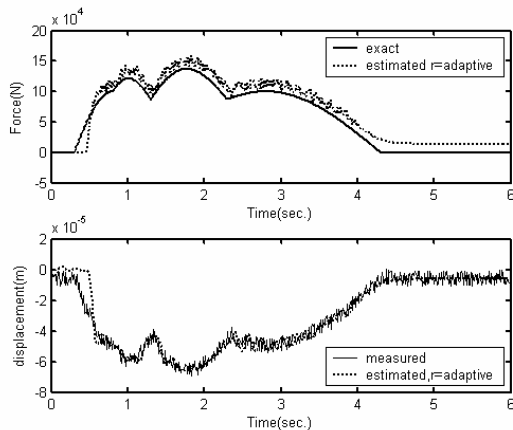


Figure 8. History of the estimated and actual multi-vehicle moving force inputs and the displacements at the middle of the bridge. ( $F_1 = F_2 = F_3 = 100(KN)$ ,  $v_1 = 30, v_2 = 20, v_3 = 10m/sec$ ,  $Q_w = 10^7$ ,  $\sigma = 10^{-5}$ , and  $error = 19.54\%$ )

Figure 8 shows the history of the multi-vehicle moving force input estimation result and the displacements. The vehicle moving force inputs are simulated by adopting the same static weights,  $F_{k=1-3} = 100KN$ , acting on the bridge structure. The velocity of the first vehicle,  $v_1 = 30m/sec$ , the second one,  $v_2 = 20m/sec$ , and the third,  $v_3 = 10m/sec$ . The Kalman estimation parameters are set that  $Q_w = 10^7$ , and  $R_v = \sigma^2 = 10^{-10}$ . The error (PRD) of the estimated multi-vehicle moving force input is approximately 19.54%. In this case, the complex

vehicle/bridge interaction influences the estimation resolution, but the result is still acceptable.

To obtain better estimation results, the values of Kalman estimation parameters can be adjusted that  $Q_w = 10^7$ , and  $R_v = \sigma^2 = 10^{-12}$ . Figure 9 shows the history of the multi-vehicle moving force input estimation result and the displacements. The result reveals a better estimating performance, and the error (PRD) of the estimated singular-vehicle moving force input is apparently reduced (5.37%). The performance of the estimator is influenced by the Kalman estimation parameter,  $R$ .

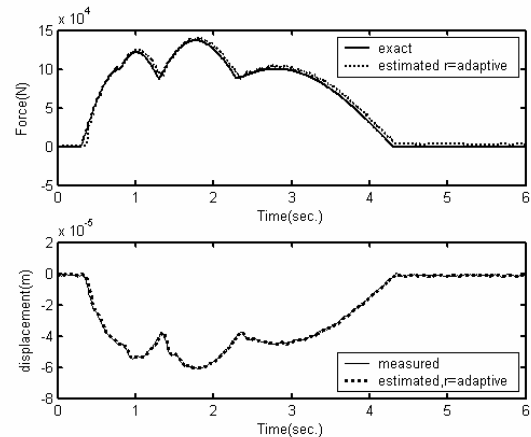


Figure 9. History of the estimated and actual multi-vehicle moving force inputs and the displacements at the middle of the bridge. ( $F_1 = F_2 = F_3 = 100(KN)$ ,  $v_1 = 30, v_2 = 20, v_3 = 10m/sec$ )

Figure 10 shows the history of the multi-vehicle moving force input estimation results and the displacements. The static weight of the first vehicle inputs moving force,  $F_1 = 200(KN)$ , to the bridge structure. The second force,  $F_2 = 150(KN)$ , and the third force,  $F_3 = 100(KN)$ . The velocity of the first vehicle,  $v_1 = 30m/sec$ , the second one,  $v_2 = 20m/sec$ , and the third one,  $v_3 = 10m/sec$ . The Kalman estimation parameters are set that,  $Q_w = 10^7$ , and  $R_v = \sigma^2 = 10^{-10}$ . The error (PRD) of the estimated multi-vehicle moving force input is about 15.70%. In this case, although the complex simulation situation influences the estimation resolution, the result is still acceptable.

The improved estimation result by using a smaller value of the Kalman estimation parameter  $R$  was shown in Figure 11. The error (PRD) of the estimated mutli-vehicle moving force input is apparently reduced (5.31%). The capability of the estimation are demonstrated through this turning Kalman parameter ( $Q_w = 10^7$ , and  $R_v = \sigma^2 = 10^{-14}$ ) example with complex multi-vehicle moving force inputs applied.

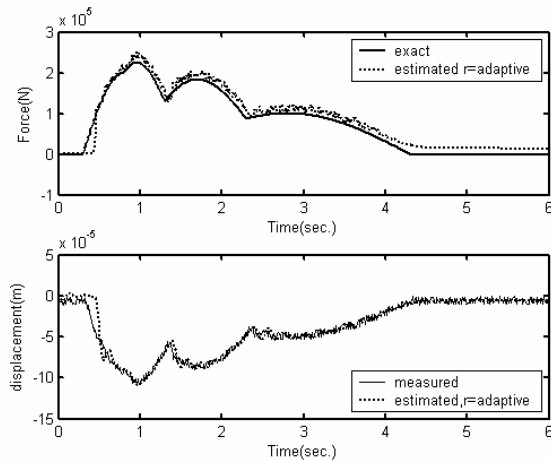


Figure 10. History of the estimated and actual multi-vehicle moving force inputs and the displacements at the middle of the bridge. ( $F_1 = 200, F_2 = 150, F_3 = 100(KN)$ ,  $v_1 = 30, v_2 = 20, v_3 = 10m/sec$ )

The above simulation results demonstrate that the system modal noise and measurement noise will influence the estimation resolution. The tracking capability of the estimator is degraded as shown in Figures 8 and 10 when applying a larger value of the measurement noise covariance,  $R_v$ . On the contrary, the opposite effect when applying a smaller measurement noise covariance is presented in Figures 9 and 11. The simulation results show that the proposed method has good performance in tracking unknown moving forces imposed on the bridge structure system.

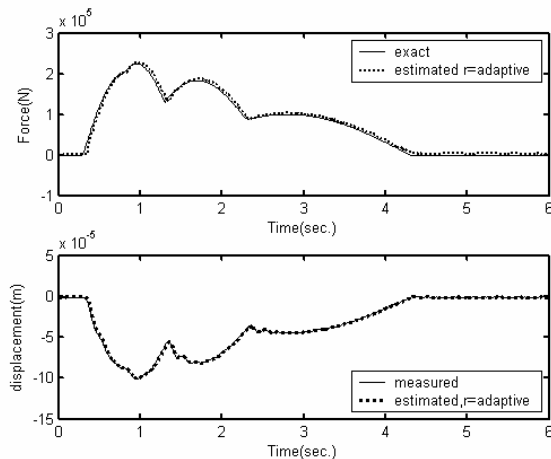


Figure 11. History of the estimated and actual multi-vehicle moving force inputs and the displacements at the middle of the bridge. ( $F_1 = 200, F_2 = 150, F_3 = 100(KN)$ ,  $v_1 = 30, v_2 = 20, v_3 = 10m/sec$ )

## 5. CONCLUSIONS

In this paper, an inverse adaptive weighted input estimation methodology is proposed to estimate the unknown time-varying moving forces produced by the vehicle in the bridge structure system. This algorithm includes the Kalman filter (KF) and the adaptive weighted recursive least square estimator (RLSE), which recursively estimates the unknown inputs under a situation that the system involves the measurement and modeling errors. The algorithm is an efficient on-line recursive inverse method to estimate the force inputs. The capabilities of the proposed algorithm are demonstrated by using two simulation examples. The method has fast adaptive capability and good performance in tracking the moving forces, by adequately choosing the smaller Kalman parameter  $R_v$ , along with the adaptive weighting factor  $r$ . The estimation method proposed in this paper can be applied to further research extensively. Future works of this study would address the problems of the force input estimation in the two or three dimensional structural system and the applications in the optimal control scope.

## 6. REFERENCES

- [1] R. Cantineni, "Dynamic behavior of highway bridges under the passage of heavy vehicles", Swiss Federal Laboratories for Materials Testing and Research (EMPA) Report, 1992, pp 220- 240.
- [2] R. J. Heywood, "Influence of truck suspensions on the dynamic response of a short span bridge", International Journal of Vehicle Design, 1994, pp 222-239.
- [3] M. F. Green and D. Cebon, "Dynamic response of highway bridges to heavy vehicle loads: theory and experimental validation", Journal of Sound and Vibration, Vol. 170, 1994, pp 51-79.
- [4] Y. B. Yang and J. D. Yau, "Vehicle-bridge interaction element for dynamic analysis", Journal of Structural Engineering, ASCE 123, 1997, pp 1512-1518,.
- [5] P. Davis and F. Sommerville, "Low-Cost Axle Load determination", Proceedings of the 13<sup>th</sup> ARRB and Fifth REAAA Combined Conference, part 6, 1986, pp 142-149.
- [6] R. J. Peters, "AXWAY-a system to obtain



- vehicle axle weights", Proceedings of the 12<sup>th</sup> ARRB Conference, Vol. 12(2), 1984, pp 10-18.
- [7] R. J. Peters, "CULWAY-an unmanned and undetectable highway speed vehicle weighting system", Proceedings of the 13<sup>th</sup> ARRB and Fifth REAAA Combined Conference, part 6, 1986, pp 70-83.
- [8] D. Cebon, "Assessment of the dynamic wheel forces generated by heavy road vehicles", Proceedings of the Symposium on Heavy Vehicle Suspension and Characteristics, Australian Road Research Board, 1987, pp 143-162.
- [9] K. Henchi, M. Fafard, G. Dhatt and M. Talbot, "Dynamic behavior of multi-span beams under moving loads", Journal of Sound and Vibration, Vol. 199(1), 1997, pp 33-50.
- [10] D. Y. Zheng, Y. K. Cheung, F. T. K. Au and Y. S. Cheng, "Vibration of multi-span non-uniform beams due to moving loads", Journal of Sound and Vibration, Vol. 212(3), 1998, pp 455-467.
- [11] Y. A. Dugush and M. Eisenberger, "Vibrations of non-uniform continuous beams under moving loads", Journal of Sound and Vibration, Vol. 254(5), 2002, pp 911-926.
- [12] J. F. Doyle and M. Eisenberger, "An experimental method for determining the location and time of initiation of an unknown dispersing pulse", Experimental Mechanics, Vol. 27, 1987, pp 229-233.
- [13] H. R. Busby and D. M. Trujillo, "Solution of an inverse dynamics problem using an eigenvalue reduction technique", Computers and Structures, Vol. 25, 1987, pp 109-117.
- [14] P. E. Hollandsworth and H. R. Busby, "Impact force identification using the general inverse technique", International Journal of Impact Engineering, Vol. 8, 1989, pp 315-322.
- [15] J. Druz, J. D. Crisp and T. Ryall, "Determining a dynamic force on a plate-an inverse problem", AIAA Journal, Vol. 29, 1991, pp 464-470.
- [16] S. S. Law, T. H. T. Chan and Q. H. Zeng, "Moving force identification: a time domain method", Journal of Sound and Vibration, Vol. 201, 1997, pp 1-22.
- [17] S. S. Law, T. H. T. Chan and Q. H. Zeng, "Moving force identification: a frequency and time domains analysis", Journal of Dynamic Systems, Measurement and Control ASME, Vol. 12, 1999, pp 394-401.
- [18] T. H. T. Chan, S. S. Law, T. H. Yung and X. R. Yung, "An interpretive method for moving force identification", Journal of Sound and Vibration, Vol. 219, 1999, pp 503-524.
- [19] P. C. Tuan, S. C. Lee and W. T. Hou, "An efficient on-line thermal input estimation method using Kalman filter and recursive least square algorithm". Inverse Problem Eng 5, 1997, pp 309-333.
- [20] P. C. Tuan and W. T. Hou, "Adaptive robust weighting input estimation method for the 1-D inverse heat conduction problem". Numerical Heat Transfer 34, 1998, pp 439-456.
- [21] C. K. Ma, P. C. Tuan, D. C. Lin and C. S. Liu, "A study of an inverse method for the estimation of impulsive loads". Int J Syst Sci 29, 1998, pp 663-672.
- [22] H. T. C. Tommy and B. A. Demeke, "Theoretical study of moving force identification on continuous bridges", Journal of Sound and Vibration, Vol. 295, 2006, pp 870-883.
- [23] T. Fun, "Random vibration of engineering", Defense industries publication, Peking, 1995.
- [24] P. L. Bogler, "Tracking a maneuvering target using input estimation", IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-23, No. 3, 1987, pp 298-310.
- [25] P. C. Tuan, C. C. Ji, L. W. Fong and W. T. Huang, "An input estimation approach to on-line two dimensional inverse heat conduction problems", Numerical Heat Transfer B, 29, 1996, pp 345-363.
- [26] C. K. Ma, J. M. Chang and D. C. Lin, "Input forces estimation of beam structures by an inverse method", Journal of Sound and Vibration, 259(2), 2003, pp 387-407.