Friction-Induced Roll Motion in Short-Crested Surface Gravity Waves

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ABSTRACT

Induced streaming due to deep water surface gravity waves propagating at oblique angles to each other is studied theoretically on the basis of a Lagrangian description. The ocean is slightly viscous, and the primary waves are maintained at constant amplitude by a suitably adjusted small wind stress distribution at the surface. The induced secondary motion in the nonrotating case consists of parallel rolls with axes aligned along the wave propagation direction, and a horizontally undulating Stokes drift. Surface convergence in the roll motion occurs at lines through nodal points of the primary wave system, and downwelling occurs below them. The surface value of the undulating Stokes drift has a minimum at these nodal points if the angle between the crossing waves is less than 76.4°. If this angle is larger than 76.4°, the Stokes drift at the surface has a maximum here. The roll motion described in the present paper is discussed in connection with the basis for the recent theoretical development of Langmuir circulations. Finally, a solution for the steady, horizontally averaged drift current in a rotating ocean is presented.

1. Introduction

Streak structures at the sea surface directed along the wind direction are commonly observed phenomena. Many attempts have been made to explain these observations; see the reviews by Faller (1971) and Leibovich (1983). It appears that the streaks, formed by collection of sea-weed, foam, etc., are due to organized motion in the water in the form of cells aligned along, or nearly along, the wind direction. The collection of surface material occurs along lines of surface convergence, and downwelling takes place below them. The cells themselves have become known as Langmuir circulations after Langmuir (1938), who was the first to investigate this phenomenon more systematically.

Among the many theories offered to explain the generation of Langmuir circulations, the theory of Craik and Leibovich (1976) seems to be one of the most successful so far. Essentially the roll motion in this theory (the CL1 mechanism; Faller and Caponi, 1978) arises from a nonlinear interaction between a horizontally undulating Stokes drift and a unidirectional shear current. In the Craik-Leibovich theory viscosity is needed to establish the shear current, while its influence is completely neglected in the determination of the wave-induced current. The latter is obtained from inviscid theory by considering the interaction between two monochromatic surface waves of equal amplitude and frequency which propagate at oblique angles to each other. Linearly this gives rise to a system of progressive short-crested surface waves. Nonlinearly a horizontally undulating Stokes drift is obtained.

At this point it is worth recalling that the effect of viscosity introduces significant changes in the wave-drift current near the surface and bottom boundaries, see for example Longuet-Higgins (1953, 1960); Ünlüata and Mei (1970); Liu and Davis (1977); Craik (1982); Weber (1983a). The cited papers consider only infinitely long-crested progressive waves or purely standing waves. A generalization of this theory to the short-crested progressive wave system considered by the Craik-Leibovich theory is therefore natural. Also from an oceanographic point of view this type of wave system is appealing, since the directional spectrum of a wind-generated sea is symmetric with respect to the wind direction (Longuet-Higgins, 1962).

According to this, we shall investigate the effect of viscosity on the wave system which is a necessary element in the CL1 generation mechanism for Langmuir circulations. We shall find that the nonlinear wave–wave interaction, due to the presence of viscosity, produces roll motions near the sea surface. The computed drift velocities in the rolls are small. However, for reasonable choices of the physical parameters, they are large enough to transport floating material towards the convergence lines at speeds not incompatible with those typically found in Langmuir circulations (Dyke and Barstow, 1983). The phenomenon we describe here is analogous to the acoustic streaming problem visualized in Kundt's dust tube (Batchelor, 1967, p. 363). More directly, similar cellular motions occur in the purely standing wave system of Liu and Davis (1977). However, in that problem, with a fluid of finite depth, the relatively strong effect of bottom friction (no-slip bottom) dominates the much weaker stress-free condition at the surface. Therefore, the induced circulations are dominated by the bottom when the layer is relatively shallow. However, when the depth increases sufficiently, as assumed in the present paper, only the surface circulations prevail.

The problem will be analysed using a Lagrangian formulation, extending the computations of the author (Weber, 1983b; hereafter refered to as I), from one single wave to a pair of crossing waves. The circulation induced by this short-crested wave system is studied in the absence of rotation. Finally a solution is given for the steady, horizontally averaged drift current in a rotating ocean.

2. Mathematical formulation

We consider an unlimited ocean of infinite depth rotating about the vertical axis with a constant angular velocity $f/2$, where f is the Coriolis parameter. When undisturbed, the surface is horizontal. The ocean water is taken to be homogeneous, incompressible and viscous. A Cartesian right-handed coordinate system is defined such that the x, y-axes are situated at the undisturbed surface, while the z-axis is directed vertically upwards.

We describe the motion by using a Lagrangian formulation. Let a fluid particle (a, b, c) have coordinates (x, y, z). The governing equations for momentum and mass may be written

$$\left.\begin{aligned} x_{tt} - fy_t &= -\frac{1}{\rho}\frac{\partial(p, y, z)}{\partial(a, b, c)} + \nu\nabla^2 x_t \\ y_{tt} + fx_t &= -\frac{1}{\rho}\frac{\partial(x, p, z)}{\partial(a, b, c)} + \nu\nabla^2 y_t \\ z_{tt} + g &= -\frac{1}{\rho}\frac{\partial(x, y, p)}{\partial(a, b, c)} + \nu\nabla^2 z_t \end{aligned}\right\}, \tag{2.1}$$

$$\frac{\partial(x, y, z)}{\partial(a, b, c)} = 1 \tag{2.2}$$

where p is the pressure, ρ the density, ν the kinematic viscosity and g the acceleration due to gravity. Subscripts denote partial differentiation, and $\partial/\partial(a, b, c)$ is the Jacobian. For the explicit form of the Laplacian ∇^2 in Lagrangian formulation, the reader is referred to Pierson (1962).

The present paper considers drift currents due to surface gravity waves. In the equations above we have replaced the initial position (x_0, y_0, z_0) of a fluid particle by its identification parameters (a, b, c); although the individual particles in the wave motion actually move around (a, b, c) in nearly closed orbits; see Pollard (1970) for the inviscid case. This discrepancy, however, does not influence the second-order mass transport solution, as shown in I.

The displacements x, y, z and dynamic pressure p will be written as series expansions (Pierson, 1962)

$$\begin{aligned} x &= a + \epsilon x^{(1)} + \epsilon^2 x^{(2)} + \cdots \\ y &= b + \epsilon y^{(1)} + \epsilon^2 y^{(2)} + \cdots \\ z &= c + \epsilon z^{(1)} + \epsilon^2 z^{(2)} + \cdots \\ p &= -\rho g c + \epsilon p^{(1)} + \epsilon^2 p^{(2)} + \cdots . \end{aligned} \tag{2.3}$$

Here ϵ is an ordering parameter proportional to the amplitude of the surface wave.

As stated in the introduction, we consider short-crested surface gravity waves. The simplest way of producing such waves is by superposing two monochromatic waves of the same amplitude and frequency propagating at oblique angles to each other. Taking the x-axis as the main propagation direction, we consider two waves with equal amplitudes and wave vectors

$$\left.\begin{aligned} \kappa_1 &= (k, l) \\ \kappa_2 &= (k, -l) \end{aligned}\right\} \tag{2.4}$$

where k and l are the wave numbers in the x and y-direction, respectively. The waves propagate on deep water, so the frequency σ, to the lowest order, is

$$\sigma = (g\kappa)^{1/2} \tag{2.5}$$

where $\kappa = (k^2 + l^2)^{1/2}$ is the overall wave number. The corresponding wavelength λ is defined by $\lambda = 2\pi/\kappa$. We assume that the wave frequency is much larger than the inertial frequency, i.e.,

$$\sigma \gg f. \tag{2.6}$$

This means that the effect of rotation, as in I, can be neglected to $O(\epsilon)$.

Since the ocean is viscous, the waves will, if let alone, attenuate in time. This may be relevant for swell in the absence of wind. However, when wind is present, there is a transfer of momentum and energy into the wave motion. We shall not go into the very complicated problem of wave growth in this paper, but merely confine ourselves to the simple case where a small variable normal wind stress transfers exactly enough energy to the wave motion to compensate for the loss due to viscous dissipation. We refer to Lamb (1932) or I for a discussion in the case of a single monochromatic wave. It is straight forward to generalize these results to the bimodal wave spectrum considered here. To $O(\epsilon)$ the displacements and pressure may be written

$$x^{(1)} = \frac{2k}{\sigma}\cos lb\left\{\left[e^{\kappa c} - \frac{\kappa}{\gamma}e^{\gamma c}(\cos\gamma c + \sin\gamma c)\right]\cos(ka - \sigma t) + \frac{\kappa}{\gamma}e^{\gamma c}(\cos\gamma c - \sin\gamma)\sin(ka - \sigma t)\right\} \tag{2.7}$$

$$y^{(1)} = \frac{2l}{\sigma}\sin lb\left\{\left[-e^{\kappa c} + \frac{\kappa}{\gamma}e^{\gamma c}(\cos\gamma c + \sin\gamma c)\right]\sin(ka - \sigma t) + \frac{\kappa}{\gamma}e^{\gamma c}(\cos\gamma c - \sin\gamma c)\cos(ka - \sigma t)\right\} \tag{2.8}$$

$$z^{(1)} = \frac{2\kappa}{\sigma}\cos lb\left\{\left[e^{\kappa c} - \frac{\kappa^2}{\gamma^2}e^{\gamma c}\sin\gamma c\right]\sin(ka-\sigma t) - \frac{\kappa^2}{\gamma^2}e^{\gamma c}\cos\gamma c\cos(ka-\sigma t)\right\} \quad (2.9)$$

$$p^{(1)} = 2\rho\sigma\cos lb\frac{\kappa^2}{\gamma^2}e^{\gamma c}[\cos\gamma c\cos(ka-\sigma t) + \sin\gamma c\sin(ka-\sigma t)]. \quad (2.10)$$

Here γ is an inverse viscous boundary layer thickness given by $\gamma = (\sigma/2\nu)^{1/2}$. The analysis assumes that

$$\frac{\kappa}{\gamma} \ll 1 \quad (2.11)$$

and higher order terms in κ/γ have been neglected in the solution above. The permanent wave solution (2.7)–(2.10) implies that the vertical component $\tau^{(z)}$ of the wind stress (the deviation from constant pressure) has a small variation of the form

$$\tau^{(z)} = -8\epsilon\rho\nu\kappa^2\cos lb\cos(ka-\sigma t), \quad c = 0. \quad (2.12)$$

This is obtained by inserting (2.9) into (A6) of the Appendix.

Assuming that each of the crossing waves has an amplitude ζ_0, we obtain analogous to I that the ordering parameter ϵ can be written

$$\epsilon = \frac{\zeta_0\sigma}{\kappa}. \quad (2.13)$$

The primary wave system defined by (2.7)–(2.10) propagates along the x-axis, while the motion in the y, z-plane is that of standing waves. For this reason we shall refer to x and y as the wave and crosswave directions, respectively.

3. Wave-drift equations

The solution to $O(\epsilon^2)$ gives the mass transport directly. We introduce

$$u^{(2)} = \bar{x}_t^{(2)}, \quad v^{(2)} = \bar{y}_t^{(2)}, \quad w^{(2)} = \bar{z}_t^{(2)}$$

$$\pi^{(2)} = \frac{1}{\rho}\bar{p}^{(2)} + g\bar{z}^{(2)} \quad (3.1)$$

where the overbar denotes average over one wavelength in the x-direction. By substituting (2.3) and (2.7)–(2.10) into the governing equations, collecting terms of $O(\epsilon^2)$, and averaging as defined above, we obtain

$$u_t^{(2)} - fv^{(2)} - \nu\nabla_L^2u^{(2)} = -\frac{8\nu k}{\sigma}[(\kappa^4 + k^4\cos 2lb)e^{2\kappa c} - \kappa\gamma(\kappa^2 + k^2\cos 2lb)e^{\gamma c}(\cos\gamma c - \sin\gamma c)] \quad (3.2)$$

$$v_t^{(2)} + fu^{(2)} + \pi_b^{(2)} - \nu\nabla_L^2v^{(2)} = -2l\left[k^2e^{2\kappa c} - \frac{\kappa^3}{\gamma}e^{\gamma c}(\sin\gamma c + \cos\gamma c)\right]\sin 2lb \quad (3.3)$$

$$w_t^{(2)} + \pi_c^{(2)} - \nu\nabla_L^2w^{(2)} = 2\kappa[(\kappa^2 + k^2\cos 2lb)e^{2\kappa c} - (\kappa^2 + (k^2 - l^2)\cos 2lb)e^{\gamma c}\cos\gamma c] \quad (3.4)$$

$$v_b^{(2)} + w_c^{(2)} = 0. \quad (3.5)$$

Here $\nabla_L^2 = \partial^2/\partial b^2 + \partial^2/\partial c^2$. Furthermore, we have assumed that there is no mean pressure gradient in the x-direction, i.e., $\bar{p}_a^{(2)} = 0$. Only the leading terms in (κ/γ) are kept on the right-hand sides of (3.2)–(3.4).

By utilizing (3.5), we may introduce a streamfunction $\psi^{(2)}$ such that

$$\left.\begin{array}{l} v^{(2)} = -\psi_c^{(2)} \\ w^{(2)} = \psi_b^{(2)} \end{array}\right\}. \quad (3.6)$$

Eliminating the pressure between (3.3) and (3.4), we obtain an equation for the vorticity in the plane perpendicular to the wave propagation direction

$$\nu\nabla_L^4\psi^{(2)} - \nabla_L^2\psi_t^{(2)} = -fu_c^{(2)} + 8\kappa l^3e^{\gamma c}\cos\gamma c\sin 2lb. \quad (3.7)$$

We note from the right-hand side of (3.7) that the effects of rotation and friction generate horizontal secondary vorticity. In particular, the viscous source is confined to a thin boundary layer near the surface.

We assume zero mean external horizontal stress to $O(\epsilon^2)$ at the free surface. By insertion from (2.7)–(2.9), the boundary conditions (A.8) and (A.9) reduce to

$$u_c^{(2)} = -\frac{4\kappa^3k}{\sigma}(1 + \cos 2lb), \quad c = 0 \quad (3.8)$$

$$v_c^{(2)} + w_b^{(2)} = \frac{4\kappa^4 l}{\sigma\gamma}\sin 2lb, \quad c = 0. \quad (3.9)$$

4. Steady solutions in a nonrotating fluid

a. Roll motion

In drift problems like the present one, where we start out with an established permanent wave field, it is no unique way of determining the initial conditions for the mean drift solution. Therefore, we shall not consider its transient development, but merely write down the asymptotic steady limit.

The vertical drift velocity $w^{(2)}$ must be zero at the free surface, or from (3.6)

$$\psi_b^{(2)} = 0, \quad c = 0. \quad (4.1)$$

By utilizing this, the condition (3.9) reduces to

$$\psi_{cc}^{(2)} = -\frac{4\kappa^4 l}{\sigma\gamma}\sin 2lb, \quad c = 0. \quad (4.2)$$

Furthermore, we assume that the mean drift velocities vanish at large depths, i.e.,

$$u^{(2)}, \psi_b^{(2)}, \psi_c^{(2)} \to 0; \quad c \to -\infty. \quad (4.3)$$

By utilizing the boundary conditions above, the solution of (3.7) in the nonrotating steady case can be written

$$\psi \equiv \epsilon^2\psi^{(2)} = \frac{\zeta_0{}^2\sigma\kappa}{\gamma}[4l^3\kappa^{-2}\gamma^{-1}(e^{2lc} - lce^{2lc} - e^{\gamma c}\cos\gamma c) - \kappa ce^{2lc}]\sin 2lb. \quad (4.4)$$

In deriving this solution we have assumed that l is the only wave number in the y-direction having energy different from zero. This is consistent with the assumption of a bimodal spectrum (2.4).

We notice that the solution above describes roll motion in the surface layer. The rolls have axes aligned along the propagation direction of the wave system, and the direction of circulation is opposite in adjacent rolls. See the conceptual sketch in Fig. 1. [In this sketch the x-axis, for aesthetic reasons, is placed along a nodal line instead of through crests and troughs as it should have been according to the primary wave solution (2.7)–(2.9).]

The speed of the circulation depends on the viscosity, since the amplitude of the mean drift currents is proportional to $\gamma^{-1} = (2\nu/\sigma)^{1/2}$. In the present analysis ν is assumed to be small, or more precisely, $\kappa/\gamma \ll 1$, as stated in (2.11). This does not mean, however, that the induced circulations are negligible. In the ocean surface layer it is reasonable to apply a turbulent eddy viscosity which is much larger than the molecular value of ν. For typical wavelengths of wind-generated surface waves, manifest circulations may occur without violating the condition (2.11).

The drift velocity components in the cross-wave plane derived from (4.4) become

$$v = \frac{\zeta_0{}^2\sigma}{\kappa\gamma}\left[4l^3e^{\gamma c}(\cos\gamma c - \sin\gamma c) + \kappa^3(1 + 2lc)e^{2lc} - \frac{4l^4}{\gamma}(1 - 2lc)e^{2lc}\right]\sin 2lb \quad (4.5)$$

$$w = \frac{2\zeta_0{}^2\sigma l}{\kappa\gamma}\left[-\kappa^3ce^{2lc} + \frac{4l^3}{\gamma}(e^{2lc} - lce^{2lc} - e^{\gamma c}\cos\gamma c)\right]\cos 2lb. \quad (4.6)$$

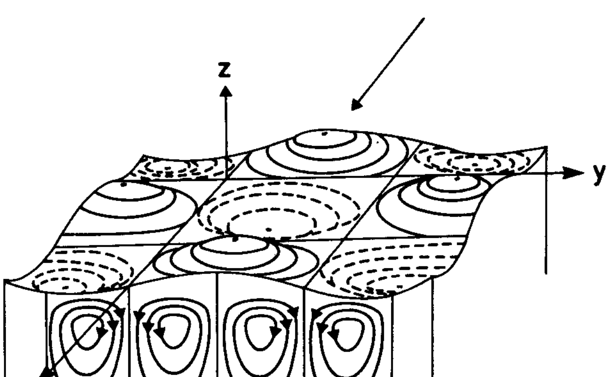

FIG. 1. Sketch of the induced roll motion together with the primary wave field (advancing in the direction of the arrow).

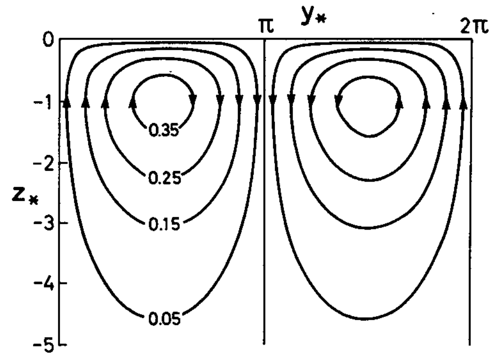

FIG. 2. Streamlines in the plane perpendicular to the wave propagation direction from the dimensionless streamfunction $\psi_* = \psi\gamma/\zeta_0{}^2\sigma\kappa$, where ψ is given by (4.4) and $y_* = 2lb$, $z_* = 2lc$; see the text for details.

As an example of a typical sea state, we take $\zeta_0 = 2$ m, $\nu = 100$ cm^2 s^{-1}, $\lambda = 30$ m and $\theta = 30°$. This means that $\sigma = 1.43$ s^{-1} and $\kappa = 0.21$ m^{-1}. Accordingly, $\kappa/\gamma = 0.025$ which should be well inside (2.11). Using these values we have plotted in Fig. 2 a pair of rolls computed from the dimensionless stream function $\psi_* = \psi\gamma/\zeta_0{}^2\sigma\kappa$, where ψ is given by (4.4). The cell width is $\pi/2l$ and maximum horizontal velocity towards the convergence zones occur at the surface in the middle of each cell. In the present example the value here is approximately 4 cm s^{-1}; see Fig. 3 where we have plotted the vertical variation of the horizontal crosswave drift velocity in the middle of a cell. Maximum downwelling/upwelling velocities occur at the vertical boundaries between the cells. In the present example they are around 1 cm s^{-1} at a depth of about ⅓ of the cell width, see Fig. 3 for details. In a recent review paper Dyke and Barstow (1983) summarize that typical upwelling velocities are 1–2 cm s^{-1} in Langmuir circulations, while downwelling velocities range from 2–6 cm s^{-1}. Although this asymmetry does not show up in the present simple second-order theory, it is worth noting that our calculated circulation velocities are of the same order of magnitude as those quoted by Dyke and Barstow, although on the lower side.

It is particularly interesting to note that the convergence zones occur at the *nodes* of the primary wave system; see the sketch in Fig. 1. This is analogous to the acoustic streaming problem visualized in Kundt's dust tube. Here the frictional influence of the walls induces secondary streaming motions which carry the dust particles towards the nodes of the standing sound wave system (Batchelor, 1967, p. 363).

In the CL-theory for the generation of Langmuir circulations, as reviewed by Leibovich (1983), one also finds that downwelling occurs below nodes of the primary wave system. Although this is the same result as obtained here, the reason for it is entirely different; since the CL-instability mechanism basically is inviscid (Leibovich and Ulrich, 1972).

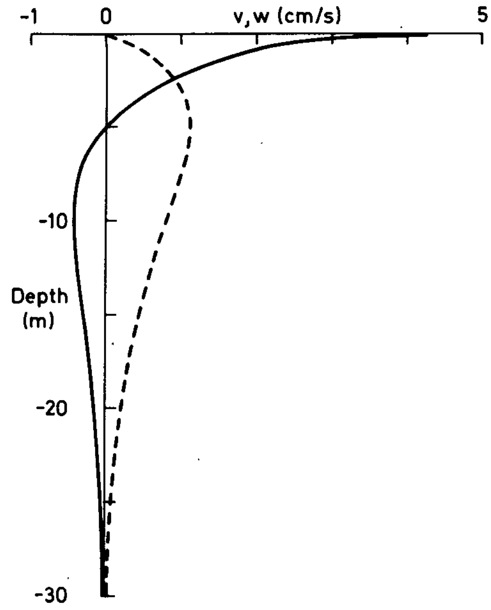

FIG. 3. Drift velocities v at $y_* = \pi/2$ (solid line) and w at $y_* = 0$ (broken line) from (4.5) and (4.6) vs depth. Values of the parameters correspond to those used in Fig. 2.

b. Horizontally undulating Stokes drift

The equation for the steady nonrotating horizontal drift, $u \equiv \epsilon^2 u^{(2)}$, is obtained from (3.2). For nonzero viscosity we find

$$u_{cc} + u_{bb} = \frac{8\zeta_0^2\sigma k}{\kappa^2}[(\kappa^4 + k^4\cos 2lb)e^{2\kappa c} - \kappa\gamma(\kappa^2 + k^2\cos 2lb)e^{\gamma c}(\cos\gamma c - \sin\gamma c)]. \quad (4.7)$$

The boundary condition at the surface is given by (3.8), or in terms of u

$$u_c = -4\zeta_0^2\sigma k\kappa(1 + \cos 2lb), \quad c = 0. \quad (4.8)$$

For $l = 0$, the problem above reduces to that solved in I for a single wave train of amplitude $2\zeta_0$. The important point in that case was the fact that a first integral of (4.7) satisfied the boundary condition (4.8) exactly. This is also the case now if we consider the horizontal drift solution averaged over one wavelength in the y-direction. If this was not so, further integration would yield infinite drift velocities in an infinitely deep ocean; see the discussion in I.

For the horizontally undulating case with $l \neq 0$, the situation is somewhat different. Now a particular solution $u^{(P)}$ of (4.7) does not fulfill the boundary condition (4.8). But since the steady problem is two-dimensional, we can always add a solution $u^{(h)}$ of the homogeneous version of (4.7) which makes the total solution finite. The homogeneous equation is

$$u_{cc}^{(h)} + u_{bb}^{(h)} = 0, \quad (4.9)$$

and the corresponding solution can be written $u^{(h)} = A\exp(2lc)\cos 2lb$. Here A is chosen such that $u = u^{(P)} + u^{(h)}$ satisfies (4.8). We then obtain

$$u = 2\zeta_0^2\sigma k\left[e^{2\kappa c} + \left(\frac{k^2}{\kappa^2}e^{2\kappa c} - \frac{l}{\kappa}e^{2lc}\right)\cos 2lb - 2\frac{\kappa}{\gamma} \times e^{\gamma c}(\cos\gamma c + \sin\gamma c)\left(1 + \frac{k^2}{\kappa^2}\cos 2lb\right)\right] + O\left(\frac{\kappa^2}{\gamma^2}\right). \quad (4.10)$$

This solution also appears to be novel, since, to the author's knowledge, the drift due to obliquely propagating wave trains in the presence of viscosity has not been reported in the literature. The undulating horizontal drift solution (4.10) has some interesting features. Consider the surface drift $u_0 = u(c = 0)$. Neglecting the small $O(\kappa/\gamma)$ term, we obtain

$$u_0 = 2\zeta_0^2\sigma k[1 + (\cos^2\theta - \sin\theta)\cos 2lb] \quad (4.11)$$

where $\cos\theta = k/\kappa$ and $\sin\theta = l/\kappa$. In the special case of $\cos^2\theta - \sin\theta = 0$, i.e. $\theta = 38.2°$, the surface drift (4.11) is uniform in the crosswave direction. For $\theta < 38.2°$ the Stokes drift at the surface has a minimum at nodes of the primary wave system, while for $\theta > 38.2°$ maximum surface velocity occurs here; see Fig. 4. This is different from the undulating drift solution obtained from inviscid theory, where the maximum drift velocity always occurs at crests or troughs of the primary wave system (Wiegel, 1964, p. 58).

5. Effect of rotation on mean drift solutions

At times comparable with, or larger than the inertial period, rotation will influence the mass transport solutions. Here we shall focus on the mean horizontal drift, and we average our solutions over one wavelength in the y-direction, in addition to the former x-averaging process. Denoting this operation by a tilde, we may define a complex mean horizontal drift velocity by

$$W = \epsilon^2\tilde{u}^{(2)} + i\epsilon^2\tilde{v}^{(2)} \quad (5.1)$$

The steady versions of (3.2) and (3.3) reduce to

$$\nu W_{cc} - ifW = 8\nu\zeta_0^2\sigma k\kappa^2[e^{2\kappa c} - \gamma\kappa^{-1}e^{\gamma c}(\cos\gamma c - \sin\gamma c)] \quad (5.2)$$

subject to

$$W_c = -4\zeta_0^2\sigma k\kappa, \quad c = 0. \quad (5.3)$$

Averaging in the y-direction means that we have

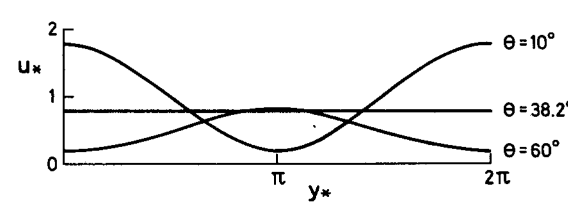

FIG. 4. Horizontal nondimensional Stokes drift $u_* = u_0/2\zeta_0^2\sigma\kappa$ at the surface from (4.11) vs $y_* = 2lb$ for three different wave angles θ.

assumed $l \neq 0$. If we had taken $l = 0$ from the outset, the factor multiplying the parenthesis on the right-hand side of (5.2) would have been $16\nu\zeta_0^2\sigma k^3$, which again is exactly the same as that obtained from I for a single wave train of amplitude $2\zeta_0$. By analogy with the results of that paper, the solution of (5.2) may be written

$$W = \frac{2\zeta_0^2\sigma k}{1+i}\frac{D}{L}\left(1 - \frac{1}{1 - 2iL^2/D^2}\right)e^{(1+i)c/D} + \frac{2\zeta_0^2\sigma k}{1 - 2iL^2/D^2}e^{c/L} - 4\zeta_0^2\sigma k\frac{\kappa}{\gamma}e^{\gamma c}(\cos\gamma c + \sin\gamma c) \tag{5.4}$$

where $L = 1/2\kappa$ and $D = (2\nu/f)^{1/2}$ are the Stokes and Ekman depths, respectively. We note from this solution that, for given overall wavenumber κ, the horizontally averaged mass transport is directly proportional to the wavenumber in the x direction. For relevant values of the parameters L and D for wind-generated waves, the surface mass transport will be slightly deflected to the right (on the Northern Hemisphere) of the wave propagation direction. For further discussion we refer to I.

6. Summary and discussion

The present paper demonstrates that two monochromatic surface waves with equal amplitude and frequency propagating at oblique angles to each other may produce roll motion near the sea surface. The rolls consist of counter-rotating pairs of cells with axes aligned along the wave propagation direction. The circulation is second order in wave amplitude and depends on the size of the kinematic (eddy) viscosity. The phenomenon is analogous to the acoustic streaming problem encountered in Kundt's dust tube experiment.

The generation mechanism does not require any mean wind stress. However, since the circulation depends on the eddy viscosity, the presence of a mean wind field may help indirectly by increasing the turbulence in the surface layer. A small value of ν ($\sim$0.01 cm^2 s^{-1}) could be the reason why these circulations were not detected in the experiments of Faller and Caponi (1978).

The streaming motion discussed here depends on the primary wave field, which we have assumed to be nondecaying in amplitude due to energy input from a weak periodic wind-stress, given by Eq. (2.12). As a result we obtain a boundary condition (3.9) for the vortex motion involving a small mean nonzero Lagrangian vorticity at the surface. By considering different bimodal wave systems (with amplitude increasing or decreasing in time), different boundary conditions for the streaming motion will result. However, this will not alter the fundamental roll producing mechanism described here, since this mechanism operates whenever one has a nonzero viscous source term on the right-hand side of the vorticity Eq. (3.7), but different circulation rates and cell depths may of course occur.

In the case of a mean wind stress in the wave direction the present generation mechanism will still operate, but now the Craik-Leibovich mechanisms (CL1 and/or CL2 in the terminology of Faller and Caponi, 1978) may be effective as well. The cell width obtained here, $L = \pi/2l$, is the same as that obtained by the CL-mechanism.

When we want to apply this sort of theory to the ocean, we are faced with the problem that the theory requires the primary wave field to be phase-locked for a considerable amount of time, a condition which is hardly met in a real sea state; see the discussion by Leibovich (1983). Now the theory may be adjusted to give room for some modification. If we allow the two wave trains to have a small phase difference α, where α could be a slowly varying function of time, computations show that the quasi-steady nonrotating roll problem in that case is governed by

$$\nu\nabla_L^4\psi^{(2)} = 8l^3\kappa e^{\gamma c}\cos\gamma c\sin(2lb - \alpha) \tag{6.1}$$

subject to

$$\left.\begin{aligned}\psi_b^{(2)} &= 0, \quad c = 0\\ \psi_{cc}^{(2)} &= -\frac{4\kappa^4 l}{\sigma\gamma}\sin(2lb - \alpha), \quad c = 0\end{aligned}\right\}. \tag{6.2}$$

We notice that this merely means a slow, small shift in the horizontal position of the parallel roll system which we already have described, i.e., solution (4.4).

Stripe structures are often observed in connection with oil slicks in the open sea. Released crude oil will relatively quickly form a very viscous "mousse" phase which will cover the sea surface. The water content of the "mousse" is 60–75%, and the viscosity may be as high as 30 000 cP (Sørstrøm *et al.*, 1984). The presence of surface contamination invalidates the use of a stress-free condition at the surface in the absence of wind, since the very viscous thin surface layer may sustain shear stress in the fluid, see the discussion by Huang (1970) or Craik (1982). The drift solutions are very sensitive to changes in the surface boundary conditions, and a nonzero mean Lagrangian stress at the fluid boundary will enhance the induced circulations. This means that the generation of rolls discussed here may be favoured by the presence of an oil film.

It is of considerable interest to predict the horizontal spread of neutrally buoyant material near the ocean surface. For the system of waves considered here, the solution for the horizontally averaged wave drift is analogous to the mean drift for single waves, as studied in I. If there is an additional mean wind stress, an Ekman current must be added to yield the total drift picture. With a suitably varying eddy viscosity, the total drift current obtained in this way may explain commonly observed drift paths at the sea surface.

In a recent note Amstutz and Samuels (1984) propose that the Langmuir circulation as developed by Craik and Leibovich should be taken into account when discussing the total wind-drift current. The idea is interesting, but the present investigation shows that the wave-drift problem first should be more carefully examined. This should, in turn, form the basis of a more comprehensive analysis of the wave-drift/wind-current interaction problem.

APPENDIX

Surface Boundary Conditions

We introduce horizontal and vertical external stresses $\tau^{(x)}$, $\tau^{(y)}$, $\tau^{(z)}$ at the sloping free surface $c = 0$. By utilizing the series expansions (2.3), the dynamic boundary conditions in Lagrangian form may be written

$$\tau^{(x)} = \epsilon\mu(x_{tc}^{(1)} + z_{ta}^{(1)}) + \epsilon^2[\mu(x_{tc}^{(2)} + z_{ta}^{(2)} + x_{tc}^{(1)}x_a^{(1)} - x_{ta}^{(1)}x_c^{(1)} + x_{tc}^{(1)}y_b^{(1)} - x_{tb}^{(1)}y_c^{(1)} + z_{ta}^{(1)}z_c^{(1)} - z_{tc}^{(1)}z_a^{(1)} + z_{ta}^{(1)}y_b^{(1)} - z_{tb}^{(1)}y_a^{(1)} - x_{tb}^{(1)}z_b^{(1)} - y_{ta}^{(1)}z_b^{(1)} - 2x_{ta}^{(1)}z_a^{(1)}) + p^{(1)}z_a^{(1)}] + O(\epsilon^3), \quad c = 0 \qquad \text{(A1)}$$

$$\tau^{(y)} = \epsilon\mu(y_{tc}^{(1)} + z_{tb}^{(1)}) + \epsilon^2[\mu(y_{tc}^{(2)} + z_{tb}^{(2)} + y_{tc}^{(1)}y_b^{(1)} - y_{tb}^{(1)}y_c^{(1)} + y_{tc}^{(1)}x_a^{(1)} - y_{ta}^{(1)}x_c^{(1)} + z_{tb}^{(1)}z_c^{(1)} - z_{tc}^{(1)}z_b^{(1)} + z_{tb}^{(1)}x_a^{(1)} - z_{ta}^{(1)}x_b^{(1)} - x_{tb}^{(1)}z_a^{(1)} - y_{ta}^{(1)}z_a^{(1)} - 2y_{tb}^{(1)}z_b^{(1)}) + p^{(1)}z_b^{(1)}] + O(\epsilon^3), \quad c = 0 \qquad \text{(A2)}$$

$$\tau^{(z)} = \epsilon(-p^{(1)} + 2\mu z_{tc}^{(1)}) + \epsilon^2[-p^{(2)} + \mu(2z_{tc}^{(2)} + 2z_{tc}^{(1)}x_a^{(1)} + 2z_{tc}^{(1)}y_b^{(1)} - 2z_{tb}^{(1)}y_c^{(1)} - 2z_{ta}^{(1)}x_c^{(1)} - x_{tc}^{(1)}z_a^{(1)} - z_{ta}^{(1)}z_a^{(1)} - y_{tc}^{(1)}z_b^{(1)} - z_{tb}^{(1)}z_b^{(1)})] + O(\epsilon^3), \quad c = 0 \qquad \text{(A3)}$$

where $\mu = \rho\nu$. We assume that the horizontal components of the external stresses are zero to $O(\epsilon)$. Hence, from (A1) and (A2):

$$\left.\begin{aligned} x_{tc}^{(1)} + z_{ta}^{(1)} = 0, \quad c = 0 \\ y_{tc}^{(1)} + z_{tb}^{(1)} = 0, \quad c = 0 \end{aligned}\right\} . \qquad \text{(A4)}$$

To sustain the wave system against friction, i.e. require that the frequency σ should be real, $p^{(1)}$ and $z^{(1)}$ at the surface must satisfy the relation

$$p^{(1)} = -2\mu z_{tc}^{(1)}, \quad c = 0 \qquad \text{(A5)}$$

see also Lamb (1932, p. 629). This means that the vertical external stress (A3) at the free surface to $O(\epsilon)$ is given by

$$\tau^{(z)} = \epsilon(-p^{(1)} + 2\mu z_{tc}^{(1)}) = 4\epsilon\mu z_{tc}^{(1)}. \qquad \text{(A6)}$$

The continuity equation (2.2) to $O(\epsilon)$ reduce to

$$x_a^{(1)} + y_b^{(1)} + z_c^{(1)} = 0. \qquad \text{(A7)}$$

By utilizing (A4), (A5), (A7), and averaging over one wavelength in the x-direction, we obtain to $O(\epsilon^2)$ from (A1)–(A3)

$$\bar{\tau}^{(x)} = \mu\epsilon^2(\bar{x}_{tc}^{(2)} - 4\overline{z_{tc}^{(1)}z_a^{(1)}}), \qquad c = 0 \qquad \text{(A8)}$$

$$\bar{\tau}^{(y)} = \mu\epsilon^2(\bar{y}_{tc}^{(2)} + \bar{z}_{tb}^{(2)} - 4\overline{z_{tc}^{(1)}z_b^{(1)}}), \quad c = 0 \qquad \text{(A9)}$$

$$\bar{\tau}^{(z)} = \epsilon^2[-\bar{p}^{(2)} + 2\mu(\bar{z}_{tc}^{(2)} + \overline{z_{ta}^{(1)}z_a^{(1)}} + \overline{z_{tb}^{(1)}z_b^{(1)}} - \overline{z_{tc}^{(1)}z_c^{(1)}})], \quad c = 0. \qquad \text{(A10)}$$

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