# Cryptanalysis of two identification schemes based on an ID-based cryptosystem

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## Abstract

Two identification schemes based on the Maurer-Yacobi ID-based cryptosystem are analysed and shown to suffer from serious security problems.

## 1 Introduction

Tseng and Jan [1] proposed a user identification scheme (referred to as the TJ scheme) based on an identity-based (ID-based) non-interactive public key distribution system due to Maurer and Yacobi [2]. Hwang, Lo and Lin [3] have proposed a further scheme, based on the TJ scheme, designed for use in the mobile environment (we refer to this as the HLL scheme). The authors of both schemes [1, 3] claim that they are secure. However, we show that both schemes suffer from serious security problems.

The reminder of this paper is organised as follows. In Section 2, we review the TJ and HLL schemes. In Section 3, we describe the security problems. In Section 4, brief conclusions are provided.

## 2 Review of the TJ and HLL schemes

Both schemes possess the same system initialisation phase. A trusted authority (TA) is required to perform the following initialisation steps.

- Select prime numbers $p_j$ $(1 \leq j \leq 4)$ each 60–70 decimal digits long,

where the values $\frac{p_j-1}{2}$ are odd and pairwise relatively prime. Put $N = p_1 p_2 p_3 p_4$.

- Select a random number $e \in Z^*_{\phi(N)}$, and compute $d$ satisfying $ed \equiv 1$ (mod $\phi(N)$). Select another random number $t$ in $Z^*_{\phi(N)}$.

- Select a value $g$ such that $g$ is a primitive element in $GF(p_j)$ for every $j$, $1 \le j \le 4$, and a one-way function $h$.

TC publishes $\{N, g, e, h\}$ and keeps $\{p_1, p_2, p_3, p_4, t, v, d\}$ secret. When a user wishes to join the system, he presents his unique identifier ID to the TC. The TC then computes

$$s = et \log_g (\mathrm{ID}^2) \bmod \phi(N)$$

and sends $s$ to the user as his private key and publishes ID as his public key.

Note that the existence of the discrete logarithm necessary to compute $s$ was established by Maurer and Yacobi [2]; computing this discrete logarithm is feasible by computing the logarithm of $\mathrm{ID}^2$ with respect to $p_j - 1$ for each $j$, and then combining the values (again as described in [2]).

## 2.1  Identification phase of the TJ scheme

Suppose Alice (with identity $\mathrm{ID}_a$) wishes to identity herself to Bob (with identity $\mathrm{ID}_b$). She follows the steps below.

1. Alice sends her identity $\mathrm{ID}_a$ to Bob.

2. Bob chooses a random integer $k$ in $Z^*_N$ and sends $Y = (\mathrm{ID}_b)^{2k} \bmod N$ to Alice.

3. On receiving $Y$, Alice computes and sends $Z = Y^{s_a} \bmod N$ to Bob.

4. Bob checks the equation $Z = (\mathrm{ID}_a)^{2ks_b} \bmod N$. If the check succeeds, then Bob has confirmed that Alice possesses the identity $\mathrm{ID}_a$.

## 2.2  Identification phase of the HLL scheme

Suppose a mobile user (with identity $\mathrm{ID}_m$) wishes to identify himself to the mobile station (with identity $\mathrm{ID}_b$). The following steps are performed.

1. The mobile user chooses a random $k$ in $Z^*_N$ and sends $\{\mathrm{ID}_m, Y, Z, T\}$ to the mobile station, where $T$ is a time-stamp and $Y$ and $Z$ are computed as follows:

$$Y = (\mathrm{ID}_m)^{2k} \bmod N$$

$$Z = (\text{ID}_b)^{2ks_mT} \bmod N$$

2. On receiving $\{\text{ID}_m, Y, Z, T\}$, the mobile station computes $Z' = Y^{s_bT}$ mod $N$. If $Z' = Z$, then the mobile station has confirmed that the mobile user possesses the identity $\text{ID}_m$.

# 3  Cryptanalysis of the TJ and HLL schemes

The following security problems exist in the TJ and HLL schemes.

1. Suppose an attacker can freely manipulate the messages sent between Alice and Bob in the TJ scheme; then he can also impersonate Alice to any valid user except Alice. To impersonate Alice to Eve, the attack can be mounted as follows.

   (a) When Alice sends her identity $\text{ID}_a$ to Bob, the attacker also sends $\text{ID}_a$ to Eve.

   (b) When the attacker intercepts the reply message from Bob to Alice, he prevents it reaching Alice.

   (c) After receiving $\text{ID}_a$ from the attacker (masquerading as Alice), Eve chooses a random integer $k$ in $Z_N^*$ and sends $Y = (\text{ID}_e)^{2k}$ mod $N$ to the attacker. After receiving $Y$, the attacker impersonates Bob to forward it to Alice.

   (d) On receiving $Y$, Alice computes and sends $Z = Y^{s_a}$ mod $N$ to Bob. The attacker intercepts this message and sends it to Eve.

   (e) Eve checks whether the equation $Z = (\text{ID}_a)^{2ks_e}$ mod $N$ holds, which it will. Eve now believes that the attacker possesses the identity $\text{ID}_a$. That is, Eve believes she is talking to Alice, whereas Alice believes she is talking to Bob.

   To mount this attack, the attacker only needs to monitor the activities of Alice. It can be mounted whenever Alice initiates a new run of the identification protocol.

2. In the HLL scheme a time-stamp $T$ is used to prevent replay attacks; however an attacker can still easily deploy a replay attack.

   Suppose the attacker has intercepted $\{\text{ID}_m, Y, Z, T\}$ in a previous run of the identification scheme. Then the attacker can initiate a new run of the identification protocol as follows.

   (a) The attacker constructs and sends $\{\text{ID}_m, Y^*, Z^*, T^*\}$ to the base station, where $T^*$ is the current time-stamp, $Y^* = Y^T$ mod $N$, and $Z^* = Z^{T^*}$ mod $N$.

(b) On receiving $\{\mathrm{ID}_m, Y^*, Z^*, T^*\}$, the mobile station computes $Z' = (Y^*)^{s_b T^*} \bmod N$. Because $Z \equiv Y^{s_b T} \pmod{N}$ in the previous run of the identification protocol, it is easy to verify that $Z' = Z^*$ will hold. The mobile station now believes that the attacker possesses the identity $\mathrm{ID}_m$.

3. In the HLL scheme, an attacker can impersonate any mobile user except Alice to the base station when Alice tries to identify herself to the base station. To mount this attack, the attacker only needs to replace $\{\mathrm{ID}_m, Y, Z, T\}$ with $\{\mathrm{ID}_x, Y, Z, T\}$ in step 1 of the protocol run, where $\mathrm{ID}_x$ is the identity of the mobile user that the attacker wishes to impersonate. The validity of the attack is obvious since the identity of the user is not involved in the verification by the base station.

## 4 Conclusion

We have shown that serious security problems exist in two identification schemes based on an ID-based cryptosystem.

## References

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