Oceanographic Implications of Laboratory Experiments on Diffusive Interfaces

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ABSTRACT

This paper contains a summary of the results from some laboratory and theoretical studies on the diffusive interface in double diffusive convection, paying particular attention to the recent work of Fernando. A simple model is developed which predicts the thicknesses of the convecting layers in a thermohaline staircase structure. The laboratory buoyancy-flux measurements and the model results are extrapolated for oceanic situations and comparisons are made with field measurements.

1. Introduction

With the development of sophisticated CTD measuring instruments, the capability of resolving minute features of the oceanic microstructure has been enhanced. At the same time, the literature emphasizing the role of double diffusion as a major oceanic mixing mechanism has markedly increased (Mack 1985). Often, oceanic mixing events contain more than one physical process, e.g., double diffusion and shear instabilities, so that the identification of the detailed physics and the relative contribution of individual processes is difficult. Controlled laboratory experiments, which are designed to study a single mixing mechanism, have proven to be useful in interpreting and modeling oceanic phenomena (Federov 1970). In this paper, the results of some theoretical and laboratory studies on diffusive interfaces are summarized. Particular attention will be given to the recent theoretical and laboratory work of Fernando (1987, 1989), which will be extrapolated to predict oceanic parameters. These predictions are also compared with those of previous investigations.

2. A review of laboratory studies on thermohaline staircases

Turner and Stommel (1964) were the first to demonstrate the formation of a "thermohaline staircase," a series of turbulently convecting layers separated by density interfaces, when a stable salinity-stratified fluid is heated from below. These interfaces are stably (unstably) stratified with respect to salt (heat), and the buoyancy transport through them, at high interfacial stabilities, occurs mainly by molecular diffusion.

The mixed layer development and the layered-structure formation in a linearly stratified fluid, subjected to a constant bottom heat flux, have been studied in a more quantitative manner by Turner (1968), Huppert and Linden (1979), and Fernando (1987). A schematic diagram of the flow configuration used in these studies is shown in Fig. 1. A nomenclature of symbols is given in the Appendix.

Turner (1968) argued that the breakdown of the thermal boundary layer, which develops above the growing convective mixed layer, is responsible for the formation of multiple layers. The thermal boundary layer was assumed to become unstable when the Rayleigh number, defined as $\mathrm{Ra}_\delta = g\alpha\Delta T\delta^3/k_h\nu$, where δ is the thermal boundary-layer thickness, α is the thermal expansivity, g is the gravitational acceleration, ΔT is the temperature jump at the edge of the mixed layer, k_h is the thermal diffusivity and ν is the kinematic viscosity, exceeds a critical value R_c. On this basis, it was shown that the critical-height of the lower layer, at which the second mixed layer appears, is

$$h_c = \left(\frac{R_c}{4}\right)^{1/4}\left(\frac{\nu q_0{}^3}{k_h{}^2 N_s{}^8}\right)^{1/4}, \qquad (1)$$

where N_s is the buoyancy frequency, defined in terms of the initial salinity gradient $d\bar{S}/dz$ as $N_s{}^2 = -g\beta(d\bar{S}/dz)$, where β is the salinity contraction coefficient, z the vertical coordinate, and q_0 the buoyancy flux due to heat, defined in terms of the bottom heat flux Q, a reference density ρ_0 and the specific heat c_p, as $q_0 = g\alpha Q/\rho_0 c_p$. By fitting laboratory data to (1), Turner (1968) found that $R_c \approx 2.5 \times 10^4$, noting that this is an order of magnitude larger than the value $R_c \approx 10^3$, expected from linear stability theory calculations. Federov (1970) and Newman (1976) have used (1) in

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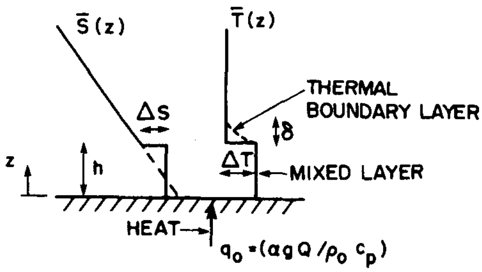

FIG. 1. Schematic diagram showing the growth of a convective mixed layer during the bottom heating of a linear salinity gradient. The symbols are defined in the nomenclature.

interpreting the data obtained during various field expeditions.

Fernando (1987) argued that the reason for high value of R_c, obtained by Turner (1968), is a misinterpretation of the laboratory data. Turner (1968) used the thickness of the lower convecting layer of the laboratory staircases as h_c in calculating R_c, from (1). Detailed temperature and salinity profile measurements showed that the thermal boundary layer breaks down at $R_c \approx 10^3$, but this does not lead to the formation of a "thermohaline" staircase structure. Rather the convective layer formed due to the break down was shown to be "engulfed" by the turbulent eddies of the lower layer, until the eddies are too "feeble" to do so. Based on the previous laboratory results of Fernando and Long (1985), it was argued that the bottom-layer eddies cannot engulf the layer above when the inertia and buoyancy forces of the eddies become of the same order, or

$$\overline{w^2} = c_1 \Delta b h, \quad (2)$$

where Δb is the buoyancy jump across the interface, $(\overline{w^2})^{1/2}$ is the rms velocity in the convecting layer, h is the mixed-layer height, and henceforth c_1, c_2, ... are used to denote constants; the estimated values of these constants are given in Table 1.

TABLE 1. Estimated values of the constants used in the double diffusion equations.

Constant	Value	Source
c_1	0.063	Fernando and Long (1985)
c_2	1.8	Hunt (1984)
c_3	41.5	Fernando (1987)
c_5	8.58×10^{-3}	Marmorino & Caldwell (1976)
c_6	0.323	Huppert (1971)
c_7	7×10^{-2}	present paper/Fernando (1989)
c_8	0.058	Linden & Shirtcliffe (1978)
c_9	0.15	Fernando (1989)
c_{10}	6.7×10^{-4}	Fernando (1989)
c_{11}	4.5×10^{-3}	Fernando (1989)
c_{12}	0.15 or $\tau^{1/2}$	Fernando (1989)
c_{13}	12.5	Present paper
c_{14}	4.7×10^{-4}	Present paper

Using the scaling for convective velocity in turbulent thermal convection (Caughey and Palmer 1979; Hunt 1984),

$$\overline{w^2} = c_2(q_0 h)^{2/3}, \quad (3)$$

it was possible to show that the thickness of the lower convecting layer in the laboratory staircases should be given by

$$h = c_3(q_0/N_s^3)^{1/2}. \quad (4)$$

The experimental results of Fernando (1987) were found to support (4) over (1). Further, the fact $h > h_c$ was used to explain why Turner (1968) obtained an anomalously high R_c value.

3. Laboratory observations on single diffusive interfaces

Although many researchers have studied heat and salt transport through single diffusive interfaces experimentally, there are no generally accepted flux laws for such transports. Turner (1965) generated a diffusive interface by heating a two-layer, salt-stratified fluid from below (Fig. 2). The buoyancy fluxes due to heat q_h and salt q_s were measured and the results indicated that for the range of stability ratios $2 < R_\rho < 7$, the flux law can be given by

$$R_F = \frac{q_s}{q_h} = c_4, \quad (5)$$

where $c_4 \approx 0.15$, $R_\rho = g\beta\Delta S/g\alpha\Delta T$, and $g\beta\Delta S$ and $g\alpha\Delta T$ are the buoyancy jumps across the interface due to salinity and temperature, respectively. For $R_\rho < 2$, the flux ratio R_F showed an abrupt increase. Later experiments, however, revealed that (5) is valid only for the experimental parameter range used by Turner (1965). Crapper (1975) demonstrated that the critical value of R_ρ, below which an abrupt increase of R_F occurs, is not a constant but varies from one experiment to another (also see Taylor 1988).

In addition to heating from below, Marmorino and Caldwell (1976) used cooling from above to achieve a quasi-stationary state in the buoyancy transfer process. They showed that R_F is a function of R_ρ and q_0. The

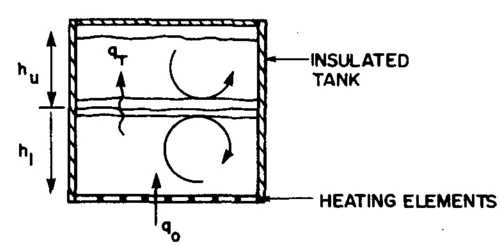

FIG. 2. Schematic diagram of the experiment of Turner (1965). Here, a two-layer salt-stratified fluid is subjected to a bottom heat flux. The symbols are defined in the nomenclature.

buoyancy (heat) flux was found to be given by the empirical relation

$$q_h \approx c_5\left(\frac{k_h^2}{\nu}\right)^{1/3}(g\alpha\Delta T)^{4/3} \times \exp\langle\{4.6\exp[-0.54(R_\rho - 1)]\}\rangle, \quad (6)$$

where $c_5 \approx 8.58 \times 10^{-3}$. Outside the range, $2 < R_\rho < 5$, Eq. (6) was noted to give significantly different heat fluxes than that predicted by the equation of Huppert (1971), which is based on Turner's (1965) data, viz.,

$$q_h = c_6\left(\frac{k_h^2}{\nu}\right)(g\alpha\Delta T)^{4/3}R_\rho^{-2}, \quad (7)$$

where $c_6 \approx 0.323$. Based on data obtained from sugar-salt double-diffusive systems, Shirtcliffe (1973) suggested that (7) may not be applicable to situations other than heat-salt systems. Shirtcliffe also proposed that the general flux law, for high stability ratios, can be written as $R_F = c_1 = \tau^{1/2}$, where $\tau = k_s/k_h$ is the Lewis number, and k_s is the molecular diffusivity of salt. The experiments of Takao and Narusawa (1980), however, did not support this postulate; instead they found $c_4 \approx 0.039\tau^{-1/2}$. Using a Turner (1965)-type experiment, Newell (1984) investigated the heat and salt transports at high R_ρ. The resulting flux law took the form

$$R_F \approx \tau R_\rho, \quad \text{for} \quad R_\rho > 7. \quad (8)$$

Turner et al. (1970) investigated the buoyancy transport through a multicomponent diffusive interface and proposed that the flux law be given by

$$R_F \approx \tau^{1/2}R_\rho. \quad (9)$$

According to Turner (1979, p. 278), Eq. (9) may have applicability to two-component systems.

Fernando (1989) assumed that the salt and heat interfacial-layer thicknesses within a diffusive interface are governed by a balance between the thickening of the interface due to diffusion and the entrainment of the thickened layer by the convective turbulence. It was shown that for $R_\rho < \tau^{-1/2}$, q_h should be given by

$$q_h = c_7\left[\frac{k_h^3(g\alpha\Delta T)^6}{h_u^2}\right]^{1/5}(1 - \tau^{1/2}R_\rho)^{1/5}, \quad (10a)$$

$$\approx c_7\left[\frac{k_h^3(g\alpha\Delta T)^6}{h_u^2}\right]^{1/5}, \quad \text{for } \tau^{1/2}R_\rho \ll 1; \quad (10b)$$

where h_u is the thickness of the upper convecting layer. The corresponding flux law is the same as (9). Fernando (1989) further carried out a Turner (1965)-type experiment to investigate the validity of the assumptions underlying (10a); they were found to be reasonably accurate. Since that paper does not contain a comparison between (10b) and the experimental data, we have presented Fig. 3, which provides support for (10b). Further, we find $c_7 = 7 \times 10^{-2}$.

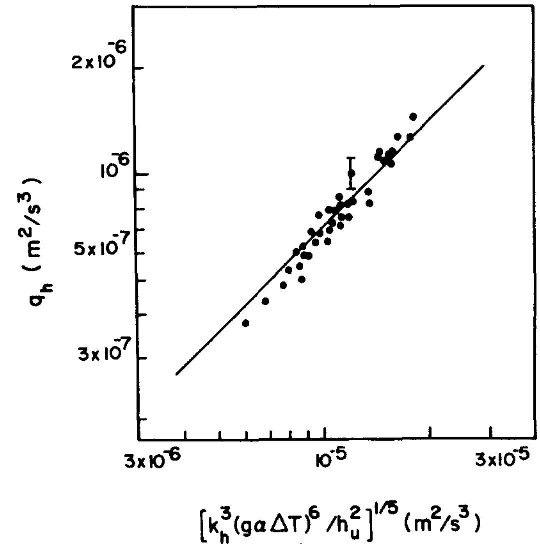

FIG. 3. Variation of the heat flux q_h transported through a purely diffusive interface with $[k_h^3(g\alpha\Delta T)^6/h_u^2]^{1/5}$ during an experiment illustrated in Fig. 2. The solid line represents the slope predicted by (10b). For the results shown here, 4.8×10^{-3} (m^2 s^{-1}) $\leq g\beta\Delta S \leq 9.6 \times 10^{-3}$ (m^2 s^{-1}) and $2 \leq R_\rho \leq 8$.

Note that (9) and (10) differ from the predictions of an earlier model of Linden and Shirtcliffe (1978). Their results are

$$q_h \approx c_8\left(\frac{k_h^2}{\nu}\right)^{1/3}(g\alpha\Delta T)^{4/3}\frac{(1 - \tau^{1/2}R_\rho)^{4/3}}{(1 - \tau^{1/2})^{1/3}}, \quad (11)$$

and

$$R_F = \tau^{1/2}, \quad (12)$$

where $c_8 \approx 0.058$. The models of Linden and Shirtcliffe (1978) and Fernando (1989) employ different physics. In the former, it is assumed that the thermals rising from the instabilities at the interface govern the convection in the upper layer, whereas the latter assumes that, once the convection is established, thermals are no longer important and turbulence dynamics govern the buoyancy transfer process.

Using measurements of rms buoyancy fluctuations and flow visualization studies, Fernando (1989) inferred that the sudden increase of heat flux, as observed by Turner (1965), occurs due to a change in the buoyancy transfer mechanism. Accordingly, at high interfacial stabilities, the turbulent eddies tend to flatten at the density interface, as demonstrated by Hannoun et al. (1988). At low stabilities, the eddies can penetrate into the interfacial layer, thus facilitating physical contact between the eddies of the two layers and increasing the surface area available for the buoyancy transfer. The criterion for the onset of "low stability" transport was shown to be

$$\underbrace{R_\rho(g\alpha\Delta T)^{4/5}\left[\frac{k_s^5h_u^4}{k_h}\right]^{1/10}\frac{[1-R_\rho^{-1}\tau^{1/2}]}{[1-R_\rho\tau^{1/2}]^{1/5}}}_{\mathcal{P}}$$

$$\approx c_9(q_0h_l)^{2/3}, \quad (13)$$

where $c_9 \approx 0.15$ and h_l is the depth of the lower convecting layer. The left-hand side of (13), $\mathcal{P}$, represents the potential energy increase associated with the distortion of the interface due to impinging eddies while the right-hand side represents the kinetic energy of the eddies of the lower layer, according to (3). If $\mathcal{P} > c_9(q_0h_l)^{2/3}$, the buoyancy transport process is considered to be diffusive; if not, it belongs to the "low stability" regime.

The flux transport expressions for the "low stability regime" were found to be

$$q_s \approx c_{10}(g\beta\Delta S)(\overline{w^2})^{1/2}, \quad (14)$$

$$q_h \approx c_{11}(g\alpha\Delta T)(\overline{w^2})^{1/2}, \quad (15)$$

where $c_{10} \approx 6.7 \times 10^{-4}$, $c_{11} \approx 4.5 \times 10^{-3}$. The flux law becomes $R_F \approx 0.15R_\rho$.

4. Extension of the Fernando (1987) model to oceanic staircases

One cannot expect exact similarity between oceanic and laboratory thermohaline staircases for several reasons. For instance, the salinity stratification in the ocean is often nonlinear and the positions of the interfaces are fixed rather than migrating since the divergence of salt and heat flux across each layer is small (Hoare 1968). Moreover, changes in the buoyancy transport process across the interfaces are so slow that it is possible to treat the oceanic layered convection as a quasi-stationary process for relatively short lengths of time. It is possible, however, to extend the model proposed by Fernando (1987), for the thickness of the bottom layer of a thermohaline staircase structure (section 2), and the results of Fernando (1989) for single diffusive interfaces (Section 3) to oceanic situations.[1]

If each individual layer of the staircase results from an initially smooth salinity profile (Fig. 4), then the observed salinity jump ΔS across the layers can be used to obtain "smoothed" initial *local* buoyancy gradient due to salinity[2]

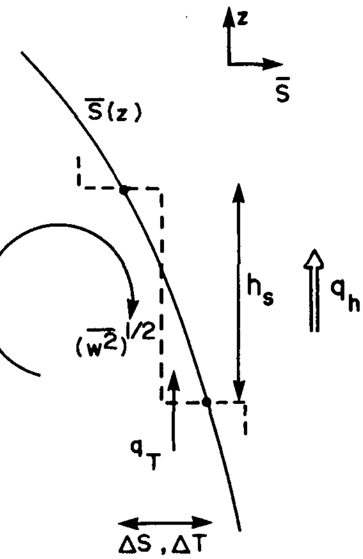

FIG. 4. Schematic diagram that shows a single convecting layer formed due to the breakdown of a nonlinear salinity gradient.

$$N_s^2 = -g\beta(d\bar{S}/dz) \approx g\beta\Delta S/h_s, \quad (16)$$

where h_s is the convective-layer thickness. The net destabilizing buoyancy flux q_T across the diffusive interface, which drives the convection, can be written as[3]

$$q_T = q_h - q_s \approx q_h[1 - c_{12}R_\rho], \quad (17a)$$

or

$$q_T \approx q_h \quad \text{when} \quad c_{12}R_\rho \ll 1, \quad (17b)$$

where $c_{12} \approx 0.15$ for the "low stability" transport, from (14) and (15), and $c_{12} = \tau^{1/2}$ for the "diffusive" transport, from (12). Also the buoyancy jump across the convective layer becomes

$$\Delta b = g\beta\Delta S(1 - R_\rho^{-1}) \approx N_s^2h_s(1 - R_\rho^{-1}). \quad (18)$$

Using (2), (3), (17a), and (18), it follows that

$$h_s = c_{13}\left(\frac{q_h}{N_s^3}\right)^{1/2}\frac{(1 - c_{12}R_\rho)^{1/2}}{(1 - R_\rho^{-1})^{3/4}}, \quad (19a)$$

$$h_s = c_{13}\left(\frac{q_h}{N_s^3}\right)^{1/2}\frac{1}{(1 - R_\rho^{-1})^{3/4}}, \quad c_{12}R_\rho \ll 1, \quad (19b)$$

where $c_{13} = (c_2/c_1)^{3/4}$, and we have assumed that the convecting-layer thicknesses are determined by a balance of kinetic and potential energies of the turbulent eddies and the layers are in a quasi-stationary state. Note that all of the quantities in (19) are local for a

[1] The Prandtl number for both laboratory and oceanic cases is of the order 10. Hence the critical Rayleigh number, at which the flow become fully turbulent, is $Ra_h = g\alpha\Delta Th_s^3/k_h\nu \approx 10^6$, where h_s is the convective-layer thickness (Turner 1979, p. 220). The fact that the laboratory (Ra $\approx 10^9$) and oceanic (Ra $\approx 10^{14}$) convective layers are fully turbulent provides a justification for the extrapolation of laboratory data to oceanic cases.

[2] Here the curvature of the density profile is neglected for motions with length-scales of the order of the layer thicknesses.

[3] Because of the assumption that, for scales of order h_s, the curvature of the buoyancy profiles is negligible, it is possible to approximate the temperature and salinity jumps across the interface as ΔT and ΔS.

particular layer. Further we obtain $c_{13} \approx 12.5$. Figure 5 shows a comparison between the geophysical data collected by previous workers and the theoretical result (19b). Note that for all cases depicted, the buoyancy (heat) fluxes have been evaluated using methods that do not employ laboratory flux laws. Although there is scatter, the data clearly follow the trend predicted by 19(b). The constant c_{13} can be evaluated, using Figure 8, as $c_{13} \approx 14$, which is close to the predicted value.

One of the problems that arises in using (19) is the uncertainty of the buoyancy (heat) flux q_h. Often q_h is not measured but is inferred from various methods, such as the laboratory flux laws for single diffusive interfaces (Kelley 1984). However, as pointed out below, for some cases, the selection of the flux law may require a priori knowledge of q_h through the interfaces.

5. Evaluation of some individual cases

Extensive field measurements have been reported in lakes and oceans in which diffusive interfaces have been found. Possible applications of the present experimental findings to interpret such geophysical observations are discussed in this section. In the process of comparison, equation (13) will be used, with h_u and h_l replaced by h_s and q_0 replaced by q_h, to investigate the nature of interfacial buoyancy transport. The value of q_h will then be checked a posteriori using (10b) if the interface belongs to the diffusive regime, or using (15), viz.

$$q_h \approx c_{14}(g\alpha\Delta T)^{3/2}h_S{}^{1/2}, \qquad (20)$$

$c_{14} \approx 4.7 \times 10^{-4}$, if the interface belongs to the "low stability" regime.

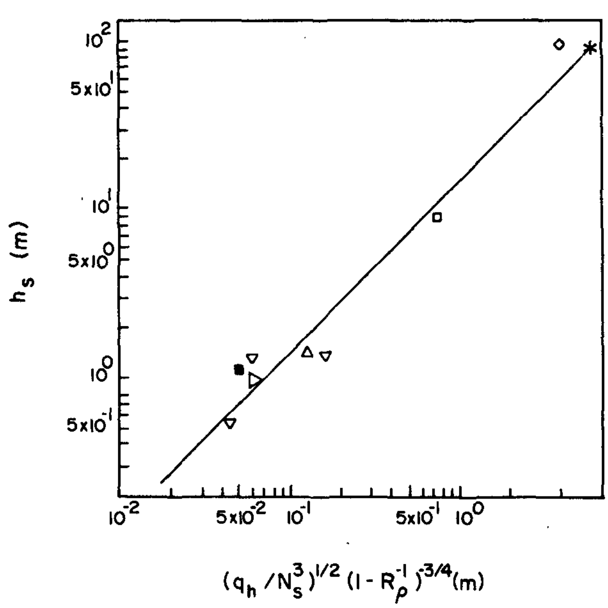

FIG. 5. The variation of the average convecting-layer thickness h_s with local $(q_h/N_s^3)^{1/2}(1 - R_\rho^{-1})^{-3/4}$, as evaluated using oceanic observations. △—Lake Vanda (Hoare 1966, 1968); □—trenches in Red Sea (Swallow and Crease 1965; Federov 1970); ◇—Mediterranean waters in Atlantic Ocean (Siedler 1968); ▽—Lake Kivu (Newman 1976); *—large steps in the Weddell Sea [step size was taken from Muench et al. (1989) and the heat flux was taken from Gordon (1981)]; ■—Lake Vanda (Shirtcliffe and Calhaem 1968); and ▷—Western Canadian Arctic (Melling et al. 1984).

Before proceeding further, it is instructive to restate the limits of applicability of the proposed models. It is assumed that, in the oceanic regions concerned, the turbulence is mainly due to the thermal convection and the Rayleigh number is sufficiently high to maintain the convection in a fully developed state; all other turbulence sources, such as velocity shear, is assumed to play an insignificant role. The predictions for heat and salt transports are valid only under certain conditions and the corresponding restrictions are tabulated in Table 2. Further we have assumed that the model constants of Fernando (1989), based on unsteady laboratory experiments, are valid for quasi-steady oceanic cases. Another important assumption is the stationarity of the interface. According to Fernando (1989), interfacial migrations due to turbulent entrainment (which give rise to additional buoyancy fluxes) are frequent under "low stability" conditions and in calculating c_{10} and c_{11} (and thus c_{14}), care has been taken to exclude the experimental cases with significant interfacial movement. Further, it was noted that the mechanisms of interfacial migration are different for different Richardson number ($\mathrm{Ri}_* = \Delta b h_s/w_*^2$) regimes. When $\mathrm{Ri}_* < 15$, the interfaces migrate rapidly due to direct engulfment of interfacial-layer fluid by the turbulent eddies (as in a nonstratified fluid) whereas at $\mathrm{Ri}_* > 15$, the stratification becomes important and the entrainment takes place due to splashing of interfacial-layer fluid into the mixed layer by the impinging eddies, as in Fernando (1987). For this case the entrainment coefficient is a decreasing function of Ri_*. Further it was found that the effects due to entrainment become vanishingly small when $\mathrm{Ri}_* > 240$.

One of the most widely investigated cases is the layered convection of polar oceans and lakes. For instance, Lake Vanda in Antarctica has been studied by Hoare (1966, 1968), Shirtcliffe and Calhaem (1968), and Federov (1970). The upward heat flux in this region, expressed in terms of a buoyancy flux, has been estimated to be 1.2×10^{-8} m^2 s^{-3} (42 W m^{-2}) and the average thickness h_s of the layers is about 1.5 m (Federov 1970). Other relevant data are $\alpha \approx 1.2 \times 10^{-4}$ K^{-1}, $g\alpha\Delta T \approx 5.8 \times 10^{-4}$ m s^{-2}, and $R_\rho \approx 1.25$ (Hoare 1966). The diffusive buoyancy flux due to heat through the interfaces can be calculated using (10b) as 5×10^{-10} m^2 s^{-3} (1.75 W m^{-2}), which is much lower than the quoted value. The right-hand side of (13) is 10^{-6} m^2 s^{-2} while the left-hand side is 6×10^{-7} m^2 s^{-2}, indicating "low stability" transport through the interface.

TABLE 2. Summary of salient parameterizations.

Predicted parameter(s)	Equation	Restrictions
Buoyancy fluxes in 'diffusive' transport regime (Fernando 1989)	$q_h \approx c_7(k_h^3(g\alpha\Delta T)^6/h_s^2)^{1/5}$ (10b) $\frac{q_s}{q_h} = \tau^{1/2}R_\rho$ (9)	$R_\rho \ll \tau^{-1/2}$ $P > c_9(q_h h_s)^{2/3}$
Buoyancy fluxes in 'low stability' transport regime (Fernando 1989)	$q_h \approx c_{14}(g\alpha\Delta T)^{3/2}h_s^{1/2}$ (20) $\frac{q_s}{q_h} = 0.15R_\rho$ (12)	$P < c_9(q_h h_s)^{2/3}$
Thickness of the convecting layers in stationary thermohaline staircases (present work)	$h_s \approx c_{13}\left(\frac{q_h}{N_s^3}\right)^{1/2}\frac{(1-c_{12}R_\rho)^{1/2}}{(1-R_\rho^{-1})^{3/4}}$ (19a) $h_s \approx c_{13}\left(\frac{q_h}{N_s^3}\right)^{1/2}\frac{1}{(1-R_\rho^{-1})^{3/4}}$ (19b)	$R_\rho < c_{12}^{-1}$ $R_\rho \ll c_{12}^{-1}$

Hence, the buoyancy flux due to heat can be evaluated, using (20), as 8.5×10^{-9} m^2 s^{-3} (30 W m^{-2}), which is in fair agreement with the quoted value.

Another example is the deep-water, near-bottom geothermal inversion (Lubimova et al. 1965; Federov 1970). The typical quoted values are $h_s \approx 10$ m, $q_h \approx 1.9 \times 10^{-11}$ m^2 s^{-3} (0.09 W m^{-2}), $\alpha = 0.9 \times 10^{-4}$ K^{-1}, $g\alpha\Delta T \approx 2.6 \times 10^{-6}$ m s^{-2}, and $R_\rho \approx 1.20$. The diffusive flux evaluated using (10b) is $q_h \approx 3.6 \times 10^{-13}$ m^2 s^{-3} (1.7×10^{-3} W m^{-2}), which is two orders of magnitude smaller than the observed value. However, the left and right hand sides of (13) become 1.5×10^{-8} m^2 s^{-2} and 5.0×10^{-8} m^2 s^{-2}, respectively, and hence, the "low stability" transport expression (20) can be used to calculate q_h. The calculated value 6.5×10^{-12} m^2 s^{-3} (0.03 W m^{-2}) is not far from the quoted value.

Newman (1976) has performed a detailed investigation on heat and salt transport in Lake Kivu. The quoted parameters (at station D1) are $h_s \approx 1.4$ m, $q_h \approx 4.0 \times 10^{-10}$ m^2 s^{-3} (0.71 W m^{-2}), $q_s \approx 1.2 \times 10^{-10}$ m^2 s^{-3}, $g\alpha\Delta T \approx 7.1 \times 10^{-5}$ m s^{-2}, and $R_\rho \approx 2$ whereas α can be evaluated as 2.4×10^{-4} K^{-1}. In this case, the diffusive buoyancy (heat) flux calculated using (10b), 4.0×10^{-11} m^2 s^{-3} (0.071 W m^{-2}), is much smaller than the quoted value. The left and right hand sides of (13) are 1.5×10^{-7} and 1.1×10^{-7} m^2 s^{-2}, respectively, suggesting the existence of marginal conditions for "low stability" buoyancy transport, and the possibility of using (14) and (15) to evaluate the buoyancy fluxes. The calculated values of buoyancy fluxes due to heat and salt, 3.4×10^{-10} m^2 s^{-3} (0.61 W m^{-2}) and 1.0×10^{-10} m^2 s^{-3}, respectively, agree well with Newman's (1976) estimations.

The role of diffusive interfaces in maintaining the marginal ice edge of the Arctic Ocean has been discussed by Stegan et al. (1985). Their measurements indicated that, under the ice edge, the ocean is stratified into two layers in such a way that overlying cold relatively fresh water is separated from the bottom warm salty water by a diffusive interface. Since the gradient Richardson number in the interfacial region is about 4, Stegan et al. (1985) concluded that there cannot be substantial turbulent mixing due to internal wave (shear) instabilities (but see Kantha 1986). The total estimated buoyancy flux due to heat required to maintain the ice edge, 2.6×10^{-8} m^2 s^{-3} (180 W m^{-2}), calculated on the basis of the ice melting rate, therefore, has to come from other sources, such as lateral advection and double diffusion. The typical values obtained, from Figure 2 of Stegan et al. (1985), are $g\alpha\Delta T \approx 5.9 \times 10^{-4}$ m s^{-2}, $R_\rho \approx 4$, $h_u \approx 35$ m, $h_l \approx 20$ m, and $\alpha \approx 6 \times 10^{-5}$ K^{-1}. The left and right hand sides of (13) become 6.7×10^{-6} m^2 s^{-2} and 10^{-5} m^2 s^{-2}, respectively, suggesting "low stability" transport through the interface. Equations (14) and (15) can be used to evaluate heat and salt fluxes through the interface as $q_h \approx 3 \times 10^{-8}$ m^2 s^{-3} (207 W m^{-2}) and $q_s \approx 1.9 \times 10^{-8}$ (1.9×10^{-10} kg m^{-2} s^{-1}). It appears that double diffusive transport through the interface can alone account for the heat flux responsible for maintaining the ice edge.

It is interesting to compare the heat-flux predictions based on Fernando (1989) with those based on previous measurements and theories. Such a comparison is given in Table 3, for the cases discussed above. It is possible to conclude that, in general, the flux-laws based on Fernando (1989) give better predictions for oceanic heat fluxes. Also it is evident that in some cases the predictions made on the basis of the present work differ from the quoted value by a factor as much as three. This discrepancy can be attributed to the low Richardson numbers Ri_* at the interface which permit interfacial migrations due to turbulent entrainment: Ri_* values for the cases given in Table 3 and the corresponding ratio between the quoted and predicted fluxes are given in Table 4. The increase in the variance be-

TABLE 3.

Location	q_h m^2 s^{-3} (W m^{-2}) quoted	q_h calculated m^2 s^{-3} (W m^{-2}) Huppert [Eq. (7)]	Marmorino and Caldwell [Eq. (6)]	Linden and Shirtcliffe [Eq. (11)]	Fernando (see text)
Lake Vanda	1.2×10^{-8} (42)	2.1×10^{-8} (73)	4.8×10^{-8} (168)	5.1×10^{-9} (18)	8.5×10^{-9} (30)
Deep water near bottom geothermal inversions	1.9×10^{-11} (0.09)	1.7×10^{-11} (0.08)	4.0×10^{-11} (0.19)	3.8×10^{-12} (0.018)	6.5×10^{-12} (0.03)
Lake Kivu	4×10^{-10} (0.71)	5×10^{-10} (0.88)	7.7×10^{-10} (1.37)	2.7×10^{-10} (0.47)	3.4×10^{-10} (0.61)
Marginal ice zone	2.6×10^{-8} (180)	2.1×10^{-9} (15)	2.3×10^{-9} (16)	3.2×10^{-9} (22)	3×10^{-8} (207)

tween the predicted and quoted values with decreasing Ri_* corraborates the notion that additional fluxes resulting from the turbulent entrainment may be responsible for the observed disparities at low Ri_*.

In all of the above comparisons with Fernando (1989), a "quoted" value of q_h was used a priori to evaluate the "regime" of interfacial buoyancy transfer. However, as pointed out by a referee, the "quoted" values are subject to a variety of uncertainties and hence must be critically judged. In the previous sections two equations, (10) and (20), to estimate the heat flux have been introduced, but to determine which equation is valid, using (13), a knowledge of q_h is required. Nevertheless, under certain circumstances, it is possible to use (10b), (20), and (13) to estimate q_h through the interface.

It is possible to recast these three equations as

$$\mathrm{Nu} \approx c_7 \, \mathrm{Ra}_h^{1/5} \, \mathrm{Pr}^{1/5}, \quad (\text{"diffusive" regime}), \quad (21)$$

$$\mathrm{Nu} \approx c_{14} \, \mathrm{Ra}_h^{1/2} \, \mathrm{Pr}^{1/2}, \quad (\text{"low stability" regime}), \quad (22)$$

and

$$\underbrace{\frac{R_\rho \, \mathrm{Ra}_h^{2/15} \, \mathrm{Pr}^{2/5} \tau^{1/2} (1 - R_\rho^{-1} \tau^{1/2})}{(1 - R_\rho \tau^{1/2})^{1/5}}}_{Y} \approx c_9 \, \mathrm{Nu}^{2/3} \quad (\text{transition}), \quad (23)$$

TABLE 4. Dependency of heat-flux estimation errors on Ri_*.

Location	$(q_h)_{\text{quoted}}/(q_h)_{\text{calculated}}$	Ri_*
Lake Vanda	1.4	32
Deep water near bottom geothermal inversions	3.0	16
Lake Kivu	1.15	146
Marginal ice zone	0.85	547

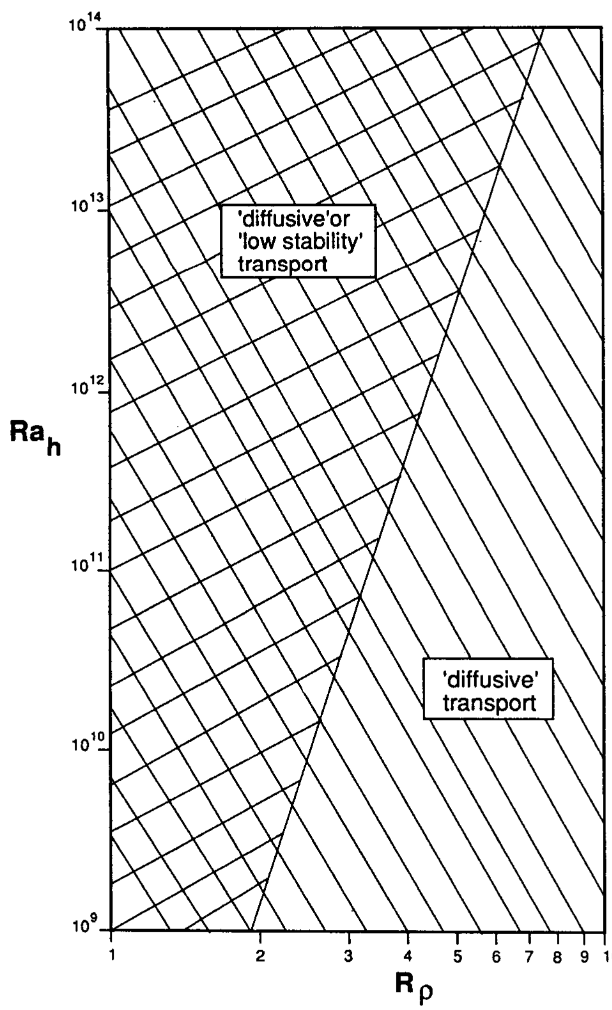

FIG. 6. A Ra_h–R_ρ diagram depicting the regimes of buoyancy transport through diffusive interfaces. The diagram shows that only the diffusive transport regime can be identified in a straightforward manner.

where $\mathrm{Nu} = q_h/(k_h g\alpha\Delta T/h_s)$ is the Nusselt number, $\mathrm{Pr} = \nu/k_h$ is the Prandtl number, and $\mathrm{Ra}_h = g\alpha\Delta T h_s^3/k_h\nu$ is the Rayleigh number. The following cases must be considered:

i) From (21) and (23), it appears that the condition for "diffusive" transport is satisfied when

$$Y > c_9(c_7\,\mathrm{Ra}_h^{1/5}\,\mathrm{Pr}^{1/5})^{2/3},$$

or

$$R_\rho \frac{(1-\tau^{1/2}R_\rho^{-1})}{(1-\tau^{1/2}R_\rho)} > c_9 c_7^{2/3}\tau^{-1/2}. \quad (24)$$

For $\tau^{1/2} \approx 0.1$, condition (24) translates to $R_\rho > 0.35$, which is satisfied by all the statically stable diffusive interfaces with $R_\rho > 1$; and

ii) From (22) and (23), the condition for 'low stability' transport can be written as

$$Y < c_9(c_{14}\,\mathrm{Ra}_h\,\mathrm{Pr}^{1/2})^{2/3},$$

or

$$R_\rho \frac{(1-\tau^{1/2}R_\rho^{-1})}{(1-\tau^{1/2}R_\rho)} \leqslant [c_9 c_{14}^{2/3}\,\mathrm{Pr}^{1/5}\tau^{-1/2}]\,\mathrm{Ra}^{1/5}. \quad (25)$$

The criteria (24) and (25) are depicted in Fig. 6, for the case $\mathrm{Pr} \approx 10$ and $\tau \approx 0.1$. Note that, in practical situations with $R_\rho > 1$, it is possible to identify the "diffusive" regime, if (25) is not satisfied. If (25) is satisfied, then the interfacial transport can be either "diffusive" or "low stability" and an idea of q_h is necessary to determine the regime. Assuming quasi-stationarity of the staircase structure, it is possible to use (19) to estimate the buoyancy (heat) flux and then to check the calculated value using (13), and (10) or (20). It should be borne in mind that in such cases (19b) and (20) give rise to the condition $R_\rho \approx 1.2$ for the steps whereas (19b) and (10b) yield $h_s = 14(k_h^2/g\alpha\Delta T)^{1/3}(R_\rho - 1)^{-5/3}$.

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APPENDIX

Nomenclature

c_p	specific heat
$c_1, c_2 \ldots$	constants (see Table 1)
g	gravitational acceleration
$g\beta\Delta S$	buoyancy jump across the interface due to salinity
$g\alpha\Delta T$	buoyancy jump across the interface due to temperature
h	thickness of the bottom mixed layer
h_c	thickness of the bottom layer at the breakdown of the thermal boundary layer
h_s	thickness of the convecting layer in a thermohaline staircase
h_u	thickness of the upper convecting layer
h_l	thickness of the lower convecting layer
k_s, k_h	diffusivities of salt and heat
N_s	buoyancy frequency based on salinity stratification
Nu	$[=q_h/(k_h g\alpha\Delta T/h_s)]$ Nusselt number
$\mathcal{P}$	$[=R_\rho(g\alpha\Delta T)^{4/5}(k_s^5 h_u^4/k_h)^{1/10}(1 - R_\rho^{-1}\tau^{1/2})(1 - R_\rho\tau^{1/2})^{-1/5}]$ increase of potential energy when turbulent eddies penetrate through the salinity interfacial layer
Pr	Prandtl number
q_h	buoyancy flux (due to heat) through the interface
q_s	buoyancy flux (due to salt) through the interface
q_T	$(=q_h - q_s)$ net destabilizing buoyancy flux
q_0	$(g\alpha Q/\rho_0 c_p)$ buoyancy flux at the heating surface
Q	heat flux at the heating surface
Ra_δ	$(=g\alpha\Delta T\delta^3/k_h\nu)$ thermal Rayleigh number
Ra_h	$(=g\alpha\Delta T h_s^3/k_h\nu)$ Rayleigh number based on the convecting layer thickness
R_c	Critical thermal Rayleigh number
Ri_*	$(=\Delta b h_s/w_*^2)$ interfacial Richardson number
R_F	$(=q_s/q_h)$ flux ratio
R_ρ	$(=g\beta\Delta S/g\alpha\Delta T)$ stability ratio
$\bar{S}$	mean salinity
$\bar{T}$	mean temperature
$\overline{w^2}$	rms velocity of the convective layers
Y	$[=R_\rho\,\mathrm{Ra}_h^{2/15}\,\mathrm{Pr}^{2/15}\tau^{1/2}(1 - R_\rho^{-1}\tau^{1/2})(1 - R_\rho\tau^{1/2})^{-1/5}]$
z	vertical coordinate
α	coefficient of volume expansion
β	salinity contraction coefficient
δ	thickness of the thermal boundary layer
ΔS	salinity jump across the interface
ΔT	temperature jump across the interface
ν	kinematic viscosity
ρ_0	reference density
Δb	$(=g\beta\Delta S - g\alpha\Delta T)$ buoyancy jump across the interface
τ	$(=k_s/k_h)$ Lewis number

REFERENCES

Caughey, S. J., and S. G. Palmer, 1979: Some aspects of turbulence structure through the depth of the convective boundary layer. *Quart. J. Roy. Meteor. Soc.*, **105**, 811–827.

Crapper, P. F., 1975: Measurements across a diffusive interface. *Deep-Sea Res.*, **22**, 537–545.

Federov, K. N., 1970: On the step-like structure of temperature inversions in the ocean. *Atmos. Oceanic Phys.*, **6**(11), 1178–1188.

Fernando, H. J. S., 1987: The formation of layered structure when a stable salinity gradient is heated from below. *J. Fluid Mech.*, **182**, 425–442.

——, 1989: Buoyancy transfer across a diffusive interface. *J. Fluid Mech.*, in press.

——, and R. R. Long, 1985: On the nature of the entrainment interface of a two-layer fluid subjected to zero-mean-shear turbulence. *J. Fluid Mech.*, **151**, 21–53.

Gordon, A. L., 1981: Seasonality of southern ocean sea-ice. *J. Geophys. Res.*, **85**(C5), 4193–4197.

Hannoun, I. A., H. J. S. Fernando and E. J. List, 1988: Turbulence structure near a sharp density interface. *J. Fluid Mech.*, **189**, 189–210.

Hoare, R. A., 1966: Problem of heat transfer in Lake Vanda, a density stratified Antarctic lake. *Nature*, **210**, 787–789.

——, 1968: Thermohaline convection in Lake Vanda, Antarctica. *J. Geophys. Res.*, **73**, 607–612.

Hunt, J. C. R., 1984: Turbulence structure in thermal convection and shear-free boundary layers. *J. Fluid Mech.*, **138**, 161–184.

Huppert, H. E., 1971: On the stability of a series of double-diffusive layers. Deep-Sea Res., **18**, 1005–1021.

——, and P. F. Linden, 1979: On heating a stable salinity gradient from below. *J. Fluid Mech.*, **95**, 431–464.

Kantha, L. H., 1986: Comments on "A heat balance for the Bering Sea ice edge." *J. Phys. Oceanogr.*, **16**(12), 2205–2207.

Kelley, D., 1984: Effective diffusivities within oceanic thermohaline staircases. *J. Geophys. Res.*, **89**(C6), 10 484–10 488.

Linden, P. F., and T. G. L. Shirtcliffe, 1978: The diffusive interface in double diffusive convection. *J. Fluid Mech.*, **87**, 417–432.

Lubimova, E. A., R. P. Von Herzen and G. B. Udintzev, 1965: On heat transfer through the ocean floor. *Terrestrial Heat Flow, Geophys. Monogr.* No. 8, 78.

Mack, S., 1985: Two-dimensional measurement of oceanic microstructure. The role of double diffusion. *J. Phys. Oceanogr.*, **15**, 1581–1604.

Marmorino, G. O., and D. R. Caldwell, 1976: Heat and salt transport through a diffusive thermohaline interface. *Deep-Sea Res.*, **23**, 59–67.

Melling, H., R. A. Lake, D. R. Topham and D. B. Fissel, 1984: Oceanic thermal structure in the western Canadian Arctic. *Continental Shelf Research*, **3**(3), 223–258.

Muench, R. D., H. J. S. Fernando and G. R. Stegan, 1989: Temperature and salinity staircases in Northwestern Weddell Sea: Remnant convective features?, *J. Phys. Oceanogr.*, in press.

Newell, T. A., 1984: Characteristics of double-diffusive interface at high density-stability ratios. *J. Fluid Mech.*, **149**, 385–401.

Newman, F. C., 1976: Temperature steps in Lake Kivu: A bottom heated saline lake. *J. Phys. Oceanogr.*, **6**, 157–163.

Shirtcliffe, T. G. L., 1973: Transport and profile measurements of the diffusive interface in double-diffusive convection with similar diffusivities. *J. Fluid Mech.*, **57**, 27–43.

——, and I. M. Calhaem, 1968: Measurements of temperature and electrical conductivity in Lake Vanda, Victoria Land, Antarctica. *N. Z. J. Geol. Geophys.* **11**, 976–981.

Siedler, G., 1968: Schichtungs- und Bewegungsverhältnisse am Südausgang des Roten Meerers. "Meteor" Forschungs-ergebnisse, Reihe A, Heft 4.

Stegan, G. R., P. J. Hendricks and R. D. Muench, 1985: Vertical mixing on the Bering Sea Shelf. *The Ocean Surface; Wave Breaking, Turbulent Mixing and Radio Probing.* Y. Toba and H. Mitsuyasu, Eds., 551–557, Reidel.

Swallow, J. C., and J. Crease, 1965: Hot salty water at the bottom of the Red Sea. *Nature*, **205**, 165–166.

Takao, S., and I. Narusawa, 1980: An experimental study of heat and mass transfer across a diffusive interface. *Int. J. Heat Mass Trans.*, **23**, 1283–1285.

Taylor, J., 1988: The flux across a diffusive interface at low values of the density ratio. *Deep-Sea Res.*, **35**, 555–567.

Turner, J. S., 1965: The coupled turbulent transports of salt and heat across a sharp density interface. *Int. J. Heat Mass Trans.*, **8**, 759–767.

——, 1968: The behavior of a salinity gradient heated from below. *J. Fluid Mech.*, **33**, 183–200.

——, 1979: *Buoyancy Effects in Fluids.* Cambridge University Press,

——, and H. Stommel, 1964: A new case of convection in the presence of combined vertical salinity and temperature gradients. *Proc. Natl. Acad. Sci.*, **52**, 49–53.

——, T. G. L. Shirtcliffe and P. G. Brewer, 1970: Elemental variations of transport coefficients across density interfaces in multiple-diffusive systems. *Nature*, **228**, 1083–1084.