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Mathematical Model of a Zigzag Sewing Machine with a Timing Belt Driven Hook

Abstract

The mathematical model of a zigzag sewing machine with a timing belt driven hook is presented in this paper. The resonance behaviour of the machine is shown graphically. The model makes it possible to choose machine parameters at the design stage.

Key words: *sewing machine, zigzag, modelling, vibration.*

Introduction

The studies of various factors affecting the performance of high-speed sewing machines can be found in papers [1,2]. Progress in improving seam appearance has been reported in paper [3]. The interaction of a fabric and a sewing machine has been investigated in works [4,5]. Mathematical modelling [6] has been found to be a very effective method for studying the dynamics of sewing machines. The effect of belt extensibility on the variation of the relative position of the needle and hook in a sewing machine has been investigated in work [7]. The vibration of a sewing machine resulting from a simultaneous zigzag motion of the needle bar and hook has been studied in works [8-10]. The purpose of this paper is to formulate a mathematical description of a sewing machine in which the zigzag motion of the needle bar does not require a zigzag motion from the hook.

Equations of Motion

The machine considered is shown in Figure 1. Only the elements having significant influence on the dynamics of the machine are taken into account, and therefore the diagram is not complete. The zigzag rocker l_1 is set in the oscillatory motion a_1 by a triangle cam and a linkage mechanism. The constant values of angles a_3 and a_4 are adjusted to control the stitch transverse location and its width. The motion Y of the needle bar with respect to the rocker is due to the slider-crank mechanism. The hook shaft is set in the rotary motion G by a timing belt. Only the needle, zigzag and hook mechanisms are taken into account in this paper. A study of the feed mechanism can be found in works [11].

Cutting the driving belts, the machine is divided into three subsystems:
1 - the crank-shaft driving the needle bar,
2 - the hook shaft and
3 - the motor.

Applying the principle of virtual work, one obtains the equation (1), where B_{01} is the mass moment of inertia of the zigzag rocker, α_1 is the zigzag angle, B_{mc} is the mass moments of inertia of the needle bar with respect to its centre of gravity, m is the mass of the needle bar, Y is a coordinate of its gravity centre, Φ is the rotation angle of the crankshaft, A - its mass moments of inertia, F_b and F_H the forces in the driving belts, and R_H & r_a are the radii of the timing belt and V-belt pulleys. By rearranging equation (1), summing up the moments of forces acting on the hook shaft and the motor and then substituting the tensile forces in belts, one obtains the set of equations governing the motion of the machine in the form of equation (2).

Here, Γ is the rotation angle of the hook, B_H is its mass moment of inertia, D_H is the coefficients of viscous damping and s_H is the stiffness of the timing belt, R_H and r_H are the radii of the timing belt pulleys, Φ_m is the rotation angle of the motor, A_m is the mass moment of inertia of its rotor, D_b is the coefficient of viscous damping, s_b is the stiffness of the V-belt, and r_a & r_b are the radii of the V-belt pulleys.

The motor torque M_m can be calculated from the set of equations (3), where u is the feed voltage vector, i is the vector of the corresponding currents, R is the diagonal matrix of the winding resistances, L is the inductance matrix, (W=1...3, K=1...3) are the row and column numbers in matrix c, and Ω_m is the synchronous circular speed.

The variable coefficients in the first equation in set (2) can be determined form the geometry of the mechanisms. The dependence of needle coordinate Y [12] and its zigzag rotation angle $a_1(F)$ on the rotation angle of the crankshaft can be found from Figure 1 in the form of the set of equations (4). The discrete solution of the set (4) are found using the Newton-Brent method. The discrete functions are then replaced by continuous ones using Lagrange polynomials.

Results and Discussion

The set of differential equations (2, 3) was solved in a step-wise manner. Integration was carried out until the difference between successive periods became negligible. The numerical data used in the example presented below are chosen to be close to those in real machines. The computations were performed for the mass moments of

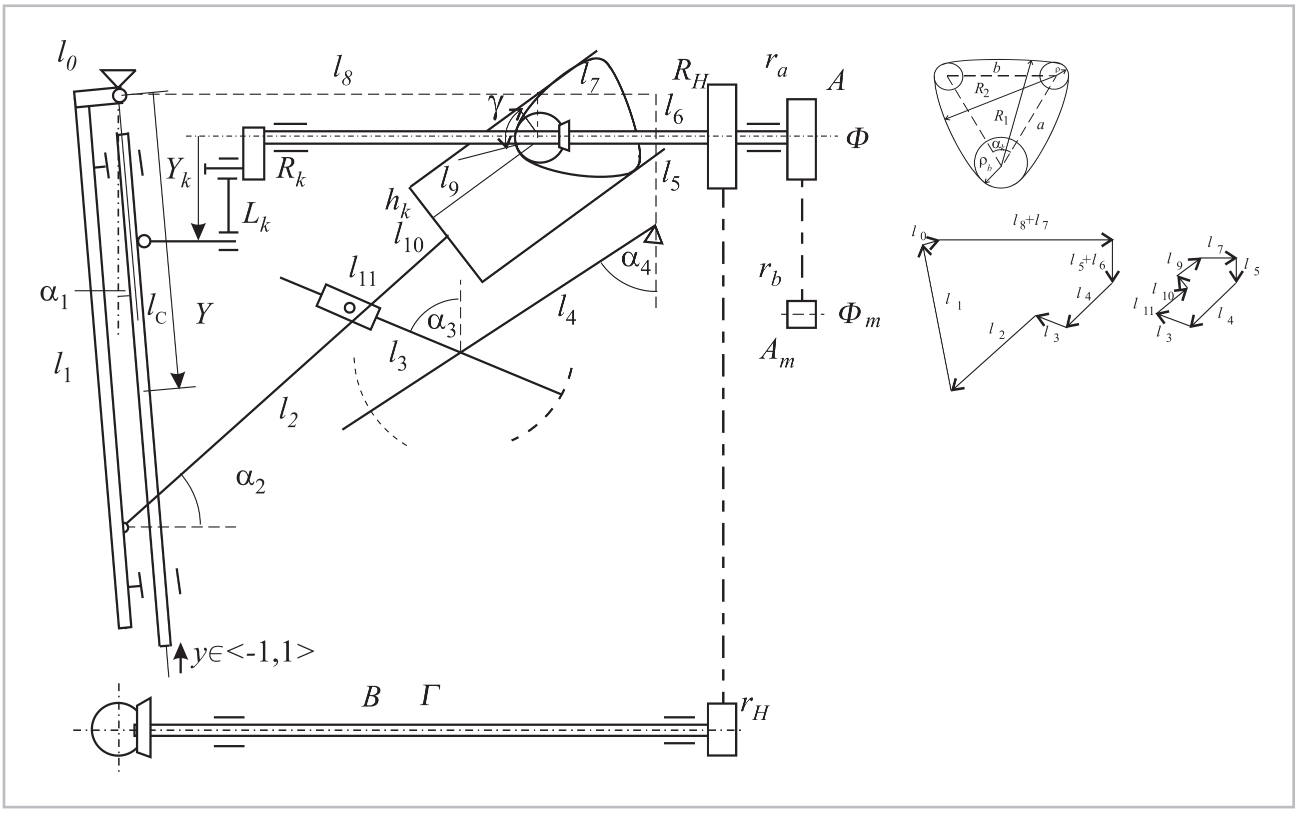

Figure 1. Mechanisms driving the needle and the hook in a sewing machine. All the symbols are explained in the text.

$$-B_{01}\frac{d^2\alpha_1}{dt^2}d\alpha_1 - B_{mc}\frac{d^2\alpha_1}{dt^2}d\alpha_1 - m\left(\frac{d^2\alpha_1}{dt^2}Y + 2\frac{d\alpha_1}{dt}\frac{dY}{dt}\right)Y d\alpha_1 - m\left(\frac{d^2Y}{dt^2} - \left(\frac{d\alpha_1}{dt}\right)^2 Y\right)dY + \left(-A\frac{d^2\Phi}{dt^2} + F_b r_a - F_H R_H\right)d\Phi = 0 \qquad (1)$$

$$\begin{aligned}
&\left(A + m\left(\frac{dY}{d\Phi}\right)^2 + \left(B_{01}+B_{mc}+mY^2\right)\left(\frac{d\alpha_1}{d\Phi}\right)^2\right)\frac{d^2\Phi}{dt^2} + \left(m\frac{dY}{d\Phi}\left(\frac{d^2Y}{d\Phi^2} + Y\left(\frac{d\alpha_1}{d\Phi}\right)^2\right) + \left(B_{01}+B_{mc}+mY^2\right)\frac{d\alpha_1}{d\Phi}\frac{d^2\alpha_1}{d\Phi^2}\right)\left(\frac{d\Phi}{dt}\right)^2 + \\
&-D_b\left(\frac{d\Phi_m}{dt}r_b - \frac{d\Phi}{dt}r_a\right)r_a - s_b\left(\Phi_m r_b - \Phi r_a\right)r_a + D_H\left(\frac{d\Phi}{dt}R_H - \frac{d\Gamma}{dt}r_H\right)R_H + s_H\left(\Phi R_H - \Gamma r_H\right)R_H = 0 \\
&B_H\frac{d^2\Gamma}{dt^2} - D_H\left(\frac{d\Phi}{dt}R_H - \frac{d\Gamma}{dt}r_H\right)r_H - s_H\left(\Phi R_H - \Gamma r_H\right)r_H = 0 \\
&A_m\frac{d^2\Phi_m}{dt^2} + D_b\left(\frac{d\Phi_m}{dt}r_b - \frac{d\Phi}{dt}r_a\right)r_b + s_b\left(\Phi_m r_b - \Phi r_a\right)r_b - M_m = 0
\end{aligned} \qquad (2)$$

$$\begin{aligned}
&M_m = \frac{1}{2}i^T\frac{\partial L}{\partial \Phi_m}i, \qquad L\frac{di}{dt} + \frac{\partial L}{\partial \Phi_m}\frac{d\Phi_m}{dt}i + Ri = u, \\
&L_{11} = L_{sr}\,diag\,(1) + L_{sg}\,c\,(0), \quad L_{12} = L_{21}^T = L_M\,c\,(\Phi_m), \quad L_{22} = L_{wr}\,diag\,(1) + L_{wg}\,c\,(0), \\
&c_{WK}(\Phi_m) = \cos\left(\Phi_m + 2(K-W)\pi/3\right), \qquad W = 1..3, \qquad K = 1..3, \\
&u = [U\cos(\Omega_m t),\ U\cos(\Omega_m t - 2\pi/3),\ U\cos(\Omega_m t - 4\pi/3),\ 0, 0, 0]^T
\end{aligned} \qquad (3)$$

$$\begin{aligned}
&\gamma = \pi/2 - \Phi/2 + \alpha_2, \qquad l_{10} = (R_1 + \rho_b)/2 - h_k \\
&h_k(\gamma) = h_k(-\gamma) = \begin{cases} \rho_b & 0 \le \gamma < \alpha_k/2 \\ R_2 - a\cos(\gamma - \alpha_k/2) & \alpha_k/2 < \gamma < \pi/2 \\ \rho + a\cos(\pi - \gamma - \alpha_k/2) & \pi/2 < \gamma < \pi - \alpha_k/2 \\ R_1 & \pi - \alpha_k/2 < \gamma < \pi \end{cases}
\qquad
\left.\begin{aligned}
&l_0\cos\alpha_1 - l_1\sin\alpha_1 + l_8 + l_7 - l_4\sin\alpha_4 - l_3\sin\alpha_3 - l_2\cos\alpha_2 = 0, \\
&l_0\sin\alpha_1 + l_1\cos\alpha_1 - l_5 - l_6 - l_4\cos\alpha_4 + l_3\cos\alpha_3 - l_2\sin\alpha_2 = 0, \\
&-l_3\sin\alpha_3 - l_4\sin\alpha_4 + l_7 + l_9\cos\alpha_2 - l_{10}\sin\alpha_2 + l_{11}\cos\alpha_2 = 0, \\
&l_3\cos\alpha_3 - l_4\cos\alpha_4 - l_5 + l_9\sin\alpha_2 + l_{10}\cos\alpha_2 + l_{11}\sin\alpha_2 = 0
\end{aligned}\right\} \\
&R_k\sin(\Phi - \pi) - L_k\sin\gamma_k = 0 \\
&-R_k\cos(\Phi - \pi) - L_k\cos\gamma_k + Y_k = 0 \qquad (Y - l_c)\cos\alpha_1 = Y_k + l_c
\end{aligned} \qquad (4)$$

Sets of equations 1-4

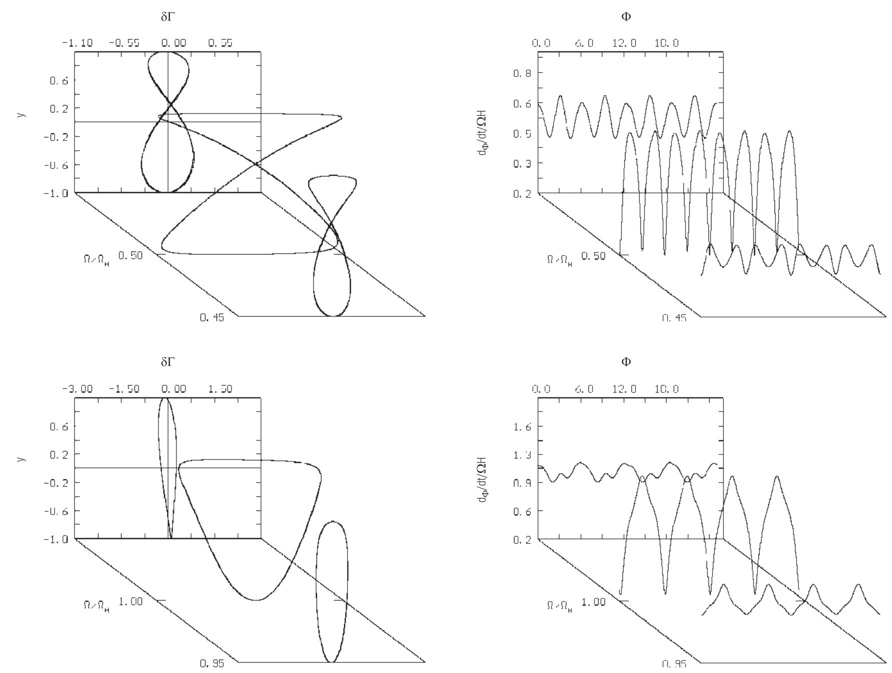

Figure 2. *Variation of hook position $\delta\Gamma$ and crankshaft speed $d\Phi/dt$ in the resonance regions; y - dimensionless coordinate of needle*

$$y = (L_k - Y_k)/R_k \in <-1,1>,$$

$\delta\Gamma$ - difference between the actual (Γ) and expected ($\Phi \frac{R_H}{r_H}$) position of hook shaft, Φ - rotation angle of crankshaft, $d\Phi/dt$ - angular speed of crankshaft, Ω_H - average natural frequency of hook belt,

Ω_m - synchronous circular speed of motor driving crankshaft,

$\Omega = \Omega_m \frac{r_b}{r_a}$ *- synchronous circular speed of crankshaft.*

inertia (all in kgm²): A=0.00015, A_m=0.023 B_H=A/3, B_{mc}=0.0002, B_{01}=0.004, the mass of the needle bar m=0.1 kg, the stiffness and damping constants of the driving belts: s_H=565,000 N/m, s_b=190,000 N/m, D_h=D_b=1 Ns/m;
the feeding voltage u=380 V; the parameters of the driving motor: the resistance and inductance of the motor windings R_w=9 Ω, R_s=9.1 Ω, L_M=0.096 H, L_{sg}=L_{wg}=L_M, L_{sr}=L_{wr}=0.034 H,
the radii of the belt pulleys (in meters): r_a=4r_b=0.04, R_H=2r_H =0.03; the length of the crank R_k=0.017 m, the length of the connecting rod L_k=0.035 m; the dimensions of the zigzag mechanism (all lengths in metres, all angles in radians): l_c=0.023, l_0=0.005, l_1= l_2=0.1, l_4=l_{11}=0.03, l_5=l_6=l_7=0, l_8=0.078, ρ_b=0.014, a=0.005/(2-2sin(α_k/2)), b=2$a$sin(α_k/2), ρ=a+ρ_b-b, R_1=a+ρ, R_2=a+ρ_b, the angles describing the positions of l_4 and l_3: α_4=40π/180, α_3=-15π/180, the angle of the triangle cam α_k=π/2.

The average natural frequencies associated with motor and hook belt were found to be Ω_b=830 and Ω_H=2590 rad/s, respectively.

The difference between the actual and expected position of the hook shaft in rad

$$\delta\Gamma = \Gamma - \Phi R_H / r_H$$

versus upward directed dimensionless coordinate of the needle

$$y = (L_k - Y_k)/R_k \in <-1,1>$$

and the crankshaft angular speed $d\Phi/dt$ versus its angle of rotation Φ in the resonance region associated with hook belt are shown in Figure 2. Here,

$$\Omega = \Omega_m r_b / r_a \text{ (rad/s)},$$

W_m is the synchronous circular speed of the motor.

From Figure 2 we can see that resonance takes place when the crankshaft speed is half or equal the circular frequency of natural vibrations associated with the elasticity of the timing belt. In the resonance region the difference between the actual and expected position of the hook is high, and the machine must be kept far away from those speeds. It was observed that the difference between consecutive cycles resulting from the zigzag mechanism was negligible.

❑

■ Conclusion

The mathematical model presented here makes it possible to study the dependence of the machine behaviour on the system parameters. It enables operators to choose machine parameters at the design stage.

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