Accelerations in Steep Gravity Waves

M. S. LONGUET-HIGGINS

Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, England, and Institute of Oceanographic Sciences, Wormley, Godalming, Surrey

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ABSTRACT

Surface accelerations can be measured in at least two ways: 1) by a fixed vertical wave gauge, 2) by a free-floating buoy. This gives rise to two different vertical accelerations, called respectively "apparent" and "real", or Langrangian. This paper presents the first accurate calculations of the two types of acceleration, for symmetric waves of finite steepness.

The apparent upwards acceleration is always less than 0.24*g*, but the apparent downwards acceleration is unlimited. The real vertical acceleration is smoother than the apparent acceleration, and always lies between 0.30*g* and −0.39*g*.

The (real) horizontal acceleration is studied, and shown to be greater in amplitude than the real vertical acceleration.

The results are discussed in relation to proposed limits on the acceleration in random seas.

1. Introduction

Various criteria for the breaking of ocean waves have been proposed. Thus Phillips (1958) suggested a limiting acceleration of $-g$ at the free surface. Snyder and Kennedy (1983) have used the value of $-\frac{1}{2}g$, the acceleration in a Stokes 120° corner flow (see Longuet-Higgins, 1963). In reinterpreting some experimental results of Ochi and Tsai (1983), Srokosz (1986) has proposed a limiting downward acceleration of $-0.4g$. In the present note we do not propose any new criterion; we simply inquire what are the actual accelerations in progressive, irrotational gravity waves of finite amplitude.

The results are paradoxical. First, a distinction has to be made between the *apparent* vertical acceleration, as measured, for example, by a fixed vertical probe, and the *real* vertical acceleration, that is the acceleration of the fluid particles, as measured ideally by a small, free-floating buoy. In linearized theory these two accelerations are equal, but in waves of finite amplitude they are not.

For example, in the limiting wave (Fig. 1), which is discussed in Section 3, the apparent acceleration is positive and approximately constant at $0.22g$ over most of the free surface, but with a sharp negative spike at the wave crest (see Fig. 2). The real acceleration, on the other hand, is almost sinusoidal, varying between $0.30g$ and $-0.25g$ (see Fig. 3). At the crest there is a nonuniformity, treated by the theory of the almost highest wave (Section 4) where the vertical acceleration has the limiting value of $-0.388g$.

Waves of arbitrary amplitude are studied in Section 4. Here the simple but accurate method of calculation introduced recently by Longuet-Higgins (1984, 1985a) is used to evaluate both the real and apparent accelerations (Figs. 4 and 5). In Section 6 the (real) horizontal accelerations are also considered, and it is found, perhaps surprisingly, that these generally exceed in amplitude the corresponding vertical components.

A discussion follows in Section 7.

2. Real and apparent accelerations

For definiteness, consider steady, uniform gravity waves in two dimensions, travelling with phase-speed c in the positive x-direction. If the surface elevation is $y = \eta(x, t)$ then the apparent vertical acceleration, as seen by a fixed vertical wave gauge, is η_{tt}. On the other hand the real, or Lagrangian, acceleration is $D\mathbf{u}/Dt$, where $\mathbf{u} = (u, v)$ is the velocity vector and $D/Dt = \partial/\partial t + \mathbf{u}\cdot\nabla$ denotes differentiation following the motion.

Transforming now to a frame of reference moving with speed c, the fluid motion becomes steady. In the new coordinates, η_t is replaced by $-c\eta_x$. The apparent vertical velocity is thus proportional to the surface slope η_x. The apparent acceleration a_E is given by

$$a_E = c^2\eta_{xx}. \tag{2.1}$$

Consider on the other hand the real, or Lagrangian acceleration. The axes being in steady motion, the particle acceleration is physically the same as in the original reference frame but is now

$$\mathbf{a}_L = \mathbf{u}\cdot\nabla\mathbf{u}. \tag{2.2}$$

We note that at the free surface this acceleration can be resolved into a tangential component $-g\sin\alpha$, where

$\alpha = \arctan\eta_x$ is the angle of inclination, together with a normal component q^2/R, where q is the particle speed $(u^2 + v^2)^{1/2}$ and R is the radius of curvature.

When the motion is irrotational with complex velocity potential $\chi = \phi + i\psi$ we have

$$\mathbf{a}_L = (u - iv)(u_x + iv_x) = \chi_z \chi_{zz}^* \tag{2.3}$$

where $z = x + iy$ and a star denotes the complex conjugate. Since it is often convenient to express the coordinates (x, y) in terms of ϕ and ψ, rather than vice-versa, we give here the corresponding expression for $\mathbf{a}$, namely

$$\mathbf{a}_L = -(z_{\chi\chi}/z_\chi{}^3)^*/z_\chi \tag{2.4}$$

as can easily be shown. This can also be written

$$\mathbf{a}_L = -q^6 z_{\chi\chi}^* z_\chi{}^2 \tag{2.5}$$

since $z_\chi z_\chi^* = 1/q^2$. Now in gravity waves we may choose the origin of y so that at the free surface Bernoulli's equation is simply

$$q^2 = -2gy. \tag{2.6}$$

Then the horizontal and vertical components of the acceleration are given by the real and imaginary parts of

$$\mathbf{a}_L = (2gy)^3(x_{\phi\phi} - iy_{\phi\phi})(x_\phi + iy_\phi)^2. \tag{2.7}$$

To plot the apparent acceleration (2.1) as a function of the time t we may use the apparent time

$$t = x/c. \tag{2.8}$$

For the Lagrangian accelerations it is more natural to use the orbital time, that is to say the elapsed time following a particle. For steady motions this is given by the simple formula

$$t_p = \int q^{-2} d\phi \tag{2.9}$$

the integral being taken along a streamline (see Longuet-Higgins 1979a). So for particles at the free surface we have

$$t_p = -\frac{1}{2g}\int \eta^{-1} d\phi. \tag{2.10}$$

3. Waves of limiting amplitude

Consider a gravity wave of given length L and of limiting amplitude a on deep water, as shown in Fig. 1. The profile being roughly parabolic, η_{xx} is nearly constant, so by Eq. (2.1) we expect that the apparent acceleration a_E will be nearly constant also. What is its value? Near the crest, the surface is inclined at an angle of $\pm 30°$ to the horizontal, hence the apparent vertical velocity is $\pm c/\sqrt{3}$. Thus in one wave period T, between the passage of one crest and the next, the velocity changes from $-c/\sqrt{3}$ to $+c/\sqrt{3}$. The mean acceleration is therefore $2c/\sqrt{3}T$. When divided by g we have for the mean upwards acceleration

$$\frac{\bar{a}_E}{g} = \frac{2}{\sqrt{3}}\frac{c}{gT} = \frac{2}{\sqrt{3}}\frac{c^2}{gL}, \tag{3.1}$$

since $T = L/c$. Introducing the linear phase speed $c_0 = (gL/2\pi)^{1/2}$ and using the value $c^2/c_0{}^2 = 1.1931$ (Longuet-Higgins 1975), one finds

$$\frac{\bar{a}_E}{g} = \frac{1}{\pi\sqrt{3}}\frac{c^2}{c_0{}^2} = 0.2193. \tag{3.2}$$

At the crest itself the velocity changes suddenly from $c/\sqrt{3}$ to $-c/\sqrt{3}$, so the acceleration is a sharp, negative delta-function, exactly counterbalancing the positive acceleration over the rest of the wave.

Accurate calculations of limiting gravity waves have been presented by many authors, particularly Williams (1981), who gives the velocity components (u, v) at points on the free surface (see his Table 12). From these values one immediately finds $\eta_x = v/u$ and hence the derivative η_{xx}. The apparent acceleration is plotted in Fig. 2 (full curve). The broken line indicates the mean upwards acceleration (3.2). It can be seen that over most of the wave the acceleration lies between $0.203g$ and $0.242g$, differing from the mean by less than 10 percent. As the crest is approached, the acceleration falls to zero, on account of the vanishing curvature there. At the crest itself the acceleration is negative and infinite.

Clearly this behavior is far from sinusoidal.

Consider on the other hand the Lagrangian accelerations. It has been shown elsewhere (Longuet-Higgins, 1979b) that the motion of a given particle in the surface of a limiting wave is approximately the same as the motion of the bob of a freely swinging pendulum of length L. In one-half swing of the pendulum the bob falls and rises from its highest level, (at which the angle of inclination α of the pendulum with the vertical is 30°) through its lowest point, where $\alpha = 0$, up to its next highest point, where $\alpha = -30°$. We may therefore expect the vertical motion to be smooth and almost sinusoidal. Near a wave crest where q vanishes, the acceleration is $-g \sin\alpha$ along the particle path, so the vertical component is

$$-g \sin^2\alpha = -0.25g. \tag{3.3}$$

At the bottom of the swing the upwards acceleration would be

$$q^2/L = 2gH/L = 2g(1 - \sqrt{3}/2) = 0.268g \tag{3.4}$$

according to our approximate model. The total time T_p is the time taken for a half-swing of the pendulum, that is to say

$$T_p = 1.074\pi(L/g)^{1/2} = 1.35T_0 \tag{3.5}$$

where $T_0 = (2\pi L/g)^{1/2}$ is the period of a linear wave of length L. The excess of T_p over T_0 can be viewed as a Doppler shift due to the mean forward displacement of particles associated with the "Stokes drift."

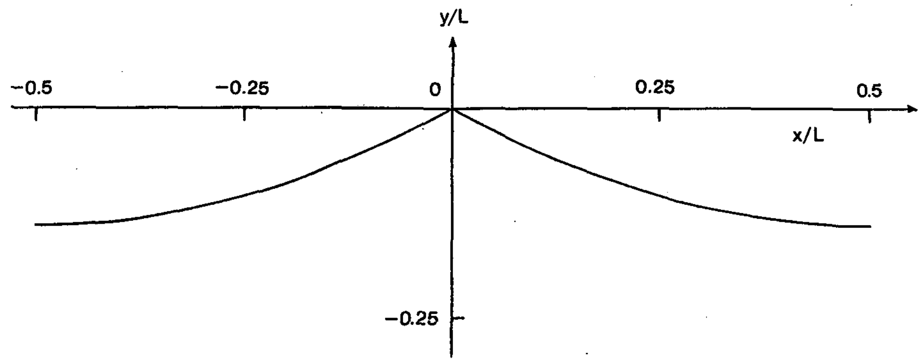

FIG. 1. Profile of a limiting wave on deep water.

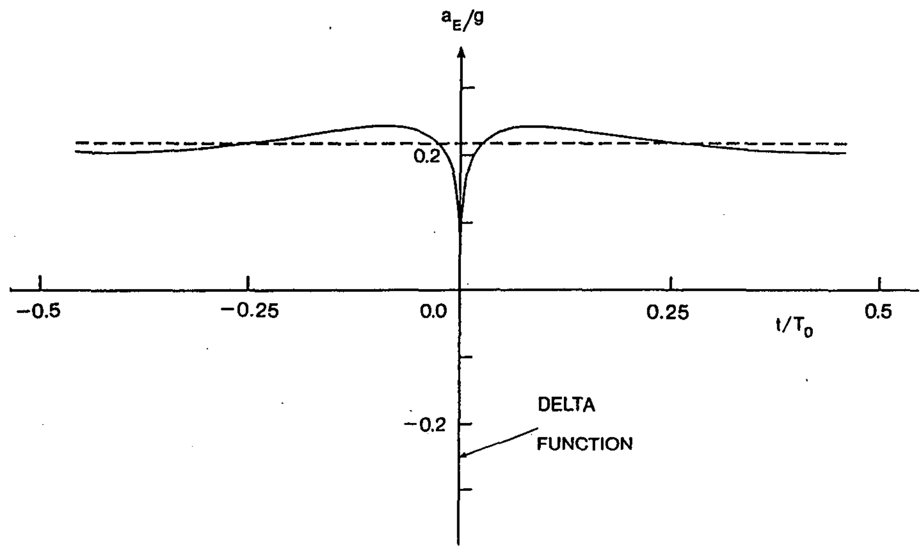

FIG. 2. Apparent vertical acceleration in a limiting wave as a function of the orbital time.

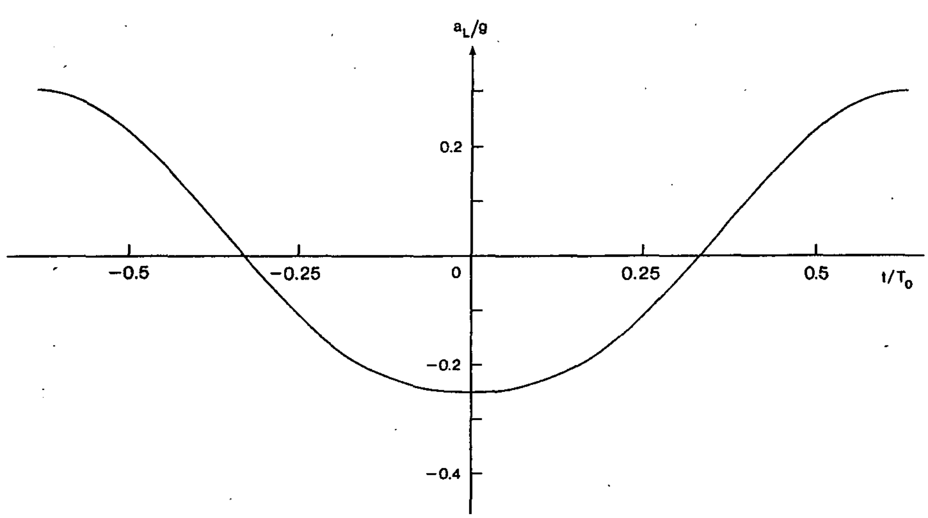

FIG. 3. As in Fig. 2 but for real (Lagrangian) vertical acceleration.

Accurate values of the Lagrangian acceleration, taken from Table 12 of Williams (1981) are plotted in Fig. 3. It will be seen that they do indeed follow a smooth, nearly sinusoidal curve which lies, in fact, between $-0.25g$ and $0.301g$, similar to the bounds in (3.2) and (3.3). The orbital times are also tabulated by Williams, the total time T_p being $1.260T_0$, somewhat less than suggested by Eq. (3.5).[1]

At the crest $t_p = 0$ there is a singularity. There, the approach to the limit is nonuniform, depending upon how it is made. The acceleration in the neighborhood of this point will be discussed in Section 5.

4. Waves of arbitrary steepness

For accurate calculations relating to waves of given length L and arbitrary height $2a$ a quick and simple method has been recently demonstrated (Longuet-Higgins 1984, 1985a). This begins with the parametric equation for the free surface

$$\left.\begin{aligned} ky &= \frac{1}{2}a_0 + \sum_1^{\infty} a_n \cos n\theta \\ kx &= \theta + \sum_1^{\infty} a_n \sin n\theta \end{aligned}\right\} \quad (4.1)$$

where $\theta = k\phi/c$, $k = 2\pi/L$ is the wavenumber and ϕ is the velocity potential. Equations (4.1) are in fact the form to which Stokes's well-known expansion is reduced on the streamline $\psi = 0$.

In the past, the coefficients a_n have usually been expanded in powers of some small parameter, representing the wave steepness. But in the papers referred to it is shown to be much simpler and more accurate to find that a_n by direct solution of the quadratic relations

$$\left.\begin{aligned} a_1b_0 + a_0b_1 + a_1b_2 + a_2b_3 + \cdots &= 0 \\ a_2b_0 + a_1b_1 + a_0b_2 + a_1b_3 + \cdots &= 0 \\ a_3b_0 + a_2b_1 + a_1b_2 + a_0b_3 + \cdots &= 0 \\ &\vdots \end{aligned}\right\} \quad (4.2)$$

where

$$b_0 = 1, \quad b_n = na_n, \quad n = 1, 2, 3, \cdots \quad (4.3)$$

for a given value of the wave steepness

$$ak = a_1 + a_3 + a_5 + \cdots. \quad (4.4)$$

The phase speed c is found from

$$a_0b_0 + a_1b_1 + a_2b_2 + a_3b_3 + \cdots = -c^2/c_0^2. \quad (4.5)$$

In this way the a_n can be determined immediately to order as high as 600, with only a few iterations. The accelerations can then be calculated from the expressions given in Section 2. In Eq. (2.1) we may use the relation

$$\eta_{xx} = \frac{1}{x_\theta}\frac{d}{d\theta}\left(\frac{y_\theta}{x_\theta}\right) = (y_{\theta\theta}x_\theta - x_{\theta\theta}y_\theta)/x_\theta^3 \quad (4.6)$$

and in (2.7) the relation

$$a_L/g = (c_0/c)^4(2gy)^3(x_{\theta\theta} - iy_{\theta\theta})(x_\theta + iy_\theta)^2. \quad (4.7)$$

The derivatives x_θ, y_θ, $x_{\theta\theta}$, $y_{\theta\theta}$ are found by summing the series

$$\left.\begin{aligned} kx_\theta &= \sum_0^N b_n \cos n\theta \\ ky_\theta &= -\sum_0^N b_n \sin n\theta \end{aligned}\right\} \quad (4.8)$$

and

$$\left.\begin{aligned} kx_{\theta\theta} &= -\sum_1^N nb_n \sin n\theta \\ ky_{\theta\theta} &= -\sum_1^N nb_n \cos n\theta \end{aligned}\right\} \quad (4.9)$$

respectively.

The rate of convergence of these series depends upon the behavior of the coefficients a_n and $b_n = na_n$ at large values of n. Elsewhere (Longuet-Higgins, 1985b) it has been shown that for steep waves, that is when

$$2.0(ak_{\max} - ak) = \epsilon^2 \ll 1 \quad (4.10)$$

the a_n behave like $n^{-5/3}$ up to values of n of order ϵ^{-3}, and then decreases exponentially like $n^{-2/3}\exp(-n\epsilon^3c_0/c)$. This ensures the ultimate convergence of all the series (4.1), (4.8) and (4.9).

For example, when $ak = 0.40$ we have from (4.10) $\epsilon^2 = 0.087$, and so the exponential behaviour sets in when $n \sim 40$. Thus 100 terms is more than adequate to insure at least four-figure accuracy in the accelerations. These conclusions are borne out by the actual numerical convergence of the solutions as N is increased.

The "apparent" and real vertical accelerations calculated in this way are shown in Figs. 4 and 5 respectively, for the representative wave steepnesses $ak = 0.1$, 0.2, 0.3 and 0.4. Consider first the apparent accelerations (Fig. 4). Though roughly sinusoidal when $ak = 0.1$, at higher values these quickly develop an up-down asymmetry. When $ak = 0.3$, the maximum downwards acceleration is already $-0.78g$, and when $ak = 0.4$ it is off-scale at $-2.78g$. When $ak = ak_{\max} = 0.4432$ it is of course infinite. In contrast, the positive accelerations never exceed the upper bound of $0.24g$ for the steepest wave (Fig. 2).

[1] For the *apparent* accelerations in a limiting wave, the pendulum model is less exact. By (2.1) the apparent acceleration in the trough would be $c^2/L = 0.19g$ but near the crest, $(4/3)^{3/2}c^2/L = 0.29g$.

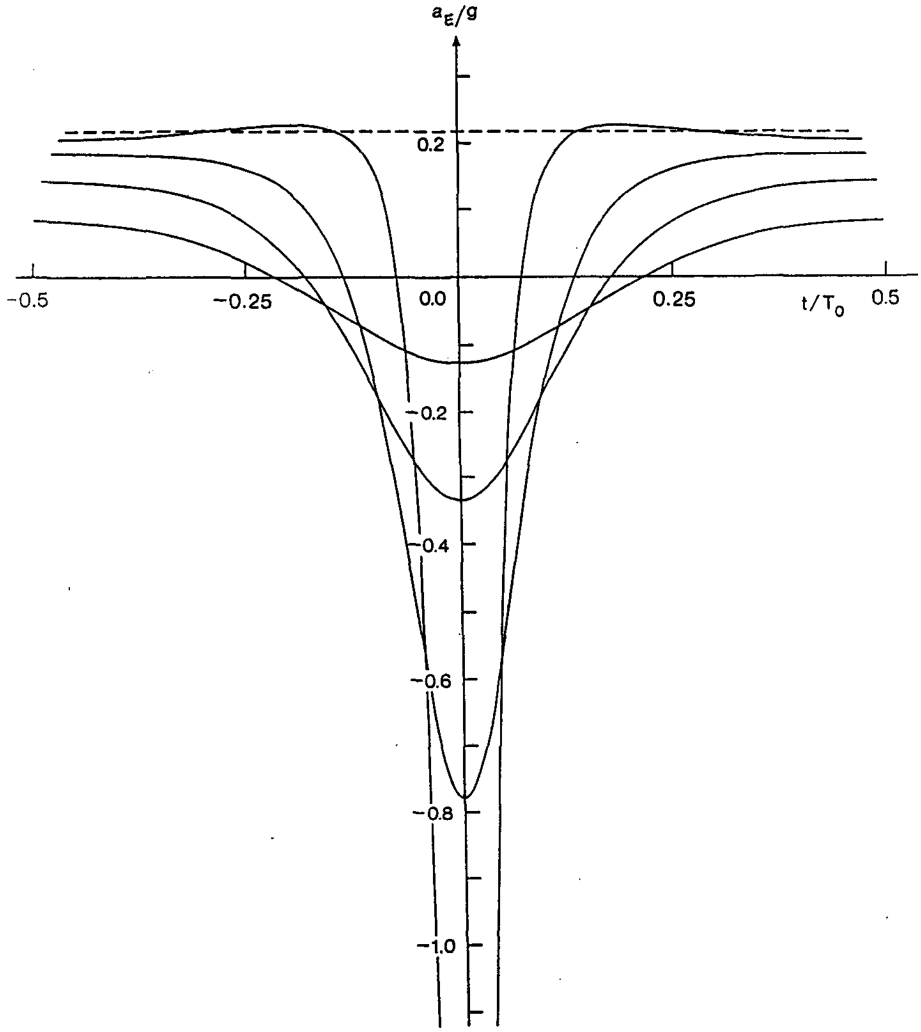

FIG. 4. Apparent vertical acceleration steep surface waves: ak = 0.1, 0.2, 0.3 and 0.4.

Consider on the other hand the real vertical accelerations, in Fig. 5. These are much more symmetric. The upwards acceleration in the trough is always less than $0.3g$, and the downwards acceleration at the crest never exceeds $0.4g$. However, the approach to the limit as $ak \rightarrow ak_{max}$ at $t_p = 0$ demands investigation and will now be discussed.

5. The almost-highest wave

The form of waves approaching their limiting steepness has been studied analytically by Longuet-Higgins and Fox (1977, 1978); see also Longuet-Higgins (1979c). For waves in deep water it is convenient to define the small parameter ϵ by

$$\epsilon = 2^{-1/2}q/c_0 \tag{5.1}$$

where q now denotes the particle speed at the wave crest, in the relative frame of reference, and $c_0 = (g/k)^{1/2}$ is the linear phase speed. As $q/c \rightarrow 0$ so $\epsilon \rightarrow 0$ and the wave crest approaches its limiting form, the Stokes 120° corner flow.

In general, when $\epsilon > 0$, it is shown that there is a small region near the crest, with dimensions of order $\epsilon^2 L$, where the flow has a characteristic form. This may be called the "inner zone" or Zone I. Beyond this is an "intermediate zone" II of dimensions O(ϵ), where the flow is fitted, or matched, to the rest of the wave (the "outer zone" III).

The form of the flow in the inner zone was calculated by Longuet-Higgins and Fox (1977) and is shown in their Fig. 7. The chosen length scale l equals $q^2/2g$ so that with the definition (5.1) for deep-water waves,

$$l = \epsilon^2 c_0{}^2/g = \epsilon^2/k = \epsilon^2 L/2\pi. \tag{5.2}$$

The radius of curvature R at the crest was shown to be $5.15l$, that is

$$R = 0.82\epsilon^2 L. \tag{5.3}$$

Moreover when defined as in (5.1), the parameter ϵ

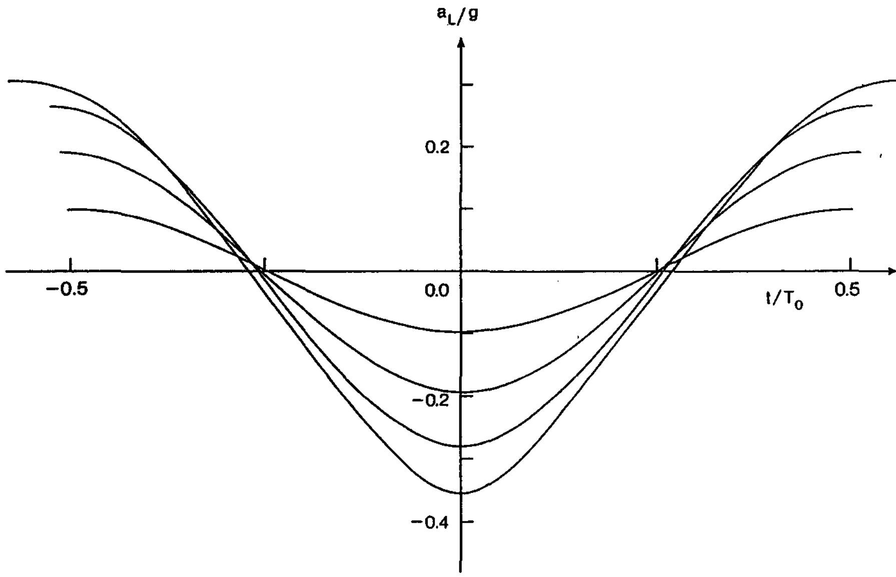

FIG. 5. As in Fig. 4 but for real (Lagrangian) vertical accelerations.

asymptotically satisfies the relation (4.10) above; [see Longuet-Higgins and Fox 1978, Eq. (5.4)].

Most interestingly, it was shown that the downwards acceleration at the crest is not $-0.5g$, as in the outer zone III, but equals $-0.39g$ approximately, a figure confirmed by Williams (1985).

From the numerical coordinates for the surface profile given in Table 3 of Longuet-Higgins and Fox (1977) we have calculated the apparent and real accelerations near the wave crest, and these are shown in Figs. 6 and 7 as functions of the scaled times $\epsilon^2 t/T_0$ and $\epsilon t_p/T_0$. From Fig. 6, as $\epsilon \rightarrow 0$ the apparent acceleration will of course tend to infinity. From Fig. 7, the real acceleration varies from $-0.39g$ at the crest to $-0.25g$ in the outer part of the inner zone, in agreement with Fig. 3.

Interpreting this result physically, we may say that the downwards acceleration $-0.39g$ is the one that would be experienced by a particle in the surface with diameter $d \ll \epsilon^2 L$. The acceleration $-0.25g$ would be experienced by the same particle when at a distance large compared to the radius of curvature but still small compared to L. On the other hand, a small *subsurface* particle may evidently experience a vertical acceleration of almost $-0.5g$.

6. Horizontal accelerations

Of equal interest is the horizontal component of the acceleration. The apparent horizontal acceleration, as seen by a fixed probe, is of course zero. However we would expect the real horizontal acceleration to be

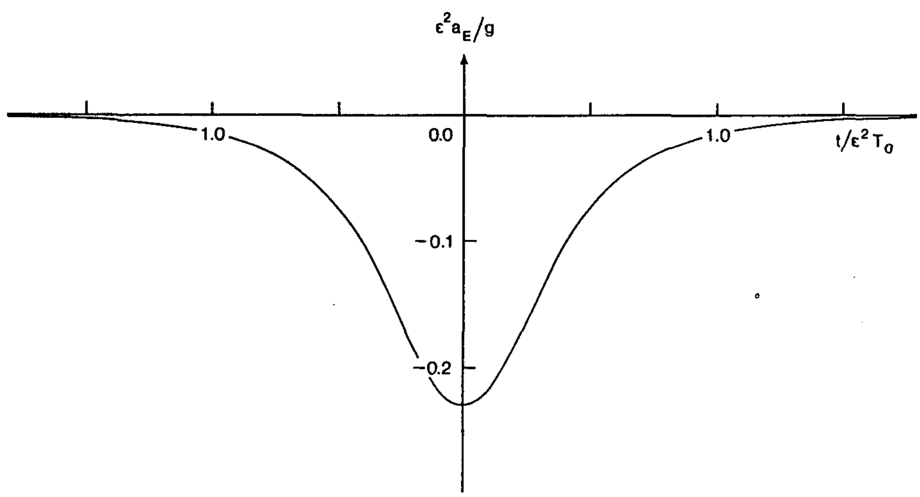

FIG. 6. Apparent vertical acceleration near the crest of an almost-highest wave.

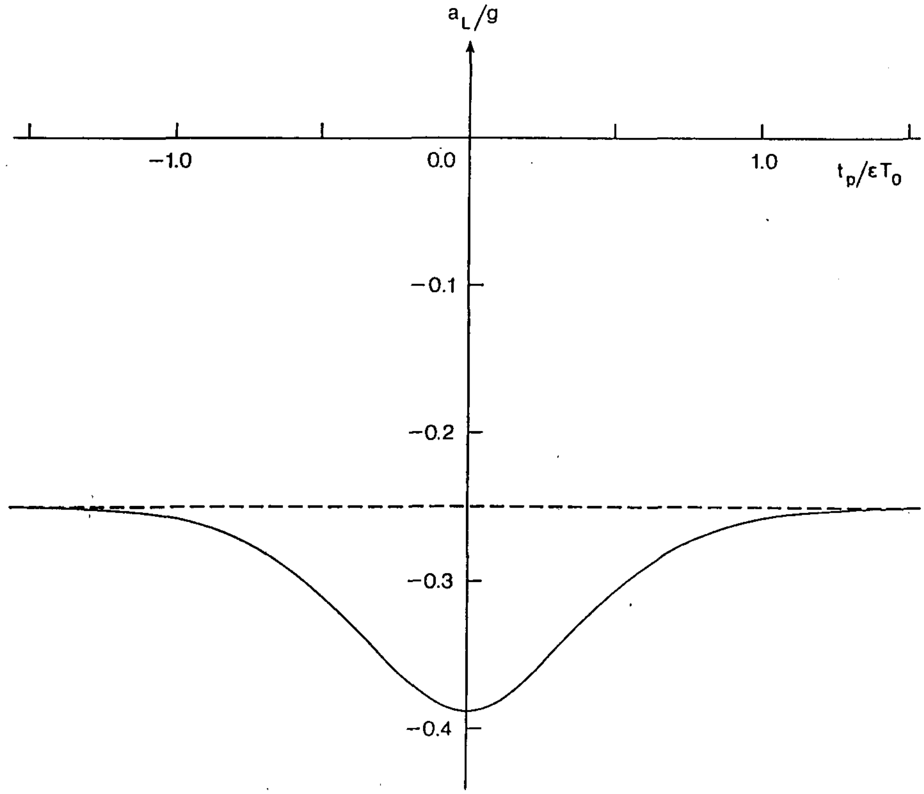

FIG. 7. As in Fig. 6 but for real vertical and horizontal accelerations.

substantial. For, a particle in the trough of a limiting wave is moving backwards, and in only half an orbital period T_p it must be accelerated up to the phase-speed c on reaching the crest.

Quantitatively, we may consider the acceleration in the moving reference frame. Using the pendulum model we see that the mean horizontal acceleration of a particle between trough and crest would be $2q/T_p$, where now q is the particle speed in the trough given by Eq. (3.4). Hence

$$\bar{a}_H \approx \frac{2(2gH)^{1/2}}{1.074\pi(L/g)^{1/2}} \approx \frac{2(2-\sqrt{3})^{1/2}}{1.074\pi} g. \quad (6.1)$$

If the acceleration varied sinusoidally with time, the peak value would be $\pi/2$ times the above value, that is

$$a_{H\,\max} \approx \frac{(2-\sqrt{3})^{1/2}}{1.074} g = 0.48g. \quad (6.2)$$

Actually the value of a_H at $t = 0$ must be $\frac{1}{2}g \cos 30°$ or $0.433g$.

In Fig. 8 are shown the horizontal Lagrangian accelerations, as a function of the orbital time, calculated accurately by the methods of Sections 3 and 4. The outermost curve represents the limiting wave $ak = 0.4432$, and it will be seen that the acceleration, though approximately sinusoidal over the interval $0 < t_p < \frac{1}{2}T_p$ lingers in the neighborhood of its maximum value $0.434g$. For all the other waves, with steepnesses $ak = 0.1$, 0.2, 0.3 and 0.4, the maximum horizontal acceleration is close to the corresponding value of ak.

The maximum values of the accelerations are tabulated in Table 1, for each value of ak. It will be seen that the maximum horizontal acceleration exceeds the maximum (real) vertical acceleration (see Fig. 5). Thus a small float would tend to be tossed to and fro horizontally as much as vertically.

7. Discussion

We have so far considered only the accelerations in regular, symmetric waves, and further thinking is necessary before applying the results to ocean waves which are generally both unsteady and asymmetric. Nevertheless it is very striking that in both the real and apparent vertical accelerations, the upwards acceleration in the trough of a steep wave is more restricted in magnitude than is the downwards acceleration at the crest. If therefore a criterion for wave breaking is to be based on some upper bound for the magnitude of the vertical acceleration it should, paradoxically, be the upwards acceleration in the trough that is restricted, not the downward acceleration at the crest.

The appropriate bound for unsteady waves may very well differ from that for steady waves. As an example,

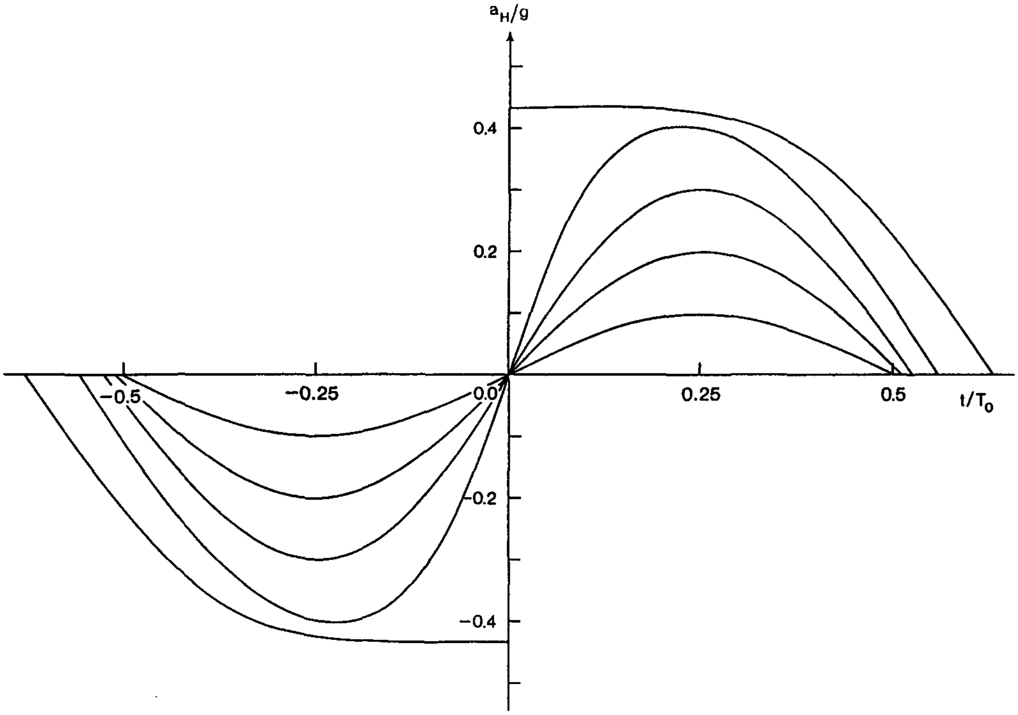

FIG. 8. Real horizontal accelerations when ak = 0.1, 0.2, 0.3, 0.4 and 0.4432.

consider a simple, two-scale model, in which a relatively short gravity waves rides on the back of a longer wave, whose steepness ak is less than the maximum value 0.4432. As is well known, the interaction of the two waves will cause the shorter wave to be at its steepest on the crest of the longer wave and less steep in the longer wave troughs. Ignoring for the present the action of the shorter wave on the longer wave, suppose that the real (Lagrangian) acceleration at the crest of the longer wave is rg, where $r = 0.2$, say. Then the effective value of gravity at the crest of the longer wave is given by

$$g' = (1 - r)g. \tag{7.1}$$

Now suppose that at the crest of the long wave the short wave is of the same steepness as the longer wave. Then at the crest of the short wave the relative downwards acceleration will be $-rg'$, that is $-r(1 - r)g$. This must be added to the previous downwards acceleration $-rg$ to give a total acceleration

$$-r[1 + (1 - r)]g. \tag{7.2}$$

The effective value of gravity at the crest of the shorter wave will thus be

$$g'' = (1 - r)^2 g = (1 - r)g'. \tag{7.3}$$

Now, allowing our imagination full play, let us suppose that on the crest of the shorter wave there is a similar but even shorter wave. This will contribute a further downwards acceleration $-rg''$, and so on. At the crest of the nth wave the effective value of gravity will be given by

$$g^{(n)} = (1 - r)g^{(n-1)} = (1 - r)^n g \rightarrow 0 \tag{7.4}$$

as $n \rightarrow \infty$. Hence it seems possible, by superposing an

TABLE 1. Maximum and minimum values of the accelerations.

			a_E/g		a_L/g		a_H/g
ak	c^2/c_0^2	H/gT^2	Max	Min	Max	Min	max, min
0.1	1.0101	0.0051	0.0833	−0.1251	0.0989	−0.0991	±0.1000
0.2	1.0408	0.0105	0.1420	−0.3357	0.1908	−0.1931	±0.2000
0.3	1.0941	0.0166	0.1831	−0.7795	0.2662	−0.2784	±0.2999
0.35	1.1300	0.0200	0.1971	−1.2814	0.2930	−0.3175	±0.3502
0.40	1.1712	0.0237	0.2050	−2.6753	0.3073	−0.3548	±0.4014
0.4432	1.1931	0.0268	0.2032	$-\infty$	0.3011	−0.388	±0.4340

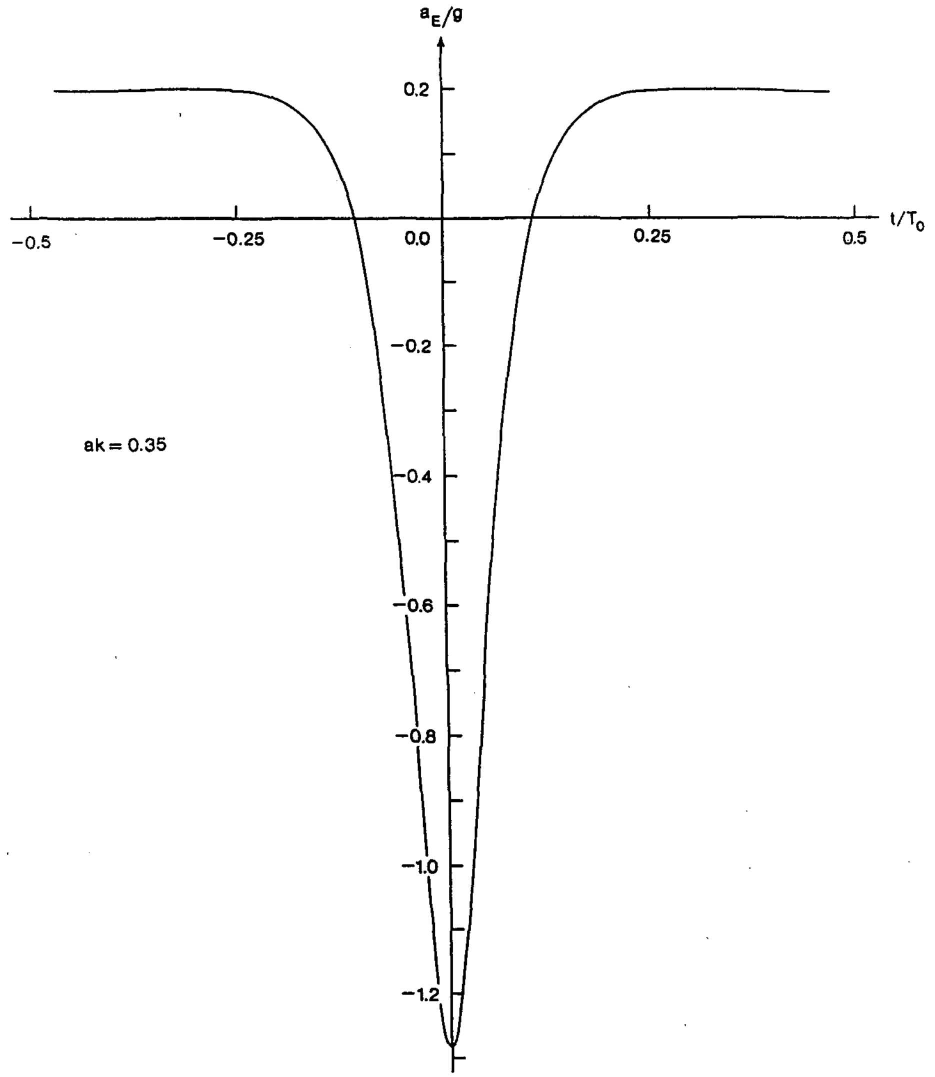

FIG. 9. Apparent vertical acceleration in a wave of steepness ak = 0.35.

infinite sequence of self-similar progressive waves, to achieve a real downwards acceleration $-g$.[2]

Although the above argument should not be taken very seriously, it does suggest that unsteady waves, regarded as steady waves on which shorter waves are superposed, may achieve higher acceleration at lower overall steepnesses. Hence they may break at lower values of the apparent steepness than do steady waves.

Accordingly some interest attaches to the observation by Ochi and Tsai (1983) that in some random waves measured by a fixed probe "breaking was imminent" when

$$H/gT^2 \geqslant 0.020 \tag{7.5}$$

where H is the local wave height and T the local wave period, as observed by a fixed wave probe. From Table 1 it will be seen that this value of H/gT^2 is equivalent to a wave steepness ak = 0.35. The corresponding curves for the apparent and real accelerations are shown in Figs. 9 and 10. The apparent acceleration (Fig. 9) is remarkable as being almost exactly constant over half the wave period T, and then smoothly negative over the rest of the wave, with a minimum $-1.27g$ at the wave crest. The real acceleration (Fig. 10) shows an upward acceleration bounded at $0.28g$, and a downward acceleration bounded at $-0.32g$. In neither case is there any clear correspondence with the bound of $-0.40g$ suggested by Srokosz (1986).

In random seas, where many different harmonic components may be present, the sharp limits presented

[2] This result appears to support the assumption made by Phillips (1958). Note that the limit (7.4) is independent of the value of r, except insofar as when r is greater, then the limit is approached more rapidly. The requirement that the waves be self-similar can also be relaxed.

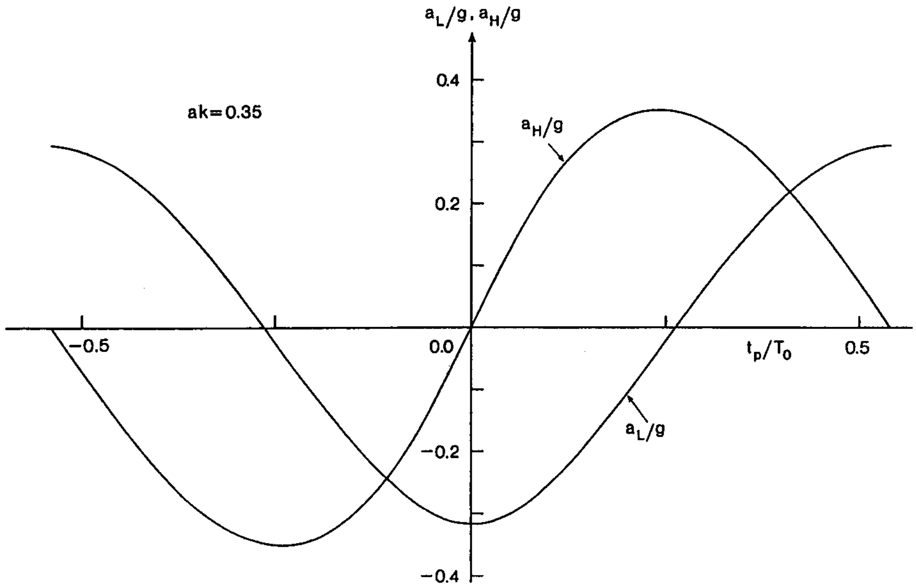

FIG. 10. Real vertical and horizontal accelerations in a wave of steepness $ak = 0.35$.

here may become blurred. For example, a short ripple superposed on a longer wave will tend to be carried past a fixed probe by the orbital motion in the long wave in such a way that the *apparent* vertical accelerations become very large, in both the upward and downward directions. However, the blurring of the upward limits would be less, because in the troughs of the long waves the steepness of the ripples, or shorter waves, is very much diminished.

For the *real* accelerations, the blurring due to high-frequency ripples would be present also, though generally less than for the apparent accelerations. Again we would expect less blurring of the positive limits than for the negative limits.

In steep waves, the superposition of different frequencies in random phase cannot be justified theoretically, on account of the strong nonlinear interactions and the presence of bound harmonics. Nevertheless a histogram of the accelerations may be approximately gaussian. The presence or absence of a cutoff would most probably depend upon the form of the frequency spectrum, and be sensitive to the existence of a high-frequency "tail".

A comparison of the accelerations in steep seas as measured by a fixed probe and by a free-floating buoy respectively could be interesting.

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