

## Effect of an Insoluble Surface Film on the Drift Velocity of Capillary-Gravity Waves

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### ABSTRACT

The drift velocity due to capillary-gravity waves in a deep ocean is investigated theoretically. The surface is covered by an insoluble, inextensible film, and the analysis is based on a Lagrangian description of motion. Attenuated as well as nondecaying, or permanent waves, are discussed. The strong temporal attenuation due to the inextensible film is shown to have a profound influence on the drift problem. It causes the induced mean virtual wave stress at the surface to decay quite rapidly, thereby limiting strongly the growth of the Eulerian part of the drift current. The drift problem for permanent waves is demonstrated to fall into two different categories, depending on the boundary conditions to second order: (i) If the mean *tangential* wind stress vanishes, our results confirm Craik's criticism of the analysis by Phillips and (ii) if the mean *horizontal* wind stress vanishes, there is an increased shear in the viscous boundary layer at the surface, as suggested by Phillips. Below the boundary layer the mean flow is essentially that of Stokes. But the surface velocity, and hence the motion of the film, is in the *opposite* direction of the waves. Finally, for temporally attenuated waves, it is demonstrated that the difference between the mean drift velocity at a clean surface and the mean drift velocity at a film-covered surface depends very much on the wavelength.

### 1. Introduction

It is well known that surface films of biogenic origin cover large areas of the ocean surface. They are particularly predominant in coastal zones (Barger et al. 1974; Brockmann et al. 1976). In addition to these natural films, or slicks, we find pollutant organic slicks from petroleum spills or municipal effluents. Such films resist the formation of wind-generated capillary waves. They also strongly enhance the attenuation of short waves. This reduces the aerodynamic roughness of the sea surface, changing its reflectance characteristics. Dynamically, the reduction of the surface roughness diminishes the wind stress and thereby the wind-induced drift current. The existence of surface films also affect the growth of longer waves, since the transfer of wind momentum to longer waves may depend on the presence of short waves riding on the backs of the longer waves (Garrett and Smith 1976; Landahl 1985). In fact, it has been anticipated since ancient times that waves in a storm could be calmed by pouring oil onto the sea (see Scott 1978, for a historical review).

The basic mechanism behind the increased damping in the presence of a surface film is now well understood

(Dorresteijn 1951). Essentially, it is related to the modification of the tangential stress boundary condition at the water-film interface. A complete treatment of this problem also involves the physical and chemical properties of the film itself. A discussion of this is beyond the scope of the present paper. There exists, however, a comprehensive literature on this subject. The interested reader is referred to Herr and Williams (1986) for review articles and references.

To the authors' knowledge published papers that consider the effect of surface film on the wave-induced mean current are not numerous. Notable exceptions are the contributions by Phillips (1977) and Craik (1982). They both, for simplicity, assume that the film is incompressible. This corresponds to the inextensible limit considered by Lamb (1932).

It is a fact that natural films contain both soluble and insoluble components. This may lead to elastic hysteresis due to different behavior of the soluble component during the compression and dilational phases of the wave cycle. Ideally such effects should be incorporated in a model of nonlinear wave drift. However, we feel that this approach is too ambitious considering the present state of the art. Instead, we focus on the more hydrodynamical aspects of the wave problem, utilizing an inextensible, insoluble, high surface-pressure film as a substitute for more realistic films. Although being an idealization, this is known to yield

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results for waves that are quite reasonable, see the discussion by Craik (1982). A more general condition at the contaminated surface was adopted by Puri and Pearce (1985). Their results subsume those obtained by Phillips (1977) in the limiting case of an inextensible film.

For a deep ocean, where the surface is covered by an insoluble, incompressible film, Lamb (1932) found that the temporal attenuation coefficient,  $\beta$ , for the waves can be written (in our notation)

$$\beta = \frac{k\omega}{4\gamma}. \tag{1.1}$$

Here  $k$  and  $\omega$  are the wavenumber and wave frequency, respectively. Furthermore,  $\gamma$  is an inverse viscous boundary-layer thickness defined by

$$\gamma = \left(\frac{\omega}{2\nu}\right)^{1/2} \tag{1.2}$$

where  $\nu$  is the kinematic viscosity of the fluid. The viscous boundary layer at the surface is always very thin, i.e.

$$\frac{k}{\gamma} \ll 1. \tag{1.3}$$

For future reference, we state the classic result of Stokes (1847) for wave drift in an irrotational inviscid fluid. Denoting the vertical coordinate by  $c \in \langle -\infty, 0 \rangle$ , and the wave amplitude by  $\zeta_0$ , Stokes' result for the mean particle velocity  $u_S$  can be written as

$$u_S = \zeta_0^2 \omega k e^{2kc}. \tag{1.4}$$

As noted, the presence of an insoluble surface film leads to increased wave damping. Phillips (1977) and Craik (1982) argue that this must promote stronger mean drift currents. Phillips (p. 58) suggested that this would manifest itself as an increased mean velocity difference across the viscous boundary layer near the surface, not altering the drift conditions in the interior. This view was opposed by Craik (1982), who argued that a large source of mean vorticity proportional to  $\zeta_0^2 \omega k^2 (\gamma/k)$  would be generated just below the surface. This would in turn induce an Eulerian mean velocity in the interior of the fluid, greatly exceeding the classic Stokes drift  $u_S$ , given by (1.4). In calculations to illustrate this point, Craik considered permanent waves. These are waves that are kept at constant amplitude by the application of a suitable stress distribution along the surface (Lamb 1932, p. 629). However, the specific form of the boundary condition that determines the development of the mean drift depends critically on how the viscous stress along the wave surface is prescribed. In particular, for a vanishing mean net horizontal stress on a fluid element at the surface, we find that the wave drift below the viscous boundary layer is essentially unaffected by the presence of a surface film.

We focus primarily on the mean drift due to waves which attenuate in time. Then second-order mean vorticity proportional to  $\gamma/k$  will diffuse into the interior, implying an increase of the mean velocity below the surface layer. However, this vorticity, or equivalently virtual wave stress (Longuet-Higgins 1969), is proportional to  $\exp(-2\beta t)$ , where  $\beta$  is given by (1.1). Hence this source of momentum will decay in time. In fact, for short capillary-gravity waves (for which the inextensible film limit is an acceptable approximation) this decay is quite rapid. This has a profound effect on the development of the drift current, as will be demonstrated for various values of the wave parameters.

## 2. Mathematical formulation

We consider waves on the surface of an incompressible, homogeneous, viscous fluid of density  $\rho$ . The depth of the fluid is infinite, or more precisely, much larger than the wavelength. The horizontal extent of the fluid is unlimited. When undisturbed, the surface is horizontal. A Cartesian coordinate system is chosen such that the  $x, y$ -axes are situated at the undisturbed surface, and the  $z$ -axis is positive upwards.

Let the wave motion be two-dimensional. In Eulerian notation the position of the surface is then given by  $z = \zeta(x, t)$ . Here we assume that the surface is covered by a very thin, insoluble film. The presence of a surface active material reduces the value of the surface tension. Let this reduced value be denoted by  $T$ . Furthermore, let the external dynamic stresses, normal and tangential to the fluid boundary, be denoted by  $\sigma$  and  $\tau$ . Expressing the effect of the surface tension explicitly, and not as a part of  $\sigma$ , the dynamic boundary conditions at  $z = \zeta$  can be written in the  $x$ - and  $z$ -directions, respectively,

$$\tau - \sigma \zeta_x = \mu(u_z + w_x) + p \zeta_x - 2\mu u_x \zeta_x + \frac{T \zeta_{xx} \zeta_x}{(1 + \zeta_x^2)^{3/2}} \tag{2.1}$$

$$\sigma + \tau \zeta_x = -p + 2\mu w_z - \mu(u_z + w_x) \zeta_x - \frac{T \zeta_{xx}}{(1 + \zeta_x^2)^{3/2}}. \tag{2.2}$$

Here  $(u, w)$  are the velocity components in the  $x$ - and  $z$ -directions, respectively,  $p$  is the dynamic pressure, and  $\mu$  the dynamic viscosity coefficient.

To deal properly with nonlinear wave motion in an Eulerian context, curvilinear coordinates should be applied (Longuet-Higgins 1953). Alternatively, the motion may be described by a direct Lagrangian approach (Pierson 1962; Weber 1983a,b). This proves to be very convenient and we shall pursue this line here.

For two-dimensional motion the fluid particles are labeled with the Lagrangian coordinates  $(a, c)$ . The displacements  $(x, z)$  and the dynamic pressure  $p$  are

written as series expansions after an ordering parameter  $\epsilon$  (Pierson 1962):

$$\begin{aligned} x &= a + \epsilon x^{(1)} + \epsilon^2 x^{(2)} + \dots \\ z &= c + \epsilon z^{(1)} + \epsilon^2 z^{(2)} + \dots \\ p &= -\rho g c + \epsilon p^{(1)} + \epsilon^2 p^{(2)} + \dots \end{aligned} \quad (2.3)$$

For wave motion with an initial amplitude  $\zeta_0$ , this parameter can be written as  $\epsilon = \zeta_0 \omega / k$  (Weber 1983a,b).

The primary wave field is described by the quantities labeled (1) in (2.3). We assume here that the surface film is inextensible in the tangential direction. This means that the tangential particle motion associated with the waves is halted at the water/film interface. Accordingly, to  $O(\epsilon)$ ,

$$x_i^{(1)} = 0, \quad c = 0 \quad (2.4)$$

where  $c = 0$  specifies the exact position of the surface in Lagrangian notation.

Formally, the external normal and tangential stresses in (2.1)–(2.2) may be written as series expansions in  $\epsilon$ :

$$\sigma = \epsilon \sigma^{(1)} + \epsilon^2 \sigma^{(2)} + \dots \quad (2.5)$$

$$\tau = \epsilon \tau^{(1)} + \epsilon^2 \tau^{(2)} + \dots \quad (2.6)$$

Utilizing (2.4), which implies that  $x_a^{(1)} = 0$  at the surface, together with the  $O(\epsilon)$  Lagrangian approximation of the continuity equation:  $x_a^{(1)} + z_c^{(1)} = 0$ , the dynamic boundary conditions (2.1) and (2.2) to  $O(\epsilon)$  and  $O(\epsilon^2)$  may be written in Lagrangian form:

$$\tau^{(1)} = \mu(x_{ic}^{(1)} + z_{ia}^{(1)}), \quad c = 0 \quad (2.7)$$

$$\sigma^{(1)} = -p^{(1)} - Tz_{aa}^{(1)}, \quad c = 0 \quad (2.8)$$

$$\begin{aligned} \tau^{(2)} - \sigma^{(1)} z_a^{(1)} &= p^{(1)} z_a^{(1)} + Tz_{aa}^{(1)} z_a^{(1)} \\ &\quad + \mu(x_{ic}^{(2)} + z_{ia}^{(2)}), \quad c = 0 \end{aligned} \quad (2.9)$$

$$\begin{aligned} \sigma^{(2)} + \tau^{(1)} z_a^{(1)} &= -p^{(2)} - Tz_{aa}^{(2)} \\ &\quad + \mu(2z_{ic}^{(2)} - 2z_{ia}^{(1)} x_c^{(1)} - x_{ic}^{(1)} z_a^{(1)} - z_{ia}^{(1)} z_a^{(1)}), \\ &\quad c = 0. \end{aligned} \quad (2.10)$$

### 3. The primary wave field

The solution to  $O(\epsilon)$ , or the primary wave field, is obtained by separating the wave field into an irrotational part and a rotational part (Lamb 1932). Utilizing (2.4), we obtain in complex notation

$$\left. \begin{aligned} x^{(1)} &= -\frac{i\kappa}{n} (e^{\kappa c} - e^{m c}) e^{i\kappa a + n t} \\ z^{(1)} &= -\frac{\kappa}{n} \left( e^{\kappa c} - \frac{\kappa}{m} e^{m c} \right) e^{i\kappa a + n t} \\ p^{(1)} &= \frac{\rho}{n} \left[ (n^2 + g\kappa) e^{\kappa c} - \frac{g\kappa^2}{m} e^{m c} \right] e^{i\kappa a + n t} \end{aligned} \right\} \quad (3.1)$$

where

$$m^2 = \kappa^2 + n/\nu. \quad (3.2)$$

Furthermore, we assume that  $\text{Re}(m), \text{Re}(\kappa) > 0$  so that the solutions vanish as  $c \rightarrow -\infty$ .

Here temporally attenuated waves are obtained when

$$\left. \begin{aligned} \kappa &= k \\ n &= -i\omega - \beta \end{aligned} \right\} \quad (3.3)$$

where the wavenumber  $k$ , the wave frequency  $\omega$  and the decay rate  $\beta$  all are assumed to be real.

Spatially attenuated waves are achieved if we take

$$\left. \begin{aligned} \kappa &= k + i\alpha \\ n &= -i\omega \end{aligned} \right\} \quad (3.4)$$

where  $\alpha$  is real.

When the viscosity is small, we obtain from (3.2):

$$m = \gamma \left[ 1 - i + (1 + i) \frac{k^2}{4\gamma^2} + O\left(\frac{k^3}{\gamma^3}\right) \right] \quad (3.5)$$

where  $\gamma$  is defined by (1.2). This result is based on the fact that  $\alpha, \beta \sim O(k/\gamma)$ , which is verified below.

The dispersion relation follows from (2.8). For attenuated waves ( $\sigma^{(1)} = 0$ ), we obtain

$$\omega = \omega_0 \left( 1 - \frac{k}{2\gamma} \right)^{1/2} \quad (3.6)$$

where  $\omega_0 = (gk + Tk^3/\rho)^{1/2}$  is the frequency of inviscid capillary-gravity waves. For temporal attenuation, the decay coefficient becomes

$$\beta = \frac{k\omega_0^2}{4\gamma\omega} \approx \frac{k\omega_0}{4\gamma} \quad (3.7)$$

as stated by Lamb (1932). For spatially attenuated waves, we find for the damping coefficient

$$\alpha = \frac{k^2 \omega_0^2}{2\gamma(\omega_0^2 + 2k^3 T/\rho)}. \quad (3.8)$$

We note from (3.7) and (3.8) that the relation  $\alpha = \beta/c_g$  (Gaster 1962), where  $c_g$  is the group velocity, also holds here.

For a suitably arranged external normal stress distribution  $\sigma^{(1)}$  along the surface, work may be done on the fluid in such a way that the loss of energy due to internal friction is exactly compensated for (Lamb 1932). This yields permanent waves, i.e., waves that propagate without change of amplitude. In the present context this means that  $\alpha = \beta = 0$ . The dispersion relation is again given by (3.6). The corresponding normal stress distribution, which somehow must arise from motions in the air, is then obtained from (2.8):

$$\sigma^{(1)} = -\frac{\rho k \omega_0^2}{2\gamma\omega} e^{i(\kappa a - \omega t)}, \quad c = 0. \quad (3.9)$$

One should note that, for attenuated as well as for per-

manent waves, there is a nonzero periodic tangential stress  $\tau^{(1)}$  acting on the fluid. This is seen by insertion of the solutions (3.1) into the boundary condition (2.7). Accordingly, an equally large and oppositely directed periodic stress  $-\tau^{(1)}$  is acting on the film. This stress is taken up by its lateral stiffness.

Obviously, the effect of rotation on short capillary-gravity waves can safely be neglected. This is because the wave frequency is very much larger than the inertial frequency.

**4. Boundary conditions for the mean motion**

We are now in the position to calculate the dynamic boundary conditions for the mean horizontal motion to  $O(\epsilon^2)$ . The mean is defined as an average value over one wavelength, and will be denoted by an overbar. For attenuated waves, there is no wind in the problem. Accordingly, the mean tangential stress to  $O(\epsilon^2)$  must vanish at the surface. This means  $\bar{\tau}^{(2)} = 0$  in (2.9). Since also  $\sigma^{(1)} = 0$ , equation (2.9) reduces to

$$\overline{p^{(1)}z_a^{(1)}} + T\overline{z_{aa}^{(1)}z_a^{(1)}} + \mu(\bar{x}_{ic}^{(2)} + \bar{z}_{ia}^{(2)}) = 0, \quad c = 0. \quad (4.1)$$

Furthermore, we assume that there is no net vertical flow at the surface, i.e.

$$\bar{z}_i^{(2)} = 0, \quad c = 0. \quad (4.2)$$

By insertion from (2.8) into (4.1), we then obtain

$$\bar{x}_{ic}^{(2)} = 0, \quad c = 0. \quad (4.3)$$

This constitutes the surface boundary condition for the mean horizontal drift current induced by temporally or spatially attenuated waves.

For permanent waves, we may arrive at two different boundary conditions. If we again assume that there is no mean tangential stress acting on the surface, (2.9) yields for the mean motion:

$$-\overline{\sigma^{(1)}z_a^{(1)}} = \overline{p^{(1)}z_a^{(1)}} + \mu\bar{x}_{ic}^{(2)}, \quad c = 0. \quad (4.4)$$

Here we have utilized (4.2) and the fact that  $\overline{z_{aa}^{(1)}z_a^{(1)}} = 0$ . By insertion from (2.8) into (4.4), we obtain

$$\bar{x}_{ic}^{(2)} = 0, \quad c = 0 \quad (4.5)$$

which is the same result as for attenuated waves.

However, since we now may choose the (weak) wind field at will, it is possible to prescribe a mean tangential wind stress  $\bar{\tau}^{(2)} \neq 0$ . Since the film is tangentially incompressible, it may transfer this mean stress to the fluid. We choose  $\bar{\tau}^{(2)}$  so that there is no net horizontal stress on a fluid element at the surface. This means from (2.9) that

$$\bar{\tau}^{(2)} = \overline{\sigma^{(1)}z_a^{(1)}}, \quad c = 0. \quad (4.6)$$

The boundary condition (2.9) then reduces to

$$\mu\bar{x}_{ic}^{(2)} = -\overline{p^{(1)}z_a^{(1)}}, \quad c = 0. \quad (4.7)$$

By insertion from (3.1), we find

$$\bar{x}_{ic}^{(2)} = -\frac{k^3\gamma}{2\omega} \left(1 - \frac{k^2}{4\gamma^2}\right), \quad c = 0. \quad (4.8)$$

The mean tangential wind stress in this case becomes from (4.6)

$$\bar{\tau}^{(2)} = -\frac{\rho k^3 \omega_0^2}{4\gamma \omega^2} \approx -\frac{\rho k^3}{4\gamma}. \quad (4.9)$$

It is obvious that the two different boundary conditions (4.5) and (4.8) lead to very different developments of the drift current induced by permanent waves. This will be discussed in some detail in section 7.

At infinite depth the mean wave drift is assumed to vanish, i.e.

$$\bar{x}_i^{(2)} \rightarrow 0, \quad c \rightarrow -\infty. \quad (4.10)$$

This must be valid for attenuated as well as for permanent waves.

**5. The mean drift due to temporally attenuated waves**

It is clear that the concept of permanent short waves is rather unrealistic in an oceanographic context. This is because it is practically impossible in a real ocean to achieve the very special surface stress distribution that is required to sustain these waves against friction. Unless the wave damping is small, the wave-induced drift due to permanent waves has therefore little practical interest. However, since this problem is often encountered in the literature, we shall discuss it briefly in section 7.

In a wave tank, it is appropriate to adopt a spatial description of wave amplitude decay. Here a wave generator is operating at a given frequency. Due to the strong damping of short waves, the wave train will be confined to a relatively small area behind the generator. In the ocean, waves are generated by the action of wind. A passing windrow generates short capillary-gravity waves over a rather large area. When the local wind forcing has disappeared, the waves in the whole area attenuate due to friction. When a new windrow comes along, the process repeats itself. Accordingly, the behavior of short waves in the open sea seems to be most adequately described by temporal attenuation. Our main attention will therefore be focused on such waves as far as the mean drift is concerned.

As already mentioned, short waves on a clean surface are subject to rapid damping. When the surface is covered with a thin, perhaps monomolecular organic film, this damping process occurs even more quickly. As an example we consider a layer of oleyl alcohol, which is often used for experimental purposes (Hühnerfuss et al. 1981). Such a film meets our requirements of insolubility and incompressibility reasonably well. The air-water surface tension,  $T$ , of a clean surface of about  $74 \text{ dyn cm}^{-1}$  may now be reduced to  $43 \text{ dyn cm}^{-1}$ .

However, one should keep in mind that for more realistic, natural films the surface tension reduction probably will be considerably smaller. Values from 0.1–5 dyn cm<sup>-1</sup> may be more appropriate, although very little data on this subject is available. Anyway, various choices of the numerical value for the surface tension reduction do not alter the basic wave drift problem. Assuming a value of 0.012 cm<sup>2</sup> s<sup>-1</sup> for the molecular viscosity of water, we obtain from (3.7) that waves with wavelength  $\lambda = 1$  cm are damped within a characteristic time  $\beta^{-1} = 0.4$  sec. For waves with wavelength 10 cm, the corresponding value of  $\beta^{-1}$  is 8 sec.

The general equation that governs the mean horizontal wave drift to  $O(\epsilon^2)$  can be written (Pierson 1962):

$$\begin{aligned} \nu \bar{x}_{icc}^{(2)} - \bar{x}_u^{(2)} = & -\frac{1}{\rho} \overline{p_a^{(1)} x_a^{(1)}} - \frac{1}{\rho} \overline{p_c^{(1)} z_a^{(1)}} \\ & + \nu [2 \overline{x_a^{(1)} x_{iaa}^{(1)}} + 2 \overline{z_c^{(1)} x_{icc}^{(1)}} + 2 \overline{z_a^{(1)} x_{iac}^{(1)}} \\ & + 2 \overline{x_c^{(1)} x_{iac}^{(1)}} + \overline{x_{ia}^{(1)} \nabla_L^2 x^{(1)}} + \overline{x_{ic}^{(1)} \nabla_L^2 z^{(1)}}] \quad (5.1) \end{aligned}$$

where  $\nabla_L^2 = \partial^2/\partial a^2 + \partial^2/\partial c^2$ . Here we have assumed that  $\bar{p}_a^{(2)} = \bar{z}_a^{(2)} = 0$ , i.e. there is no mean horizontal pressure gradient in the fluid. This is a reasonable assumption for an ocean of unlimited lateral extent. Furthermore, in (5.1) we have neglected the effect of rotation on the mean flow. This is justified a posteriori from the results. They show that the mean drift vanishes due to friction on a time scale that is very much smaller than the inertial period.

We introduce a mean drift velocity  $u = \epsilon^2 \bar{x}_t^{(2)}$ , where  $\epsilon = \zeta_0 \omega/k$  (Weber 1983a,b). Then, insertion of real parts from (3.1) into (5.1) yields

$$\begin{aligned} \nu u_{cc} - u_t = & \nu \zeta_0^2 \omega k^3 e^{-2\beta t} \left[ \frac{\gamma}{k} e^{2kc} + 4 \frac{\gamma^2}{k^2} e^{(\gamma+k)c} \sin \gamma c \right. \\ & \left. + \frac{3\gamma^2}{k^2} e^{2\gamma c} + O(e^{2kc}) + O\left(\frac{\gamma}{k} e^{\gamma c}\right) \right]. \quad (5.2) \end{aligned}$$

The boundary conditions (4.3) and (4.10) become

$$u_c = 0, \quad c = 0 \quad (5.3)$$

$$u \rightarrow 0, \quad c \rightarrow -\infty. \quad (5.4)$$

A complete solution of (5.2) can be obtained as a particular solution  $u^{(p)}$  of (5.2) plus a solution  $u^{(h)}$  of the homogeneous version of this equation. Hence the Lagrangian mean velocity  $u$  is given by  $u^{(p)} + u^{(h)}$ . To compare with more common ways of dealing with wave-induced drift, e.g., Longuet-Higgins (1953), or more recently Craik (1982), we recall that the Lagrangian mean velocity can be written as a sum of an Eulerian mean velocity  $u_E$  plus the Stokes drift  $u_S$  (for example, see Phillips 1977, p. 43). The Stokes drift is obtained from the inviscid irrotational wave field. To compare with our ab initio Lagrangian approach, we note that the particular solution  $u^{(p)}$  essentially yields the Stokes drift, but with the important difference that

it is modified by friction in the surface boundary layer. The homogeneous solution  $u^{(h)}$ , which must be added to  $u^{(p)}$  in cases where  $u^{(p)}$  does not fulfill the surface boundary condition (5.3), corresponds approximately to the Eulerian mean velocity  $u_E$ . It is straightforward to integrate (5.2) in the vertical. Since  $k \ll \gamma$ ,  $k$  can be neglected in the exponent of the second term on the right-hand side of (5.2). We find

$$\begin{aligned} u^{(p)} = & \zeta_0^2 \omega k e^{-2\beta t} \left[ e^{2kc} - 2e^{\gamma c} \cos \gamma c \right. \\ & \left. + \frac{3}{4} e^{2\gamma c} + O\left(\frac{k}{\gamma}\right) \right]. \quad (5.5) \end{aligned}$$

Here we have assumed that  $u_c^{(p)}, u^{(p)} \rightarrow 0, c \rightarrow -\infty$ .

The surface boundary condition for the homogeneous problem can now be derived from (5.3):

$$u_c^{(h)} = -u_c^{(p)}, \quad c = 0. \quad (5.6)$$

Calculating  $u_c^{(p)}$  ( $c = 0$ ) from (5.5), the condition above can be written

$$u_c^{(h)} = \frac{1}{2} \zeta_0^2 \omega k \gamma e^{-2\beta t} \left[ 1 + O\left(\frac{k}{\gamma}\right) \right], \quad c = 0. \quad (5.7)$$

Hence we find, for temporally damped waves, that mean secondary vorticity proportional to  $\gamma/k$  will diffuse inwards from the surface into the interior of the fluid. Apart from the exponential damping, Eq. (5.7) is equivalent to Craik's condition (4.12) for the mean Eulerian velocity gradient in the limit of infinite depth.

As already mentioned, the effect of damping is very important in the present problem. Analogous to Longuet-Higgins (1969), we may define a virtual wave stress  $\tau_w$  at the surface:

$$\tau_w = \rho \nu u_c^{(h)}, \quad c = 0. \quad (5.8)$$

Initially, this quantity is much larger here than for a clean surface. However, from the calculated values of  $\beta^{-1}$  in the beginning of this paragraph, we see that  $\tau_w$  decays quite rapidly with time. This implies that there is a limit to the growth of  $u^{(h)}$ , or the Eulerian mean flow.

We assume that the mean Lagrangian drift equals the classic Stokes drift,  $u_S$ , at time  $t = 0$ . The homogeneous version of (5.1) is solved by Laplace transforms. A complete solution of the drift problem  $u = u^{(p)} + u^{(h)}$  can then be written:

$$\begin{aligned} u = & \zeta_0^2 \omega k \left[ e^{-2\beta t} \left\{ e^{2kc} - 2e^{\gamma c} \cos \gamma c + \frac{3}{4} e^{2\gamma c} \right\} \right. \\ & + \frac{1}{2} \left( \frac{\omega}{2\pi} \right)^{1/2} \int_0^t \frac{e^{2\beta(\xi-t) - c^2/(4\nu\xi)}}{\xi^{1/2}} d\xi \\ & + \frac{(2\omega)^{1/2}}{\pi} \int_0^\infty \left( \frac{\xi + \omega}{\xi^{1/2}(\xi^2 + \omega^2)} \right. \\ & \left. \left. - \frac{3}{4\xi^{1/2}(\xi + 2\omega)} \right) e^{-\xi t} \cos(c\xi^{1/2}/\nu^{1/2}) d\xi \right]. \quad (5.9) \end{aligned}$$

Here the first integral on the right-hand side stems from the large, but gradually damped virtual wave stress (5.8). The second integral arises from our particular choice of boundary conditions.

It is of interest to compare the magnitude of  $u$  from (5.9) with the surface value of  $u_S$  given by (1.4). For future reference we therefore define a dimensionless drift velocity,  $u_*$ , by

$$u_* = \frac{u}{\xi_0^2 \omega k}. \quad (5.10)$$

As an example we let the surface be covered by a film of oylel alcohol, for which  $T = 43 \text{ dyn cm}^{-1}$ . When  $\lambda = 1 \text{ cm}$ , capillarity dominates the dispersion relation (3.6). For water, with  $\nu = 0.012 \text{ cm}^2 \text{ s}^{-1}$ , we obtain from (3.6) and (3.7)

$$\omega = 126 \text{ rad s}^{-1}, \quad \beta = 2.8 \text{ s}^{-1}. \quad (5.11)$$

To get an idea of the vertical length scales involved, we recall that  $u_S$ , which we have chosen as the initial mean velocity distribution of the problem, reaches down to a depth of about  $\pi/(2k)$ . This "Stokes depth" is here 0.25 cm. Furthermore, we take the outer edge of the surface boundary layer to be situated at  $c = -\pi/\gamma$ . The boundary-layer thickness defined in this way becomes 0.04 cm in the present example. Defining a nondimensional vertical coordinate  $c_*$  by

$$c_* = -2kc, \quad (5.12)$$

the dimensionless boundary-layer thickness  $\delta$  may be written

$$\delta = 2\pi k/\gamma. \quad (5.13)$$

In Fig. 1 we have plotted the dimensionless wave drift  $u_*$  from (5.10) as function of time at  $c_* = 0, \delta, 1, \pi$ . As mentioned above, the initial dimensionless drift velocity distribution with depth is given by  $u_* = \exp(-c_*)$ . For the surface velocity this is not easily recognized from the figure. This is due to a rapid variation of the drift velocity during the first few tenths of

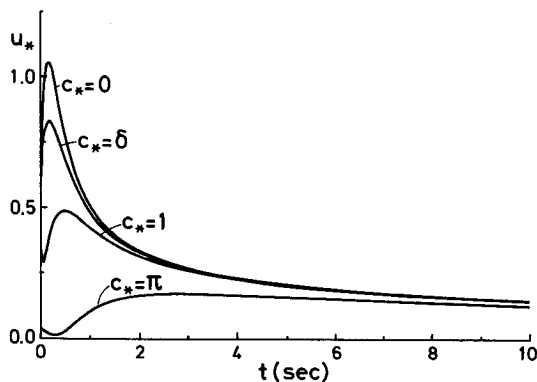


FIG. 1. Dimensionless wave drift velocity  $u_*$  from (5.10) as function of time at various depths. Here  $\nu = 0.012 \text{ cm}^2 \text{ s}^{-1}$ ,  $T = 43 \text{ dyn cm}^{-1}$ , and  $\lambda = 1 \text{ cm}$ .

a second. It comes from the imposed initial condition, and is a result of the evaluation of the second integral on the right-hand side of (5.9). Very soon, however, the first integral in (5.9) starts to dominate. As already mentioned, this term essentially comes from the presence of a virtual wave stress at the surface. It causes the drift velocity to exceed  $u_S$  at any particular depth, before damping halts the increase and finally takes over. We note that the position of the maximum drift velocity at larger depths systematically is shifted towards larger times. This reflects the fact that the mean momentum is transported downwards by diffusion. Although the drift velocity below the viscous surface boundary layer tends to exceed  $u_S$ , it never becomes very much larger, contrary to the prediction by Craik (1982). This is due to the strong damping in the present case.

As a second example we consider waves with wavelength  $\lambda = 10 \text{ cm}$ . Now gravity dominates the dispersion relation. Equations (3.6) and (3.7) then yield

$$\omega = 25 \text{ rad s}^{-1}, \quad \beta = 0.12 \text{ s}^{-1}. \quad (5.14)$$

The Stokes depth,  $\pi/(2k)$ , and the boundary layer thickness,  $\pi/\gamma$ , in this example become 2.5 and 0.1 cm, respectively. Figure 2 exhibits  $u_*$  plotted as function of time for  $\lambda = 10 \text{ cm}$ . Results are displayed at five different dimensionless depths: 0,  $\delta$ ,  $1/2$ , 1,  $\pi$ . Qualitatively the results are the same as for  $\lambda = 1 \text{ cm}$ . Now damping is less pronounced. Accordingly, the maximum mean velocity exceeds  $u_S$  more strongly than before. At the surface this excess amounts to a factor of about 2. Again, the local minima in the curves at the early stages of the time development are due to the imposed initial conditions.

Finally, we consider waves with wavelength  $\lambda = 100 \text{ cm}$ . According to Lucassen (1982) this seems to be as far as we can stretch the theory for the influence of a surface active material. If we compute the relative increase in damping coefficients, i.e. the ratio of (1.1) to the corresponding value of  $\tilde{\beta} = 2\nu k^2$  for a clean surface (Lamb 1932), we find  $\beta/\tilde{\beta} = 72$ . This will imply a (perhaps) unrealistically high value of the surface dilational modulus, see Lucassen (1982, Fig. 1). Now, for this gravity wave, we obtain

$$\omega = 7.8 \text{ rad s}^{-1}, \quad \beta = 0.007 \text{ s}^{-1} \quad (5.15)$$

from (3.6) and (3.7). In Fig. 3 we have depicted  $u_*$  from (5.10) as function of time for  $\lambda = 100 \text{ cm}$  at four different dimensionless depths: 0,  $\delta$ ,  $1/2$ , 1. The curves exhibit the same qualitative behavior as before, with a further increase of the maximum drift velocity. At the surface this maximum occurs after about 100 seconds, exceeding  $u_S$  by a factor of 5.

The Stokes depth,  $\pi/(2k)$ , in this example is 25 cm. One might speculate that at such length scales, one should perhaps introduce a turbulent eddy viscosity to account for the downward diffusion of momentum. It would then be appropriate to assume a depth-increasing

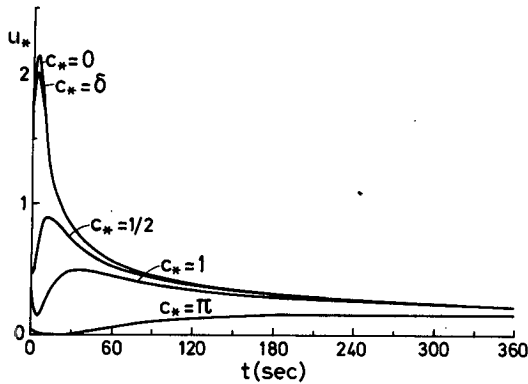


FIG. 2. As in fig. 1, but now for  $\lambda = 10$  cm.

eddy viscosity. Qualitatively, the effect of turbulent diffusion can be obtained by a depth averaged value. Assuming a constant eddy viscosity of  $1 \text{ cm}^2 \text{ s}^{-1}$ , which is two orders of magnitude larger than the molecular value, the drift velocity becomes considerably reduced. At the surface one then finds a maximum value of 1.7 occurring after about 15 seconds in this example.

The primary wave field studied here attenuates on a characteristic time scale  $\beta^{-1}$ . It is interesting to note from Figs. 1–3 that the induced secondary current exists over a much longer period of time. Even at the surface, the time required for reducing the induced current to about  $1/3$  of its maximum value, is much larger than  $\beta^{-1}$ . When the waves have disappeared, all the original wave momentum has been transferred to the mean current. By diffusion, mean momentum is transported downward, and will gradually be distributed over the entire water column. At the bottom (infinity) there is a sink. Since no additional momentum is supplied at the surface, the mean current finally will tend to zero everywhere. The discussion of this problem continues in section 8.

It is straightforward to assess the importance of rotation on the drift problem. It turns out that this effect manifests itself through the dimensionless parameter

$$G \equiv \frac{f}{2\beta} = \frac{2f\gamma}{\omega k}. \quad (5.16)$$

Rotation is not important if  $G \ll 1$ . This is easily seen to be the case for the examples considered.

As mentioned earlier in this paragraph, spatial attenuation of short waves is relevant for laboratory experiments. The drift current in this case may be found from Weber [1987, Eq. (6.5)], by applying the boundary conditions (5.3) and (5.4) of the present paper.

**6. Comparison with wave drift at a clean surface**

The theory developed in the present paper is valid for inextensible surface films of horizontal extent much larger than the wavelength. The mean wave-induced

surface velocity equals the drift velocity  $u_0$  of the film. Nondimensionally this is given by (5.10):

$$u_0 = u_*(0, t). \quad (6.1)$$

The mean wave-induced surface velocity in the absence of a film may be obtained from Longuet-Higgins (1960, 1969) for a nonrotating ocean. More directly, the Lagrangian formulation of Weber (1983b) yields in the limit of vanishing rotation:

$$\begin{aligned} \tilde{u}(0, t) = & \zeta_0^2 \tilde{\omega} k \left[ e^{-2\tilde{\beta}t} \right. \\ & \left. + 2k \left( \frac{\nu}{\pi} \right)^{1/2} \int_0^t \frac{e^{2\tilde{\beta}(\xi-t)}}{\xi^{1/2}} d\xi + O\left(\frac{k}{\gamma}\right) \right]. \quad (6.2) \end{aligned}$$

Here  $\tilde{\omega}$  is the frequency of waves on a clean surface, and  $\tilde{\beta} = 2\nu k^2$  is the corresponding attenuation coefficient. Due to the reduction of the surface tension caused by a surface active material, short waves at a given wavelength will experience a frequency decrease upon entering an area covered by a film slick. In order to compare the wave-induced drift in the two cases directly, we define a dimensionless drift velocity at a clean surface in accordance with (5.10) as

$$\tilde{u}_0 = \frac{\tilde{u}(0, t)}{\zeta_0^2 \omega k} \quad (6.3)$$

where  $\omega$  is the frequency in the presence of contamination and  $\tilde{u}(0, t)$  is given by (6.2).

Equation (6.2) is valid in a fluid where the surface is uncontaminated, and where the viscous effect of the air is neglected. Since the density of the air is so much smaller than the density of water, this is equivalent to assuming vacuum above the water. The term ‘‘clean surface’’ is in the present paper used in this context only. It should be remembered, however, that the viscous effect of the air may influence the drift velocity of water waves, as pointed out by Dore (1978a,b). Dore finds that only for waves with wavelengths of a fraction of a meter, the effect of the air may be neglected. This applies to the case studied here. In Fig. 4 we have de-

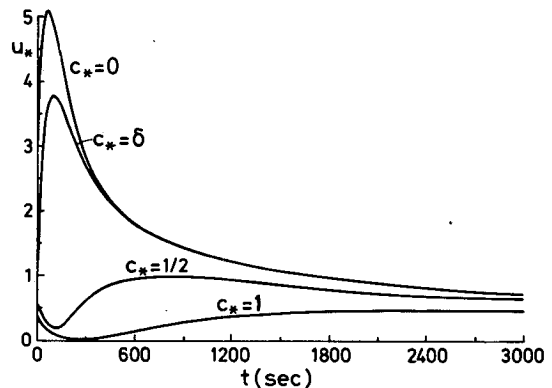


FIG. 3. As in fig. 1, but now for  $\lambda = 100$  cm.

picted  $u_0$  from (6.1) and  $\tilde{u}_0$  from (6.3) as functions of time when  $\lambda = 1$  cm. We note the somewhat surprising result that the value for a clean surface (broken line) is always larger than that for a surface film. This means that for these capillary-dominated waves, a large slick will move slower than the surrounding water. This is a consequence of the fact that the initial virtual wave stresses in these two cases are comparable in magnitude. Hence the slower decay in the free surface case induces the largest surface current at subsequent times. At larger wavelengths the situation changes somewhat. In Fig. 5 we have plotted the surface drift when  $\lambda = 10$  cm. Now the virtual wave stress at the film covered surface initially is about a factor of ten larger than that at a clean surface. Accordingly, in the early stages of the development of the surface velocity, the largest velocity occurs in the presence of a film (solid line). At larger times, however, the effect of the smaller damping of the virtual wave stress in the clean surface case tends to take over, so the clean surface velocity (broken line) becomes the largest. The areas under the curves in Fig. 5 represent the length of two trajectories from the same starting point. Obviously, a particle at a clean surface will move the longest distance in this example.

At even longer wavelengths the tendency towards higher velocities for a film covered surface is strengthened. This is evident from Fig. 6, where  $u_0$  from (6.1) (solid line) and  $\tilde{u}_0$  from (6.3) (broken line) are depicted for  $\lambda = 100$  cm.

**7. The mean drift due to permanent waves**

Although permanent waves require a very special surface wind-stress distribution to exist, they have often been used as a substitute for more realistic wave conditions when the wave-induced drift is concerned. Inserting from the real parts of (3.1) with  $\alpha = \beta = 0$  into (5.1), we obtain for permanent waves

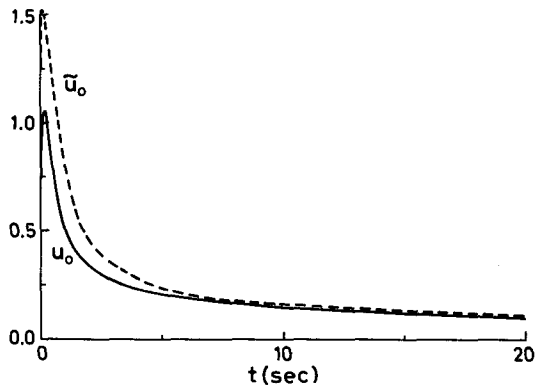


FIG. 4. Dimensionless surface drift velocity as function of time. Solid line: Film covered surface (6.1). Broken line: Clean surface (6.3). Furthermore,  $\nu = 0.012 \text{ cm}^2 \text{ s}^{-1}$ ,  $T = 43 \text{ dyn cm}^{-1}$ , and  $\lambda = 1$  cm.

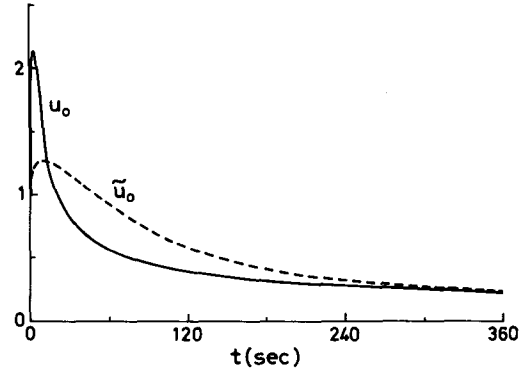


FIG. 5. As in Fig. 4, but now for  $\lambda = 10$  cm.

$$\nu u_{cc} - u_t = \nu \zeta_0^2 \omega k^3 \left[ 4e^{2kc} - \frac{4\gamma}{k} e^{(\gamma+k)c} (\cos \gamma c - \sin \gamma c) + \frac{4\gamma^2}{k^2} e^{(\gamma+k)c} \sin \gamma c + \frac{3\gamma^2}{k^2} e^{2\gamma c} \right] \quad (7.1)$$

where again  $u = \epsilon^2 \bar{x}_t^{(2)}$ .

As discussed in section 4, the boundary condition for the mean drift depends on how the wind stress is specified. For a vanishing mean *tangential* wind stress to  $O(\epsilon^2)$ , we find from (4.5)

$$u_c = 0, \quad c = 0. \quad (7.2a)$$

The condition for a vanishing mean *horizontal* stress to  $O(\epsilon^2)$  is given by (4.8). In terms of the drift velocity  $u$ , it becomes

$$u_c = -\frac{1}{2} \zeta_0^2 \omega k \gamma \left[ 1 - \frac{k^2}{4\gamma^2} \right], \quad c = 0. \quad (7.2b)$$

Concerning the mean drift equation (7.1), it has been necessary to compute the right-hand side to a greater accuracy in the parameter  $\gamma/k$  than for attenuated waves [cf. (5.2)]. This is done in order to match the boundary condition (7.2b).

It is straightforward to find a particular solution  $u^{(p)}$  of (7.1). Again  $k \ll \gamma$ , so  $k$  can be neglected in the exponent of the second term on the right-hand side of (7.1), which is the smallest of the boundary-layer terms. This cannot be done in the third term, however, since we then will lose a term of order unity in the velocity gradient at the surface. By a series expansion of the expression for  $u_c^{(p)}$  after  $k/\gamma$  as a small parameter, we find

$$u_c^{(p)} = \zeta_0^2 \omega k^2 \left[ 2e^{2kc} - 2e^{\gamma c} \cos \gamma c + \frac{2\gamma}{k} e^{\gamma c} (\sin \gamma c - \cos \gamma c) + \frac{3\gamma}{2k} e^{2\gamma c} + O\left(\frac{k}{\gamma}\right) \right]. \quad (7.3)$$



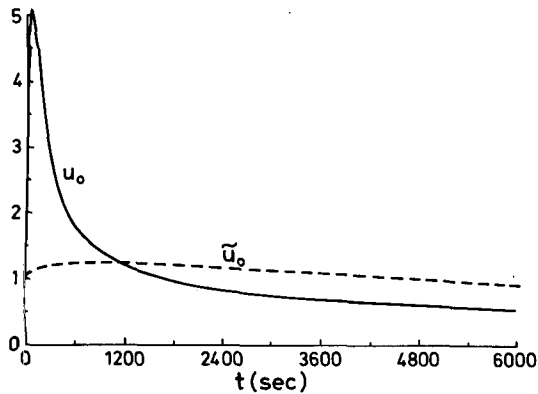


FIG. 6. As in Fig. 4, but now for  $\lambda = 100$  cm.

By further integration, we obtain

$$u^{(p)} = \zeta_0^2 \omega k \left[ e^{2kc} - 2e^{\gamma c} \cos \gamma c + \frac{3}{4} e^{2\gamma c} + O\left(\frac{k}{\gamma}\right) \right] \quad (7.4)$$

This is just the same expression as (5.5), if we put  $\beta = 0$ . The surface boundary conditions for the solution  $u^{(h)}$  of the homogeneous equation may be written from (7.2a), (7.2b) and (7.3), respectively:

$$u_c^{(h)} = \frac{1}{2} \zeta_0^2 \omega k \gamma \left[ 1 + O\left(\frac{k^2}{\gamma^2}\right) \right], \quad c = 0 \quad (7.5a)$$

$$u_c^{(h)} \sim \zeta_0^2 \omega k^2 O\left(\frac{k}{\gamma}\right), \quad c = 0. \quad (7.5b)$$

The first condition here is equivalent to that given by Craik [his Eq. (4.12)] in the limit of infinite depth). It implies a large constant virtual wave stress at the surface. This will in turn yield infinitely large drift velocities in the fluid, when time tends to infinity. The complete solution in this case is simply obtained by putting  $\beta = 0$  in (5.9).

When (7.5b) applies, the virtual wave stress is small and can be neglected. Hence the particular solution (7.4) represents the complete solution as time tends to infinity. We note that  $u$  from (7.4) coincides with the classic inviscid Stokes drift,  $u_S$ , below the viscous surface layer. However, at the surface the value of the drift current equals  $-u_S(0)/4$ . Hence the film in this case will move in opposite direction of the waves. This is related to the presence of a constant mean tangential stress,  $\bar{\tau}^{(2)}$ . Here this stress has to be imposed to ensure that there is no net horizontal stress on a fluid element at the surface. From (4.9) we note that this stress is directed in the negative  $x$ -direction.

For gravity waves on a clean surface Weber (1983a) has studied a similar problem. Here the effect of friction in the surface layer is smaller. Consequently, the surface drift velocity only differs from Stokes classic result by a term of order  $k/\gamma$ .

Formally, to avoid the infinite drift velocities obtained from the solution of (7.1) and (7.2a) as  $t \rightarrow \infty$ , rotation must be included in the problem, see Madsen (1978) and Weber (1983a,b). Then the balance between the viscous stress and the Coriolis force will imply finite drift velocities everywhere. The procedure is straightforward. However, this requires that the waves persist on a time scale comparable to the inertial period. For short capillary-gravity waves in the presence of a surface film, this is highly unlikely. We will therefore not pursue this problem here.

### 8. Discussion

In an infinitely deep ocean the mean drift velocity as well as the viscous stress tend to vanish as  $c \rightarrow -\infty$ . With no external forcing at the surface, it follows from the basic principles that the total mean momentum for the entire water column must be conserved, i.e.,

$$\frac{dU}{dt} = 0 \quad (8.1)$$

where  $U \equiv \int_{-\infty}^0 udc$ .

This is also confirmed by our approximate analysis in the limit of small viscosity. By integrating (5.2), utilizing (5.3) and assuming that  $u_c \rightarrow 0$  as  $c \rightarrow -\infty$ , we find that the mean total momentum obtained in that way satisfies (8.1). Accordingly,  $U$  is entirely determined by the initial distribution of the mean velocity. This is also verified by integrating the drift solution (5.9) (Jenkins, private communication, 1989).

One should perhaps intuitively think that stronger viscous damping of the waves would be associated with larger mean drift currents in the fluid. This is not necessarily so, as seen from Figs. 1-3 for nondimensional drift in a film covered ocean. Here the viscosity dependence of the virtual wave stress is the same in all three examples ( $\tau_w \sim \nu^{1/2}$ ). The initial value of  $\tau_w$  increases as the wavelength decreases. As a result, the early time development of the drift current shows a more rapid increase for short waves. On the other hand, the larger damping of short waves more effectively limits the growth of the current. This yields a lower maximum value for the dimensionless drift current in the case of larger damping.

The comparison between the drift current obtained for a film covered surface and that obtained for a clean surface is interesting. In the latter case the frictional influence is smaller ( $\tau_w \sim \nu$ ) and the damping is less pronounced. But for high frequency (capillary) waves, the virtual wave stresses in these two cases are comparable in magnitude. The slower damping in the free surface case then leads to a larger surface velocity, as seen from Fig. 4.

It would be of interest to apply the present theory to the wave-induced drift of large oil slicks in a fully developed sea. But in such a sea state most of the energy will be concentrated at lower frequencies. These fre-

quencies fall outside the range for which the inextensible film limit seems to be an acceptable approximation. Therefore, for a fully developed sea, the theory presented here should only be considered in a qualitative sense.

As already mentioned, our analysis neglects entirely the influence of the air on the drift velocity of water waves. For laminar flow and an uncontaminated surface, Dore (1978a,b) concludes that this is only correct when the wavelength is much less than a metre. This is the range of waves we are considering here (the results displayed for  $\lambda = 1$  m are done merely to show the trend for longer waves). In the light of the present results, however, it appears that the water wave problem with an uncontaminated surface should be studied afresh. The effect of the air resembles that of a film, although weaker. It increases the shear near the air-water interface, and thereby enhances the dissipation. Nonlinearly, the source terms for the induced second order mean motion are increased initially, but are then subject to stronger damping as time progresses. This yields a tendency towards stronger drift currents at short times and weaker drift currents at larger times as compared to the free surface case. A more thorough study of this problem is now in progress.

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