On the Circulation of the Warm Water of the Subtropical Gyres[1]

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ABSTRACT

A ventilated thermocline model is used to discuss the circulation of the warm water of the subtropical gyre. It is suggested on theoretical grounds that the warm water layers that outcrop well south of the zero wind-stress curl line are completely replenished by the mass flux pumped down from the upper Ekman layer while recirculation of mass through the western boundary current plays a relatively insignificant role for these layers. The recirculation seems, instead, to be confined to the deeper layers of the thermocline.

1. Introduction

The water circulating in the thermocline region of a midoceanic subtropical gyre can be traced to basically two quite distinct sources. One source is water which has recirculated through the western boundary-current region and then reenters the thermocline Sverdrup domain. The other source is the upper Ekman layer which pumps water downward into the subtropical gyre.

These two separate cycles form the basis of two quite distinct elementary pictures of the general circulation. The first is a largely unventilated circulation which emphasizes the quasi-horizontal, recirculating character of the circulation in which the Ekman pumping, although the driving agent for the circulation, plays an insignificant role in the mass balance. The second pattern emphasizes the flow in a meridional, vertical plane in which the refreshment of the water mass by the Ekman layer is significant to the mass balance as well as to the budget of potential vorticity. The issue of the relative proportion of the volume of water in each cycle has long been a matter of discussion and the determination of the proportion observationally has led to sometimes conflicting claims. For example, Sarmiento (1983) has argued that for the thermocline region as a whole, the Sverdrup transport exceeds the Ekman pumping by about a factor of 3. On the other hand, Montgomery (1938) and Stommel (1979) have argued that the horizontal flow in at least portions of the thermocline can be understood as the simple redirection of flow from the Ekman layer.

The recent model of the thermocline described by Luyten *et al.* (1983) allows a direct examination of the question raised above. In fact, all of the results to be presented in this paper are implicit in that model and no novel dynamical questions are addressed here. There are, however, certain consequences of the ventilated thermocline model, especially as concern the mass balance of the general circulation of the subtropical gyre that are illuminating when described explicitly and are rather surprising.

In particular, I believe that it is especially useful to distinguish between the circulation of the warmer waters whose isopycnal surfaces have their outcrop lines well within the subtropical gyre and those isopycnal surfaces which outcrop either to the north or only slightly south of the northern boundary of the gyre. For the former it is easy to show that the mass balance is dominated largely by the flux from the Ekman layer, i.e., that, in fact, the Sverdrup flow of the warm water is of the same order as the volume of fluid pumped down from the Ekman layer. The recirculation domain is seen to be limited to deeper layers which are unventilated. Overall, the Sverdrup transport is of the same order in both density intervals and the question of whether recirculation dominates Ekman ventilation depends entirely on which density interval is examined. Hence, the suggestion is made here that the contradictions between the conceptual and descriptive pictures described above are quite likely more apparent than real and are largely due to insufficient attention paid to the density domain of the flows under consideration.

2. The model

The model is identical to that used by Luyten *et al.* (1983, hereafter LPS) and the reader is referred to that paper for a description of the formulation of the model and the analysis of the thermocline theory presented there. In this paper I will use freely the results of their analysis and their notations. Figure 1

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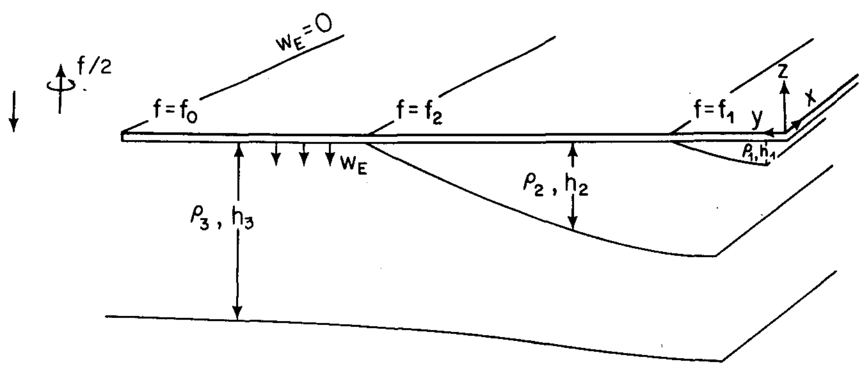

FIG. 1. A schematic model of the ventilated thermocline showing only the moving layers. In the present study, layer 1 is absent.

shows the basic model. As analyzed in LPS there were three moving layers. To keep the present discussion as simple as possible, I will use the simplest possible resolution of the warm water region by considering only a two-moving-layer model, i.e., layer 1 will be absent. This can be achieved formally by setting $\rho_1 = \rho_2$. All the Sverdrup transport in the region $f_2 < f \leq f_0$ (where f_0 is the Coriolis parameter at the latitude where the Ekman pumping vanishes) is carried by layer three although as shown by Pedlosky and Young (1983) a significant portion of the Sverdrup flow can be carried by the deeper unventilated layers. Hence, in this model I lump all the latter in layer 3 and all the warm water transport in layer 2. Although there is no layer 1 in the present case, I will stick to this notation to facilitate reference to the formulas in LPS.

In the region $f < f_2$, LPS show that

$$h_2 = (1 - f/f_2)h, \tag{2.1a}$$

$$h_3 = f/f_2 h, \tag{2.1b}$$

while

$$h = \frac{(D_0^2 + H_0^2)^{1/2}}{[1 + \Gamma(1 - f/f_0)^2]^{1/2}}, \tag{2.1c}$$

where

$$\left.\begin{aligned} h &= h_2 + h_3 \\ D_0^2 &= -\frac{2f^2}{\gamma_3 \beta}\int_x^{x_E} w_E(x', y)dx' \\ \gamma_n &= (\rho_{n+1} - \rho_n)/\rho_0 g \\ \Gamma &= \gamma_2/\gamma_3 \end{aligned}\right\} \tag{2.2}$$

and where H_0 is the thickness of layer 3 on the eastern wall at $x = x_E$. The solution given by (2.1) is valid in the ventilated region of the gyre, west of the shadow zone, whose boundary $\tilde{x}_3(y)$ is given by

$$D_0^2(\tilde{x}_3, y) = \frac{\gamma_2}{\gamma_3} H_0^2(1 - f/f_2)^2 \tag{2.3}$$

and east of the constant potential-vorticity pool near the western boundary. The Ekman pumping velocity w_E is negative in the subtropical gyre.

In layer 2 the meridional velocity, which is geostrophic, is given by

$$v_2 = \frac{\gamma_3}{f}\frac{\partial}{\partial x}(h + \Gamma h_2) = \frac{\gamma_3}{f}[1 + \Gamma(1 - f/f_2)]\frac{\partial h}{\partial x}, \tag{2.4}$$

while

$$u_2 = -\frac{\gamma_3}{f}\frac{\partial}{\partial y}\{h[1 + \Gamma(1 - f/f_2)]\}. \tag{2.5}$$

Thus the projection on the horizontal plane of the trajectories of the flow in layer 2 are lines of constant $h[1 + \Gamma(1 - f/f_2)]$.

3. The Meridional flux of warm water

Consider now the meridional velocity in layer 2. From (2.1c), (2.2) and (2.4) it follows that

$$v_2 = \frac{f w_E}{\beta h}\frac{[1 + \Gamma(1 - f/f_2)]}{[1 + \Gamma(1 - f/f_2)^2]}. \tag{3.1}$$

Note that $v_2 \neq 0$ right at the outcrop line where $f = f_2$, but let us momentarily defer a discussion of the actual particle trajectories. From (3.1) and (2.1a), the meridional mass flux per unit longitudinal extent is

$$v_2 h_2 = \frac{f w_E}{\beta}(1 - f/f_2)\left[\frac{1 + \Gamma(1 - f/f_2)}{1 + \Gamma(1 - f/f_2)^2}\right], \tag{3.2}$$

which vanishes at $f = f_2$.

Consider now the meridional mass flux at the apex of the wedge angle formed by the interface between layers 2 and 3 and the base of the mixed layer (see Fig. 2). For values of $(y_2 - y) \ll f_2/\beta_2$ where $\beta_2 = \beta(f_2)$, (3.2) implies that

$$v_2 h_2 \sim (y_2 - y) w_E. \tag{3.3}$$

Thus, within the β-plane approximation, the mass balance near the outcrop latitude is entirely a balance

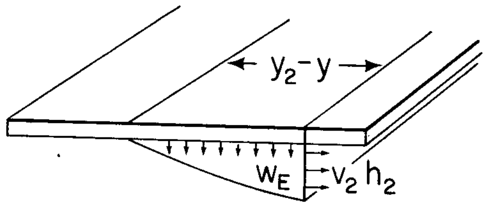

FIG. 2. The mass balance in the wedge formed by the base of layer 2 and the Ekman layer. The distance $y_2 - y$ is $\ll f_2/\beta$.

between Ekman pumping and the meridional Sverdrup flux. Any zonal flux associated with recirculation is negligible in this region.

Equation (3.2) can be rewritten as

$$v_2 h_2 = \frac{f w_E}{\beta}\left[1 - \frac{f/f_2}{1 + \Gamma(1 - f/f_2)^2}\right]. \tag{3.4}$$

For values of $(1 - f/f_2) \ll 1$, i.e., near the outcrop latitude, the meridional mass flux per unit longitude is precisely balanced by the Ekman downwelling as (3.3) shows. However, the size of the transport as given by (3.3) is less than the characteristic Sverdrup transport which is $O(f w_E/\beta)$. Equation (3.4) shows that the meridional transport in the warm water layer grows with decreasing latitude. When $(1 - f/f_2)$ is O(1), the meridional flux of warm water is of the same order as the characteristic Sverdrup transport of the cold water layer associated with recirculation. The ratio of the two can be shown to be, in the region $0 \leqslant f \leqslant f_2$,

$$\frac{v_2 h_2}{v_3 h_3} = (1 - f/f_2)\frac{f_2}{f}[1 + \Gamma(1 - f/f_2)]. \tag{3.5}$$

This ratio is plotted in Fig. 3.

This ratio reaches unity for f/f_2 equal to 0.586. Thus the meridional mass flux in the warm water does, in fact, grow to the same order as the transport of the deeper recirculating flow. What is its source?

4. The mass balance of the warm-water layer

To answer the question raised at the end of the preceding section, it is useful to consider a particular example. Suppose we choose a form of w_E as follows:

$$w_E = -W_0 \frac{\cos\theta}{\cos\theta_0} \sin\pi(f/f_0), \tag{4.1}$$

whose shape as a function of $f/f_0 = \sin\theta/\sin\theta_0$ is shown in Fig. 4 for the case $\theta_0 = 45°$. The factor $\cos\theta/\cos\theta_0 = \beta/\beta(f_0)$ is introduced simply for technical convenience for the calculation to be described below. The Ekman pumping given by (4.1) vanishes at $f = f_0$ and at the equator.

In a particular longitudinal strip the southward transport of warm water is, from (3.2),

$$v_2 h_2 = -w_0 \frac{f_0}{\beta_0}\cdot\frac{f}{f_0}\left[1 - \frac{f/f_2}{1 + \Gamma(1 - f/f_2)^2}\right]\sin\pi f/f_0. \tag{4.2}$$

In that same unit strip the volume of fluid pumped down from the Ekman layer from any latitude north to the outcrop line is

$$\int_y^{y_2} w_E dy = \int_f^{f_2} w_E \frac{df}{\beta} = -\frac{W_0}{2\pi}\frac{f_0}{\beta_0}\left[\cos\pi\frac{f}{f_0} - \cos\pi\frac{f_2}{f_0}\right]. \tag{4.3}$$

Note that the characteristic meridional scale of the Ekman pumping is

$$L_y = \frac{w_E}{\partial w_E/\partial y}$$

or

$$L_y/R = O(\sin\theta_0/\pi\cos\theta). \tag{4.4}$$

If θ_0 is 45°, then L_y/R is never greater than 0.32, while if the outcrop latitude is farther south, say at 30°, the ratio falls to 0.25.

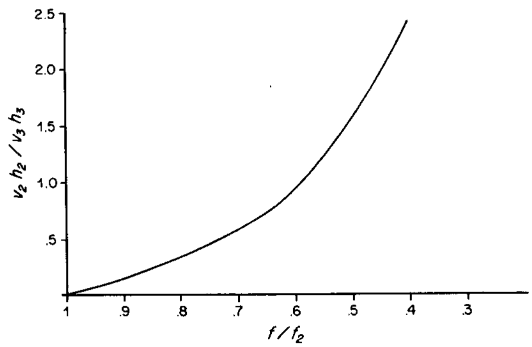

FIG. 3. The ratio of the meridional transport in layer 2 and layer 3 as a function of distance of the outcrop latitude, for the case $\Gamma = 1$.

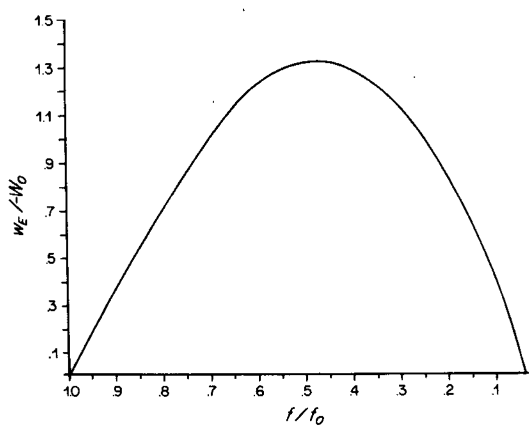

FIG. 4. The form of the Ekman pumping used in Section 4.

TABLE 1. Mass balance for the ventilated warm water layer.

f/f_2	v_2h_2	$\int_y^{y_2} w_E dy$	Difference
	$f_2/f_0 = 0.9$		
1	0	0	0
0.95	0.0197	0.0168	0.0029
0.9	0.0495	0.039	0.01
0.8	0.1275	0.0998	0.0277
0.7	0.2068	0.1763	0.0305
0.5	0.2666	0.353	−0.087
0.4	0.229	0.438	−0.209
	$f_2/f_0 = 0.75$		
1	0	0	0
0.95	0.0293	0.0283	0.001
0.9	0.0627	0.059	0.0037
0.8	0.1317	0.127	0.005
0.7	0.187	0.200	−0.013
0.6	0.215	0.295	−0.08
0.5	0.208	0.347	−0.139
0.4	0.1589	0.412	−0.25
	$f_2/f_0 = 0.4$		
1	0	0	0
0.95	0.0185	0.0189	−0.005
0.9	0.0354	0.0373	−0.0019
0.8	0.0623	0.0723	−0.01
0.7	0.0772	0.1046	−0.0274
0.5	0.070	0.159	−0.0892

Table 1 shows the result of a calculation of the southward flux of warm water, the volume of fluid delivered from the Ekman layer, and (in the last column) the difference between them. The calculation is repeated for three different positions of the outcrop latitude with respect to the latitude of the zero wind-stress curl. *Each column is scaled by* $(-W_0 f_0/\beta_0)$. In each case I have chosen $\Gamma = 1$. The table demonstrates that, except far south of the outcrop latitude, the mass balance is quasi-two dimensional, with the Ekman layer delivering the necessary fluid to feed the Sverdrup transport of warm water. This is especially true for those warm water layers that outcrop in the southern part of the subtropical gyre (e.g., the case $f_2/f_0 = 0.4$). The relatively small contribution made to the mass balance by the zonal flow is due to two effects. Near the outcrop line the layer is too thin for the zonal transport to be significant. Somewhat farther south the direction of the flow tends to be more meridional. Only for fairly small values of f/f_2 is the layer thickness of order of the total thermocline depth with a significant zonal component to the velocity.

The same qualitative point is made in Fig. 5. This figure shows the streamlines of flow in the upper layer for the same three choices of outcrop latitude. As before, I have chosen $\Gamma = 1$, and $\theta_0 = 45°$. At the same time I have chosen the extreme solution, $H_0 = 0$, which (LPS) maximally enlarges the nonventilated zone in layer 3 which LPS identify as a constant potential-vorticity pool. In Fig. 5 I have shown isopleths of $h + \Gamma h_2$, which are coincident with the projection on the horizontal plane of the trajectories of flow in the warm water layer. Only trajectories that emanate from the outcrop line are shown. In each panel the region marked R represents a domain where warm water must be fed from the western boundary. For the trajectories shown the *only* possible source of warm water is the Ekman

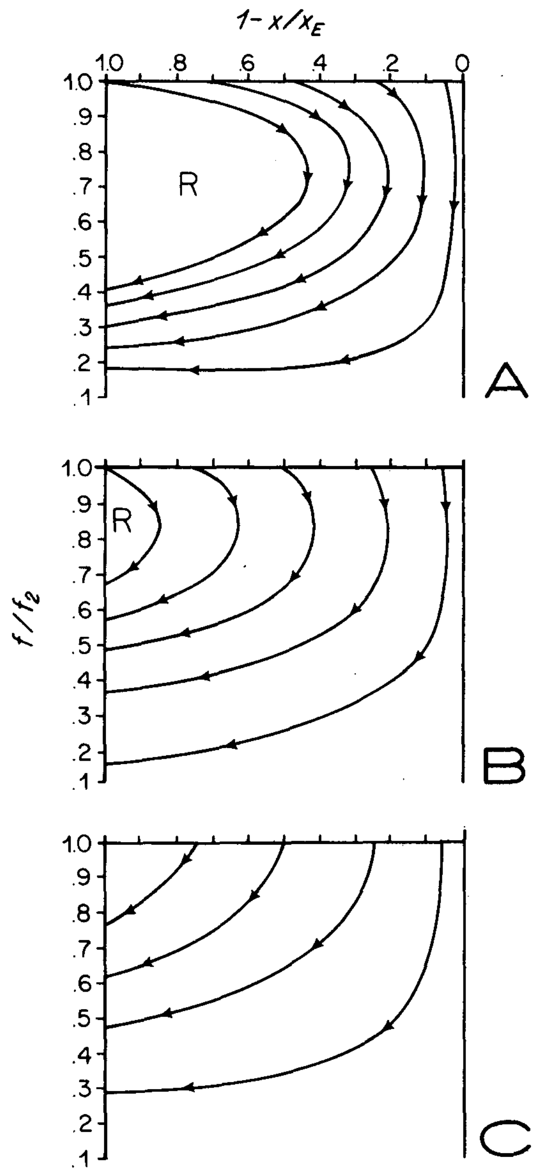

FIG. 5. The horizontal projection of the trajectories of the motion in layer 2 are shown for three different outcrop latitudes. The line of zero wind stress is chosen to be at 45°N. (a) $f_2/f_0 = 0.9$, (b) $f_2/f_0 = 0.75$, (c) $f_2/f_0 = 0.4$. In (a) and (b) the region marked R is covered by trajectories (not shown) which emanate from the western boundary.

layer. Hence a subjective measure of the contribution of the Ekman pumping is the degree to which the displayed trajectories cover the warm water area. Figure 5 shows that the closer the outcrop latitude is to the zero wind-stress curl line, the larger is the region of recirculation. However, even in the case where $f_2/f_0 = 0.9$, the *maximum* longitudinal extent of the recirculation region is only half of the region fed by the Ekman layer. This shrinks considerably in the case $f_2/f_0 = 0.75$ while, for the case shown in Fig. 5c, where $f_2/f_0 = 0.4$, the recirculation zone is entirely absent. The fluid columns in traveling southward in the upper layer are completely filled by water pumped downward from the Ekman layer.

To examine this filling process, consider the vertical velocity in the warm-water layer. Since in this (or any layer)

$$\frac{\partial w_2}{\partial z} = \frac{\beta}{f} v_2, \tag{4.5}$$

it follows that

$$w_2 = w_E + z\beta v_2/f. \tag{4.6}$$

However,

$$w_2 = \frac{dz}{dt};$$

hence in the warm water layer

$$\frac{dz}{dt} = w_E + z\beta v_2/f. \tag{4.7}$$

The first term in (4.7) represents the feeder source of water from the Ekman layer. By itself, it would tend to fill the fluid column rather rapidly. The second term in (4.7) represents the horizontal divergence of the column, i.e., its widening as it moves southward. This tends partially to compensate the Ekman pumping term. This latter term was neglected by LPS and they therefore overestimated the rate of filling of layer 3 north of the first outcrop latitude. On a trajectory,

$$dt = dy/v_2; \tag{4.8}$$

thus

$$dz = z\frac{\beta}{f}dy + w_E dy/v_2. \tag{4.9}$$

Changing dependent variables from y to f yields

$$\frac{d(z/f)}{df} = \frac{w_E}{f\beta v_2}, \tag{4.10}$$

which, with (3.1) becomes simply

$$\frac{d(z/f)}{df} = \frac{h}{f^2}\frac{[1 + \Gamma(1 - f/f_2)^2]}{[1 + \Gamma(1 - f/f_2)]}. \tag{4.11}$$

Now, if (4.11) is integrated along a trajectory in the warm water layer, the factor $h[1 + \Gamma(1 - f/f_2)]$ is constant. This allows (4.11) to be integrated directly to yield, using (2.1a),

$$z = z_* \frac{f}{f_*} - h_2\left[1 - \frac{f}{f_*}\frac{(1 - f_*/f_2)}{(1 - f/f_2)}\frac{[1 + \Gamma(1 - f/f_2)]}{[1 + \Gamma(1 - f_*/f_2)]}\right], \tag{4.12}$$

where z_* is the depth of the fluid element at the starting point, $f = f_* < f_2$. (If $f_* = f_2$, $z_* = 0$ and $z = -h_2$).

Figure 6 shows the motion in the vertical plane, along trajectories of fluid elements emanating from the Ekman layer at different f_*, each of which has $z_* = 0$. One sees immediately that at any given latitude *the fluid column is filled completely by water pumped down from the Ekman layer.* Such columns are therefore completely ventilated and all tracer properties should be completely ventilated by the mixed layer.

This differs completely from the situation in layer 3, the deeper, cold layer. Consider (4.6) applied to layer 3 in the region $f_2 \leq f \leq f_0$, i.e., where layer 3 is exposed to Ekman pumping. Then

$$w_3 = w_E + z\beta v_3/f, \tag{4.13}$$

and from (LPS) in this region

$$v_3 = h_3 \frac{f}{\beta} w_E; \tag{4.14}$$

hence in that region

$$\frac{dz/f}{df} = \frac{h_3}{f^2}, \tag{4.15}$$

which should replace equation (3.4) of LPS (where the divergence term is erroneously ignored). Since the trajectories in layer 3 in the region $f_2 \leq f \leq f_0$ are lines of constant h_3, (4.15), when integrated along a trajectory yields, for $f \leq f_*$,

$$z = z_* \frac{f}{f_*} - h_3(1 - f/f_*). \tag{4.16}$$

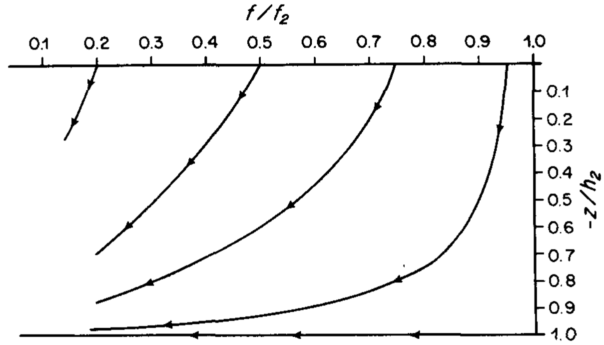

FIG. 6. The vertical trajectories (scaled by layer depth) for fluid elements inserted into the warm water layer from the Ekman layer. The trajectories are projected on a vertical cylindrical surface whose elements follow the horizontal streamlines.

Those trajectories that emanate from the mixed layer have $z_* = 0$, as in the cold water layer

$$\frac{z}{h_3} = -(1 - f/f_*); \quad f \leqslant f_*. \tag{4.17}$$

Thus only a portion of the cold water column can be ventilated by the downwelling from the Ekman layer. Since f/f_* cannot become too small before $f = f_2$ is reached, and layer 3 subducts, the natural conclusion to draw is that columns in layer 3 must be fed primarily by the western boundary region, i.e., by the recirculation.

5. Conclusions

The ventilated thermocline model of LPS has been used to discuss the mass budget of the warm water layers that outcrop in the subtropical gyre. Although the volume of flux of the warm water layer is of the same order as the total Sverdrup flux, a variety of simple calculations suggests that the warm water layer is completely ventilated from the Ekman layer aside from a western zone of recirculation whose eastward extent is a continuously decreasing function of the distance between the outcrop line and the line of zero wind-stress curl. Thus the horizontal volume flux of much of the warm water can be found as the efflux of a long tube leading back to a source in the upper mixed layer. This is precisely the result Montgomery (1938) suggested in his observational isentropic analysis of the subtropical North Atlantic. The picture is different if one wishes to discuss the circulation of the entire thermocline region for, as shown above, the deeper, colder thermocline waters are dominated by recirculation and the Ekman downwelling penetrates only slightly into this domain as suggested by Sarmiento's (1983) discussion of tritium tracer data.

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