

## On the Potential Energy of Baroclinic Rossby Waves in the North Pacific

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### ABSTRACT

Estimates of baroclinic Rossby wave potential energy spectra for various parts of the North Pacific were calculated from published material containing information about this energy in many different formats, definitions and units. The standardized results lead to frequency spectra of potential energy, ranging from 0.1 to 0.45 cpy (cycles per year), in 34 5° squares between 20 and 30°N; to energy spectra, ranging from 0.28 to 2.11 cpy, in eight subsections of the great circle route from Honolulu to San Francisco; and to a map of annual Rossby wave energy between 30 and 40°N. The most remarkable finding is a consistent sequence of interannual Rossby wave spectra, with a peak at 0.15 cpy (i.e., at a wave period of 6.7 years) covering the area 20–25°N and 175–130°W. The most complete information is available for a 5° square east of the Hawaiian Islands. There the spectrum shows, besides the 0.15 cpy peak, another broad peak ranging from about 0.4 to 1.4 cycles per year.

### 1. Introduction

During the last six years a series of papers has been devoted to the analysis of North Pacific temperature data with respect to baroclinic Rossby waves. Monthly XBT and hydrographic data from the great circle route between Honolulu and San Francisco (Dorman and Saur, 1978) have been analyzed by Emery and Maggaard (1976), Maggaard and Price (1977), Price (1981), White and Saur (1981) and Price and Maggaard (1983). Time series of annual vertically integrated density fluctuations from the upper 500 m of the North Pacific (20–50°N, 145°E–130°W) prepared by White (1977) were studied by Price and Maggaard (1980) and Price (1981). Monthly XBT data collected in the central North Pacific (30–40°N, 160°E–130°W) under the NORPAX program (TRANSPAC) by White and Bernstein (1979) were investigated by Kang (1980), Kang and Maggaard (1980) and White (1982). Most of these papers dealt mainly with the identification of Rossby waves in the data. Thus, length and time scales as well as propagation characteristics of the fluctuations and their compatibility with Rossby wave dispersion features were the main topics. The basic result was that first-mode baroclinic Rossby waves dominate the internal temperature fluctuations of the Pacific between 20 and 30°N for the whole period range from the cut-off period (~5 months) to 10 years. Between 30 and 40°N, however, only annual or near-annual first-mode baroclinic Rossby waves were found in most cases. The energy of the analyzed baroclinic Rossby waves was also determined but little attention was paid to it. Many different definitions of energies and different units were used.

I have tried to unify and combine all the information about baroclinic Rossby wave energy from the papers referred to above. In doing so I found that in some cases the description of the work was not complete enough to derive unambiguous results about the energy. In other cases I found obvious errors. In all these cases I have turned to the authors and was able to obtain the missing information and to straighten out errors.

The result of this work is a description of the distribution of potential energy of baroclinic Rossby waves in physical and frequency space. I believe it is the first time that estimates of Rossby wave spectra in the frequency range from 0.1 to 2 cycles per year have been published.

### 2. Definitions and units

The mean potential energy of a baroclinic wave contained in a water column of unit surface area is

$$E_{\text{pot}}^* = \frac{\rho_0}{2} \int_{-H}^0 N^2(z) \overline{\zeta^2(z)} dz, \quad (1)$$

where  $\overline{\zeta^2(z)}$  is the variance of the wave-induced vertical particle displacement  $\zeta(x, y, z, t)$ ,  $N(z)$  the Brunt-Väisälä frequency,  $\rho_0$  a constant reference density, and  $H$  the depth of the water;  $x, y, z$  are Cartesian coordinates directed eastward, northward and upward, respectively; and  $t$  is the time. Introducing the frequency spectrum  $S(\nu, z)$  of  $\zeta$ , where  $\nu$  is the cyclic frequency ( $\nu = T^{-1}$ , where  $T$  is the wave period), we have, for the frequency spectrum  $E_{\text{pot}}(\nu)$ ,

$$E_{\text{pot}}(\nu) = \frac{\rho_0}{2} \int_{-H}^0 N^2(z) S(\nu, z) dz. \quad (2)$$

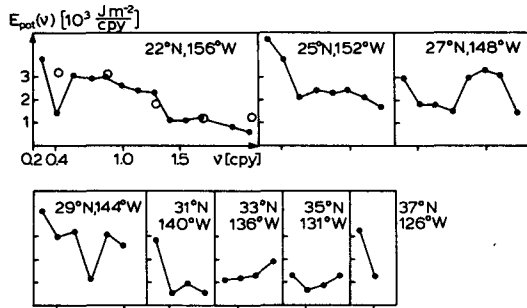


FIG. 1. Rossby wave potential energy spectra  $E_{pot}(v)$  at subsections of the Honolulu-San Francisco great circle route. Relative standard errors are 25–50%. The full lines show the standardized results derived from Price (1981) and Price and Magaard (1983). The open circles in the upper left spectrum indicate the corresponding results from Emery and Magaard (1976).

I have chosen to express all information about Rossby wave energy in terms of  $E_{pot}(v)$ . Since all analyzed Rossby waves are of the first baroclinic mode,<sup>1</sup>  $S(v, z)$  is always proportional to  $\zeta_1^2(z)$ , where  $\zeta_1(z)$  is the vertical displacement amplitude of the first mode.

The unit of  $E_{pot}^*$  is  $J m^{-2}$ . I have expressed cyclic frequencies in cycles per year (cpy). Hence the unit of  $E_{pot}(v)$  is  $J m^{-2} (cpy)^{-1}$ .

**3. Results**

The results from the great circle route Honolulu-San Francisco are displayed in Fig. 1. Rossby wave potential energy spectra  $E_{pot}(v)$  are given for eight subsections of the route at 13 frequencies ranging from 0.28 to 2.11 cpy. Whenever no value for  $E_{pot}(v)$  is given, no Rossby waves were found at that subsection and frequency (by means of a model assuming random amplitudes and phases of the waves). The full lines represent the results from the 85-month series as analyzed by Price (1981) and Price and Magaard (1983). The open circles in the upper left portion of Fig. 1 indicate the results of Emery and Magaard (1976) who analyzed 56-month series from a subsection of the route.

The results from the 20-year series as analyzed by Price and Magaard (1980) and Price (1981) are given in Figs. 2a and 2b. The figures show Rossby wave potential energy spectra  $E_{pot}(v)$  for 34 5° squares between 20 and 30°N at eight frequencies ranging from 0.1 to 0.45 cpy. No error bars are given in Figs. 1 and 2. The relative standard error of the spectral estimates is generally in the 25–50% range.

Fig. 3 shows the geographical distribution of the potential energy density  $E_{pot}(v)$  of the annual Rossby waves ( $v = 1$  cpy) as analyzed by Kang (1980) and Kang and Magaard (1980) from the 40-month series of the TRANSPAC data.

<sup>1</sup> The energy of higher modes could not be proved to be significantly different from zero.

**4. Discussion**

In my opinion, the most remarkable result is the sequence of interannual Rossby wave spectra in the area 20–25°N, 175–130°W (Fig. 2b). Maximum energy, with a peak at 0.15 cpy (i.e., at a period of 6.7 years), occurs in the square 20–25°N, 155–150°W next to but not containing the Hawaiian Islands. No attempt is made in this paper to explain the generation of the interannual Rossby waves. I will leave it with the following remarks: The fact that the energy increases monotonically with distance from the eastern boundary from 130 to 155°W seems to call for local generation of the waves. Local atmospheric forcing has not been studied yet in the open ocean at the time scales under consideration. Willmott and Mysak (1980) have developed a model for wind-generated low-frequency (period of 6 years) Rossby wave motions in the Gulf of Alaska, where the special shape of the coastline has a decisive influence on the nature of the fluctuations. Bryan and Ripa (1978), in a theoretical investigation, also considered 6-year Rossby waves in the North Pacific generated by the reflection of a wind-driven fluctuation at the eastern coast. An interesting aspect of their model, in connection with the observational findings under discussion, is the fact that it produces internal fluctuations with amplitude modulations similar to the observed ones. On the other hand, several other features of their model are inconsistent with our observational findings, especially the latitudinal dependence. It may be worthwhile to try to improve and refine the Bryan and Ripa model to make it consistent with our observations as well as more recent results concerning the sea surface temperature anomalies (e.g., Barnett, 1981). But such an attempt is beyond the scope of this paper. The idea that atmospheric forcing at time scales around 6 years (Southern Oscillation) has anything to do with the observed spectra would not be more than a speculation at this time.

Another important aspect for the sequence of spectra under discussion is the existence of the Hawaiian Ridge, that runs from about 19°N, 155°W to about 30°N, 180°. For the waves in question the Hawaiian Ridge should act as a vertical semi-infinite barrier. Some of the wave energy will be reflected, a small portion will be scattered as cylindrical waves near the tip of the barrier (Mysak and LeBlond, 1972), and some of it may travel westward underneath the tip of the barrier. Since the group velocity of the incident waves is westward (deviations from that direction are smaller than 2°) and the barrier makes an angle of about 25° with the circles of latitude, the shadow zone behind the barrier is small. The observed total potential Rossby wave energy in the first square (20–25°N, 165–160°W) west of the Hawaiian Ridge is 49% of that in the square with maximum energy east of the ridge. Hence the Hawaiian Ridge does not seem

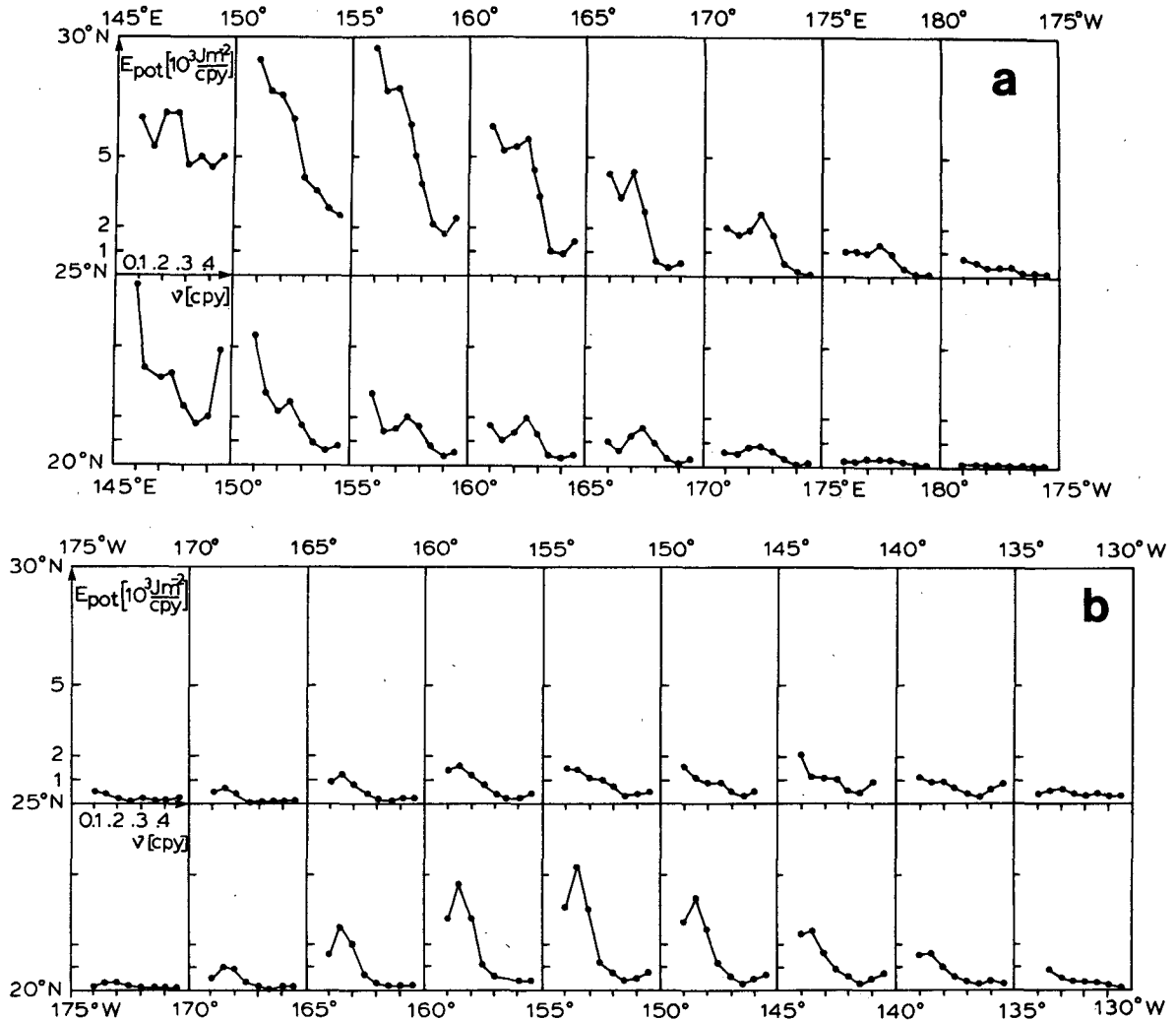


FIG. 2. Rossby wave potential energy spectra  $E_{pot}(v)$  at  $5^\circ$  squares between  $20^\circ$  and  $30^\circ\text{N}$ ,  $145^\circ\text{E}$  and  $175^\circ\text{W}$  (a) and  $175^\circ\text{W}$  and  $130^\circ\text{W}$  (b). Relative standard errors are 25 to 50%. These standardized results are derived from Price and Magaard (1980) and Price (1981).

to be a problem for the interpretation of the observed spectra.

The Honolulu–San Francisco great circle route intersects the area between  $20^\circ$  and  $30^\circ\text{N}$  where we have spectra ranging from 0.1 to 0.45 cpy (Figs. 2a and 2b). Comparisons between the results from the great circle route (Fig. 1) and the results displayed in Figs. 2a and 2b are possible at two frequencies: 0.28 and 0.42 cpy. Between  $20^\circ$  and  $25^\circ\text{N}$  the results agree at the level of standard error; north of  $25^\circ\text{N}$  the energy values obtained from the great circle route appear to be systematically higher. West of  $140^\circ\text{W}$ , our previous picture that there are no interannual Rossby waves north of  $30^\circ\text{N}$  (Kang and Magaard, 1980), is violated by the results from the great circle route even though they show a general decay of interannual Rossby wave energy toward the north. Such a decay is shown much more drastically in the results dis-

played in Figs. 2a and 2b east of the date line. In general, I would consider the findings east of the date line more reliable than the ones west of that meridian because, in the models, the mean oceanic conditions were taken into consideration more carefully on the eastern side.

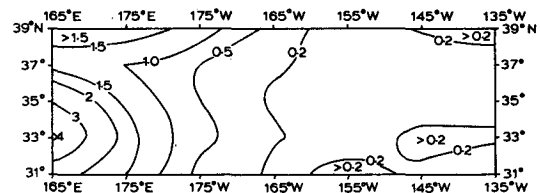


FIG. 3. Distribution of Rossby wave potential energy density  $E_{pot}(v)$  at  $\nu = 1$  cpy. The unit of  $E_{pot}$  is  $10^3 \text{ J m}^{-2} \text{ cpy}^{-1}$ . These standardized results are derived from Kang (1980) and Kang and Magaard (1980).

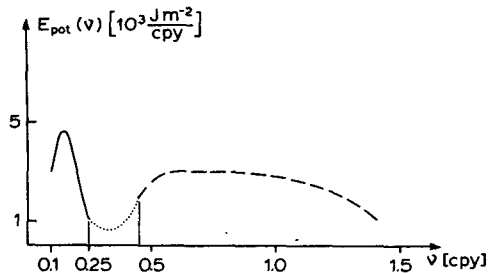


FIG. 4. Smoothed Rossby wave potential energy spectrum  $E_{\text{pot}}(\nu)$  for the square  $20\text{--}25^\circ\text{N}$ ,  $160\text{--}155^\circ\text{W}$ . Full line: after Price and Magaard (1980) and Price (1981); dashed line: after Price (1981) and Price and Magaard (1983); dotted line: hypothetical.

The geographical extent to which Rossby waves were found (by means of random-phase models) along the great circle route decreases from the whole Honolulu-San Francisco section at frequencies of 0.28 and 0.42 cpy to only its westernmost portion (centered at  $22^\circ\text{N}$ ,  $156^\circ\text{W}$ ) for frequencies  $> 1.28$  cpy (Fig. 1). Using a deterministic-phase model, White and Saur (1981) found annual baroclinic waves all along the great circle route. A comparison between the results of random- and deterministic-phase models for the annual waves along the great circle route is given in Price and Magaard (1983). The following reasons may account for the fact that, in the eastern half of the circle, no Rossby wave energy was found at frequencies  $> \nu = 1$  cpy:

- 1) The critical latitude for such waves may have been shifted southward under the influence of the mean flow and the bottom topography.
- 2) In the models used for the analysis it was assumed that the waves can only propagate in one direction per frequency. For frequencies  $< 1$  cpy this has proven to be a useful concept. For higher frequencies this may not be the case. Indications for that are found in Emery and Magaard (1976).

Concerning the energy of the annual Rossby wave between  $30$  and  $40^\circ\text{N}$  (Fig. 3) two regimes can be distinguished: West of the Emperor Seamount Chain (about  $175^\circ\text{E}$ ) we find relatively high energy. Values of relative standard error (not shown in Fig. 3) are generally between 30 and 50%. East of the Emperor Seamount Chain the energy drops rapidly to very small values. Small energy of annual Rossby waves in the area  $30\text{--}45^\circ\text{N}$ ,  $180\text{--}120^\circ\text{W}$  was also found by White (1982). In addition he found waves with a period scale of 1–2 years, which have not been detected by means of random-phase models. The small energies displayed in Fig. 3 east of the Emperor Seamount Chain have large errors. Relative standard errors range from 50 to 100%. Amazingly, the annual fluctuations turn out to show Rossby wave dispersion most convincingly east of the Emperor Seamount Chain (Kang and Magaard, 1980).

The largest amount of information about the Rossby wave potential energy spectrum is available for the  $20\text{--}25^\circ\text{N}$ ,  $160\text{--}155^\circ\text{W}$  square. Combining the information from Fig. 1 (upper left part) and Fig. 2b, one obtains a spectrum in the frequency range 0.1–2.11 cpy. It is clear that this spectrum has a peak at 0.15 cpy (6.7 years) and a broad peak ranging from about 0.5–1.4 cpy (2.0–0.7 years). Between 0.25 and 0.45 cpy the picture is less clear. I hypothesize that there is a spectral gap between 0.3 and 0.4 cpy. A smoothed spectrum containing all these elements is shown in Fig. 4.

I am aware of the fact that the picture of the distribution of baroclinic Rossby wave energy, as I have drawn it in this paper, is gappy and preliminary. The results are based on six years of work analyzing oceanic Rossby waves from data sets which are only marginally suitable for such an analysis. I expect significant changes of this picture to occur as more complete data become available. In view of the large time scales of the phenomena under consideration it may take many years to modify or confirm these spectra. Therefore it may be justified to paint this preliminary picture now. I hope it can stimulate further measurements and theoretical studies.

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