Scattering of Shelf Waves by Islands

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ABSTRACT

The domain exterior to an island in a channel with topography is not simply connected and so the circulation about the island is indeterminate. Rhines shows that, in a rotating flow with a rigid lid, requiring the pressure to be continuous forces the circulation to be constant in time. It is shown here that the constant-circulation continuous-pressure solution also conserves energy and, moreover, the circulation associated with scattering of incident shelf waves is identically zero. The scattering problem is then well posed and a method is given for constructing the scattered field at arbitrary frequencies.

The problem simplifies greatly in the low frequency limit and an explicit solution for waves scattered by a thin barrier follows by decomposing the motion into propagating modes and a geostrophic current, following Hsieh and Buchwald. Explicit values are given for the round-island flux and the distribution of scattered wave energy for a long, thin island in the center of a channel. It is shown that with increasing island length the round-island flux decreases rapidly from the value determined by requiring the volume flux to be continuous at the leading edge of the island towards a solution with little flux between the island and a coastal boundary, as in Wilkin and Chapman.

1. Introduction

The problem of the scattering of continental shelf waves by islands or isolated, finite barriers has received considerable attention of late in, for example, Hsieh and Buchwald (1985, denoted by HB herein), Wilkin and Chapman (1988, denoted by WC herein), and Buchwald and Hsieh (1988). The domain exterior to an island is not simply connected and so the circulation round the island is indeterminate in inviscid flow. It is the purpose of the present note to show that following Rhines (1969b) and requiring the pressure to be continuous round the island forces this circulation to be constant in time. Moreover, the solution with constant circulation has the same down-channel energy flux both before and after the island. Section 2 gives these results, showing also that the circulation associated with an incident shelf wave is identically zero and giving a method for constructing the scattered field at arbitrary frequencies. Section 3 discusses the considerable simplifications in the low-frequency or long-wave limit of the problem, pointing out, however, that in general even vanishingly small viscosity causes a dissipative boundary layer at the leading edge of an island (Johnson 1989b). This layer, which absorbs energy and makes a nonzero contribution to the circulation, is absent if the scattering region about the leading edge of the island is "conservative" (Johnson 1989c) and in particular for the thin island treated by HB and WC. A solution with a continuous pressure field is thus obtained by forming the linear combination of the HB and WC expressions that has zero round-island circulation. Section 4 presents explicit results for a barrier in the center of a channel with rigid walls and exponential topography. The results are discussed briefly in section 5.

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2. Governing equations

The topographic wave equation can be written (Rhines 1969a)

$$\nabla \cdot (H^{-1}\nabla \Psi_t) + \hat{\mathbf{z}} \cdot [\nabla \Psi \times \nabla (f/H)] = 0, \quad (2.1)$$

where $H(x, y)$ is the local depth, $\hat{\mathbf{z}}$ a unit vertical vector, f the Coriolis parameter and Ψ the depth-averaged streamfunction, giving the horizontal velocity field,

$$H\mathbf{u} = \hat{\mathbf{z}} \times \nabla \Psi. \quad (2.2)$$

The momentum equation corresponding to (2.1) can be written

$$H\nabla P = f\nabla \Psi - \hat{\mathbf{z}} \times \nabla \Psi_t, \quad (2.3)$$

where ρP is the departure of the pressure from hydrostatic and ρ is the constant density. Consider a channel given by $-\infty < x < \infty$, $0 \leqslant y \leqslant l$ containing an island with boundary C (Fig. 1). Following previous discussion of this geometry by HB and WC, the boundary at $y = l$ is supposed to border a flat open ocean region,

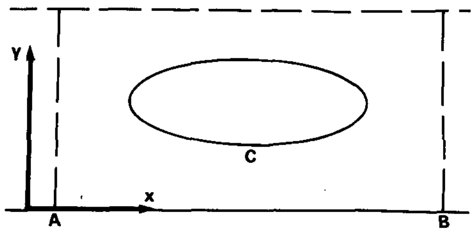

FIG. 1. A plan view of a rectilinear channel containing an island. There is an impervious coastal boundary at $y = 0$ and a boundary to the open ocean at $y = l$. It is shown in the text that the pressure or surface elevation is single-valued and the energy flux across any uninterrupted cross-channel plane (such as those at A and B) is independent of its position provided the net circulation round the island vanishes.

modeled by taking $u = 0$ there. The condition on Ψ is then

$$\Psi_y = 0, \quad \text{for} \quad y = l. \qquad (2.4)$$

The boundaries at C and $y = 0$ are solid and so the conditions on Ψ can be written

$$\Psi = 0, \quad \text{for} \quad y = 0, \qquad (2.5)$$

$$\Psi = \alpha(t) \quad \text{on } C. \qquad (2.6)$$

The function $\alpha(t)$ is determined by requiring the pressure to be continuous, corresponding in flows with a free surface to requiring the surface elevation to be continuous. Integrating (2.3) around C and using (2.6) gives

$$\frac{d}{dt}\oint_C H^{-1}\Psi_n ds = 0, \qquad (2.7)$$

where ∂_n is the normal derivative and s the arc length. Condition (2.7) determines $\alpha(t)$ and is a restatement of Kelvin's theorem that the circulation around a closed material contour is constant (Rhines 1969b). The flux round the island is a globally determined quantity, and varies with the dimensions of the island and the composition of the incident flow. Equation (2.1) with conditions (2.4), (2.5), (2.6), (2.7) gives a well-posed problem with a unique solution.

The solutions for scattering by islands in WC and HB do not satisfy (2.7). Thus the pressure, or free-surface elevation, is not a single-valued function of position in their solutions. Integrating round the island yields a jump in pressure at some point and so unphysical infinite pressure gradients and infinite accelerations there. When (2.7) is satisfied these difficulties are removed.

Multiplying (2.1) by Ψ gives the energy conservation relation

$$E_t + \nabla\cdot\mathbf{F} = 0, \qquad (2.8)$$

where $E = \frac{1}{2}H^{-1}|\nabla\psi|^2$ is the local energy density and $\mathbf{F}$ the energy flux. Several forms for $\mathbf{F}$, differing from each other by quantities whose divergence is zero, are given in Johnson (1989a). A convenient choice is

$$\mathbf{F} = (\Psi/H)\nabla\Psi_t - \frac{1}{2}\Psi^2\hat{\mathbf{z}}\times\nabla(f/H). \qquad (2.9)$$

Consider a region bounded by cross-channel planes lying either side of the island (A and B in Fig. 1). Integrating (2.8) over this region gives

$$\frac{d}{dt}\int Edxdy = \left[\int_0^l \mathbf{F}\cdot\mathbf{i}dy\right]_A^B - \alpha(t)\frac{d}{dt}\oint_C H^{-1}\Psi_n ds, \qquad (2.10)$$

using the no-flux condition (2.6) on C and noting that as H is constant on $y = l$ so $\mathbf{F}\cdot\mathbf{n}$ vanishes there. Condition (2.7) shows the final term in (2.10) to be identically zero. The circulation condition means there is no net flux of energy into the region through the boundary of the island. For periodic motions integrating (2.10) over one temporal period shows that the time-averaged flux over a cross-channel plane is independent of the position of the plane and given by

$$\int_0^l [-(\Psi/H)\Psi_{xt} + (f/H)_y\Psi^2]dy, \qquad (2.11)$$

whether the plane lies before or after the island. It is only the net flux through C that vanishes: $\mathbf{F}$ is, in general, nonzero on C, there is a local flux of energy through the boundary of the island, and the flux over planes joining the island to boundaries varies with their position. A choice for the flux which vanishes on solid boundaries is $H\mathbf{u}P$ and this is indeed independent of x on planes joining the island to the coast. It does not however vanish on $y = l$ for condition (2.4). If instead the boundary $y = l$ were solid, then both forms of the flux would coincide away from the island and be independent of x.

A prototype for a wide class of problems is given by taking the island to be the finite barrier of zero thickness:

$$0 \leqslant x \leqslant 2a, \quad y = lL, \quad \text{for} \quad 0 < L < 1 \qquad (2.12)$$

and the topography to be rectilinear [i.e. $H = H(y)$].

For periodic motion, as associated with wave scattering, write

$$\Psi(x, y, t) = \mathrm{Re}\{\psi(x, y)e^{-i\omega ft}\}. \qquad (2.13)$$

Then (2.1) becomes

$$i\omega\nabla^2\psi - G(y)(i\omega\psi_y - \psi_x) = 0, \qquad (2.14)$$

where $G(y) = (\log H)_y$. The circulation round the island associated with scattering is time periodic, given by $\mathrm{Re}\{\Gamma e^{-i\omega ft}\}$ for the complex constant

$$\Gamma = \int_0^{2a} H^{-1}[\psi_y(x, lL^+) - \psi_y(x, lL^-)]dx. \qquad (2.15)$$

(An additional circulation, constant in time, may be present representing steady irrotational flow round the island. This flow plays no part in scattering.) The constraints on ψ are thus (2.4) and (2.5), and from (2.6):

$$\psi = \alpha, \quad \text{for} \quad 0 \leq x \leq 2a, \quad y = lL, \qquad (2.16)$$

where α is a second complex constant, determined by the circulation condition (2.7), which becomes

$$\Gamma = 0. \qquad (2.17)$$

Equations (2.14)–(2.17) can be solved directly with an appropriate upstream condition, but perhaps simpler conceptually is to find the solutions $\psi^{(0)}$ and $\psi^{(1)}$, of (2.14)–(2.17) for two arbitrary values of α, $\alpha^{(0)}$ and $\alpha^{(1)}$, and compute the corresponding circulations about the island, $\Gamma^{(0)}$ and $\Gamma^{(1)}$. Then the continuous-pressure zero-circulation solution is given by

$$\psi = (\Gamma^{(1)}\psi^{(0)} - \Gamma^{(0)}\psi^{(1)})/(\Gamma^{(1)} - \Gamma^{(0)}), \qquad (2.18)$$

with

$$\alpha = (\Gamma^{(1)}\alpha^{(0)} - \Gamma^{(0)}\alpha^{(1)})/(\Gamma^{(1)} - \Gamma^{(0)}). \qquad (2.19)$$

Each of the subsidiary problems can be solved straightforwardly by matching cross-channel modal expansions at $x = 0$ and $x = 2a$. The above ideas are, however, most clearly illustrated in the low frequency limit.

3. The low-frequency or long-wave limit

Introduce the nondimensional variables

$$(x', y') = (x/l, y/l), \qquad (3.1)$$

and the slowly varying scale $X = \omega x'$. Then (2.14) becomes, dropping dashes,

$$\omega^2\psi_{XX} + \psi_{yy} - G(y)(\psi_y + i\psi_X) = 0, \qquad (3.2)$$

where $G(y)$ is taken to be a smooth function of y of order unity. Now consider the limit $\omega \to 0$, $a/l \to \infty$ such that $x_0 = 2a\omega/l$ remains constant. Then the first term is absent from (3.2), (2.4) is applied on $y = 1$ and (2.16) on $0 \leq X \leq x_0$, $y = L$. This system is valid away from the planes $X = 0, x_0$. Near these planes but away from $y = L$, x is order unity and the sole surviving term of (3.2) in the present limit is

$$\psi_x = 0, \qquad (3.3)$$

where $x = \omega^{-1}X$ or $\omega^{-1}(X - x_0)$. Integrating from $-\infty$ to ∞ with respect to x gives the condition

$$0 = [\psi]_{x=-\infty}^{\infty} = [\psi]_{x=0^-}^{0^+}. \qquad (3.4)$$

The streamfunction is continuous across $X = 0, x_0$. It remains to consider the equations in the neighborhood of the points $(0, L)$ and (x_0, L). The solution is well behaved in the neighborhood of the trailing edge, but at the leading edge $\psi(0^-, L)$ will, in general differ from α. In this region the appropriate scale for x and y is ω, the scale of short waves in (2.1). The leading order terms in (3.2) then give

$$\psi_{\xi\xi} + \psi_{\eta\eta} - iG(L)\psi_\xi = 0, \qquad (3.5)$$

where $\xi = x/\omega$ and $\eta = (y - L)/\omega$. The boundary conditions on (3.5) are

$$\psi \to \psi(0^-, L) \quad \text{as} \quad \xi \to -\infty \quad \text{for each } \eta, \qquad (3.6)$$

$$\psi = \alpha \quad \text{on } C. \qquad (3.7)$$

Equation (3.5), with (3.6) and (3.7), defines a Rossby wave scattering problem on a β-plane. Short waves scattered from the island in general carry energy in the negative ξ direction. It is shown, however, in Johnson (1989b) that in the present low-frequency limit even vanishingly small viscosity dissipates these waves and they have no signature outside the neighborhood of $(0, L)$. The energy reflected in the waves in inviscid flow at nonzero frequencies is destroyed in this neighborhood. In the present example of an island of zero width compared to the cross-channel width, this dissipation is zero to leading order in ω and so energy is conserved to leading order. The thinness of the island also means that the neighborhood of $(0, L)$ makes no contribution to the circulation. For wider islands, of width of order the channel width, the amount of dissipation and the contribution to the circulation depend on the type of scattering region at the leading edge of the island. If the scattering region is "conservative" (Johnson 1989c) then energy is conserved and there is no contribution to the circulation. Otherwise an intense boundary layer of finite cross-stream extent is present, dissipating energy and making a nonzero contribution to the circulation. Such a dissipative system is discussed at greater length in Johnson (1989d).

The continuous-pressure zero-circulation low-frequency problem for a narrow island can thus be written

$$\psi_{yy} - G(y)(\psi_y + i\psi_X) = 0, \qquad (3.8)$$

$$\psi = 0 \quad \text{for} \quad y = 0, \quad \psi_y = 0 \quad \text{for} \quad y = 1, \qquad (3.9\text{a,b})$$

$$\psi = \alpha \quad \text{for} \quad 0 < X < x_0, \quad y = L, \qquad (3.10)$$

$$\Gamma = \int_0^{x_0} H^{-1}[\psi_y(X, L^+) - \psi_y(X, L^-)]dX = 0, \qquad (3.11)$$

ψ continuous at $X = 0, x_0$

$$(0 \leq y < L, L < y \leq 1). \qquad (3.12)$$

The time-averaged energy flux over a cross-channel plane is given by

$$F = \frac{1}{4}\int_0^1 (H^{-1})_y\psi\psi^*\,dy,$$

(where the asterisk denotes complex conjugate) and is independent of X.

Hsieh and Buchwald (1985) note that (3.8) can be solved for general $H(y)$ by separation of variables and that for $G \neq 0$ the boundary conditions (3.9) and (3.10) with α specified give for the cross-stream structure a Sturm–Liouville problem with a complete set of orthogonal eigenfunctions. Solutions satisfying (3.12) follow by matching at $X = 0, x_0$. Complete expressions for the various normalization factors and expansion coefficients for general $H(y)$ can be found in HB. Wilkin and Chapman (1988) use the same procedure, differing from HB solely in the choice of α. Hsieh and Buchwald choose $\alpha = \psi(0^-, L)$ giving a continuous-streamfunction solution and in $0 \leqslant X \leqslant x_0$ an X-independent mode and a rapidly converging eigenfunction expansion. Wilkin and Chapman choose $\alpha = 0$, giving a zero-flux solution with the X-independent mode absent and the expressions for the expansion coefficients simplified although with the expansion itself converging more slowly. In neither of these solutions is the pressure or surface elevation single-valued, but once the round-island circulation associated with these solutions is calculated, the continuous-pressure zero-circulation solution follows using (2.18) and (2.19).

4. Exponential topography

Consider the depth profile given by

$$H = H_0 e^{2by}, \quad H_0, b \text{ constants.} \tag{4.1}$$

Full details of the continuous-streamfunction and zero-flux expressions can be found in HB and WC for arbitrary b, L and cross-channel incident mode number. The continuous-pressure zero-circulation solution then follows as above. To illustrate the nature of the results consider the simpler problem obtained by taking the island to be in the center of the channel ($L = ½$) and replacing the open ocean boundary condition at $y = 1$ with a rigid wall so ψ vanishes there. Introduce $\Phi = \exp[-b(y - ½)]\psi$.

Modes incident on the barrier are given by

$$\Phi = \sin n\pi y \exp[i(n^2\pi^2 + b^2)(X/2b)], \quad X < 0. \tag{4.2}$$

Modes for even n are odd about $y = ½$, and the continuous-streamfunction, zero-flux and continuous-pressure zero-circulation determinations coincide, each giving $\alpha = 0$. The wave is unaffected by the barrier. Modes for odd n are even about $y = ½$ and so therefore is Φ. It is sufficient to consider the half channel $0 \leqslant y \leqslant ½$. Here

$$\Phi_{yy} - 2ib\Phi_X - b^2\Phi = 0, \tag{4.3}$$

$$\Phi = 0 \quad \text{for} \quad y = 0, \tag{4.4}$$

$$\Phi = \alpha \quad \text{for} \quad 0 < X < x_0, \quad y = \frac{1}{2}, \tag{4.5a}$$

$$\Phi_y = 0 \quad \text{for} \quad X > x_0, \quad y = \frac{1}{2}, \tag{4.5b}$$

$$\Gamma = 2\int_0^{x_0} \Phi_y\left(X, \frac{1}{2}\right)dX + 2b\alpha x_0, \tag{4.6}$$

$$\Phi = \sin\pi y \quad \text{for} \quad X = 0, \quad 0 \leqslant y < \frac{1}{2}, \tag{4.7}$$

where for definiteness the incident mode is taken to be the fundamental. Denote the solution in $0 < X \leqslant x_0$, decomposed into waves of amplitude a_n by Φ^w, with

$$\Phi^w = \sum_{n=1}^{\infty} a_n \sin 2n\pi y \exp(ik_n X), \tag{4.8}$$

where $k_n = (4n^2\pi^2 + b^2)/2b$. The corresponding circulation is given by

$$\Gamma^w(x_0) = 2\pi i \sum_{n=1}^{\infty} (-1)^{n+1} n a_n[\exp(ik_n x_0) - 1]/k_n. \tag{4.9}$$

The zero-flux solution $\Phi^{(0)}$ and its circulation $\Gamma^{(0)}$ are then given by (4.8)–(4.9) with

$$a_n = a_n^{(0)} = \frac{2}{\pi}\frac{(-1)^{n+1}n}{n^2 - 1/4}. \tag{4.10}$$

A convenient second solution is that for a unit round-island flux but zero energy incident from $X < 0$, i.e., satisfying $\Phi = 1$ on C and $\Phi = 0$ on $X = 0$. The solution and its circulation are

$$\Phi^{(1)} = \sinh by \Big/ \sinh\frac{1}{2}b - \Phi^w, \tag{4.11}$$

$$\Gamma^{(1)} = 2bx_0/(1 - e^{-b}) - \Gamma^w, \tag{4.12}$$

where Φ^w and Γ^w are given by (4.8)–(4.9) with coefficients

$$a_n = a_n^{(1)} = (-1)^{n+1}4\pi n/bk_n. \tag{4.13}$$

The value of α in the continuous-pressure zero-circulation solution is given explicitly by

$$\alpha = -\Gamma^{(0)}/\Gamma^{(1)}, \tag{4.14}$$

and the solution itself by

$$\Phi = \alpha \sinh by \Big/ \sinh\frac{1}{2}b + \Phi^w, \tag{4.15}$$

where the coefficients in Φ^w are $a_n = a_n^{(0)} - \alpha a_n^{(1)}$. The a_n are in general complex as are the streamfunction and the round-island flux. Figure 2 shows the variation in $|\alpha|$, the round-island flux scaled on the flux incident on the island, as a function of x_0 for $b = 2$. As $x_0 \to 0$, $\alpha \to 1$: the streamfunction is continuous for barriers of length vanishing on the scale ω^{-1}, i.e., $a \ll l/\omega$. As x_0 increases, α rapidly approaches zero, i.e., the flux becomes close to that given by the zero-flux solution. The decrease is not monotonic and much high-frequency structure is visible, superposed on a decaying oscillation of wavelength approximately 0.60. This

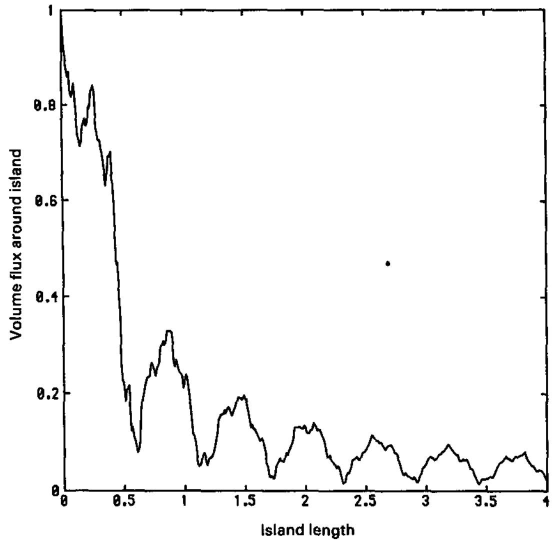

FIG. 2. The round-island volume flux $|\alpha|$ (scaled on the incident flux) between a centrally placed long thin island and a channel wall as a function of the length x_0 of the island in the low frequency limit for an incident shelf wave of mode 1. The flux is initially continuous but drops rapidly towards zero with increasing barrier length. The logarithmic slope b is 2.

compares with the long-channel wavelength, $k_1 \approx 0.58$, of the lowest mode between the island and the coast and represents the resonance present in the geometry for the particular incident wave frequency. In the limit of a semi-infinite barrier ($x_0 \rightarrow \infty$), $\alpha = 0$: the round-barrier volume flux vanishes. This is the determination used by WC, and their Fig. 2 can be reinterpreted as giving the distribution of energy flux among modes propagating beside a semi-infinite island.

The solution in $X \geqslant x_0$ follows by matching across $X = x_0$, giving

$$\Phi = \sum_{m=1}^{\infty} (-1)^{m+1} d_m \sin[(2m-1)\pi y] \times \exp\{i[(2m-1)^2\pi^2 + b^2][(X - x_0)/2b]\}, \tag{4.16}$$

where only the odd modes appear as Φ is even about $y = 1/2$, and the coefficients d_m are given by

$$d_m = 4\alpha b \coth \frac{1}{2} b \Big/ [(2m-1)^2\pi^2 + b^2] + \frac{2}{\pi} \sum_{n=1}^{\infty} (-1)^{n+1} n a_n \exp(ik_n x_0) \Big/ \left[n^2 - \left(m - \frac{1}{2}\right)^2\right]. \tag{4.17}$$

The time-averaged energy flux associated with mode $2m - 1$ is given as a fraction of the incident flux by $d_m d_m^*$. Figure 3 shows for $b = 2$, as a function of x_0 and scaled on the incident flux, the individual contributions of the first three nonzero modes ($m = 1, 2, 3$) and the total time-averaged flux. The contributions are once again strongly modulated by geometric resonance. The total flux is unity as required.

5. Discussion

It has been shown, in the context of the scattering of continental shelf waves, that requiring the pressure to be single-valued (or the surface elevation in a model with a free surface) forces an otherwise arbitrary circulation about any island to vanish. This condition then implies the constancy of the net energy flux down the channel. Solutions of a given problem follow most straightforwardly by solving for two arbitrary values of the round-island volume flux and then forming the linear combination with zero circulation.

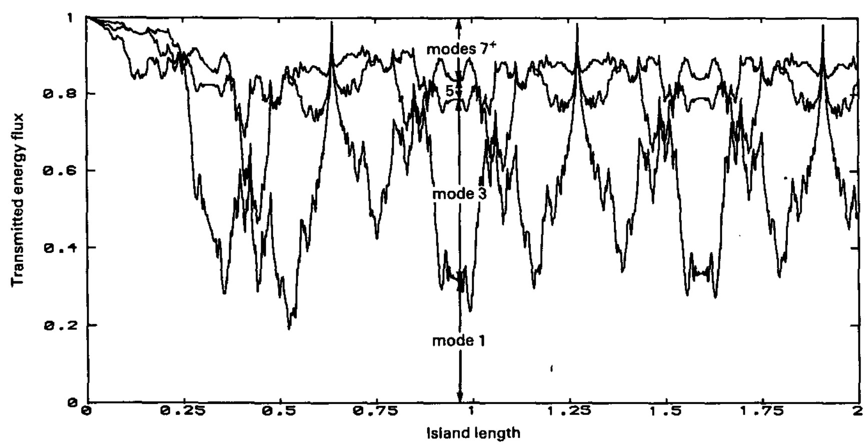

FIG. 3. Time-averaged energy fluxes downstream of the island as a function of the island length x_0 for an incident mode one wave with unit flux. The total flux is unity as required by energy conservation for a thin island. The logarithmic slope b is 2.

This construction can be shown most straightforwardly in the low-frequency limit. In general, in this limit even vanishingly small viscosity dissipates energy in a boundary layer at the leading edge of obstacle (Johnson 1989b). If the scattering region is conservative (Johnson 1989c) or the island occupies a negligible fraction of the shelf-width, however, then this boundary layer is absent and energy is conserved. Such an example is the thin barrier in a channel with rectilinear topography, discussed in the low frequency limit by HB and WC, and a continuous-pressure zero-circulation solution can be constructed from a linear combination of their results. A particular example has been given for exponential topography in a channel with rigid walls and a central island. The round-island flux of the geostrophic current is shown to decrease rapidly from that forced by the oncoming flow (for barriers with $a \ll l/\omega$) to near zero with increasing barrier length. Both the volume flux and the energy flux of the scattered field exhibit fluctuations caused by resonances in the domain bounded by the island and the coastal boundary.

In the limit of a semi-infinite barrier, $a\omega/l \rightarrow \infty$, the present results agree with those obtained from radiation conditions on free surface flows. If the surface is free, two Kelvin waves are supported by a semi-infinite barrier—one incident from $x = +\infty$ supported by the side $y = lL^-$ and the other generated at the origin and propagating to $x = +\infty$ supported by the side $y = lL^+$. The radiation condition for scattering of a long shelf wave incident from $x = -\infty$ requires that the amplitude of the Kelvin wave incident from $x = +\infty$ vanishes. In the rigid-lid limit this is equivalent to requiring the round-island volume flux to vanish, i.e., $\alpha = 0$, precisely the result obtained from the present analysis in the limit $x_0 \rightarrow \infty$.

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