Steady Coastal Circulation Due to Oceanic Alongshore Pressure Gradients

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ABSTRACT

A depth-averaged barotropic model is used to investigate the steady response of the coastal ocean to alongshore pressure gradients imposed by the deep ocean. Solutions indicate that the dimensionless continental margin width δ is the appropriate parameter determining the effectiveness of the transmission of the alongshore pressure field from ocean to coast. For linear depth profiles having depth $h = h_0 + h_1x$ to the abyssal plane at $x = l$, $\delta = (fk/r)^{1/2}(h_1 l^2/2)^{1/2}$ where f is the Coriolis parameter, r is the linear friction coefficient for alongshore flow and k is the wavenumber of the alongshore pressure perturbation. For parabolic depth profiles having $h = h_0 + h_2x^2$ to $x = l$, $\delta = (3fk/2r)^{1/3}(h_2 l^3/3)^{1/3}$. On narrow continental margins with $\delta \ll 1$, oceanic pressure fields are almost completely transmitted to the coast causing substantial near-coastal currents, while on wide continental margins with $\delta \gg 1$ the near coastal ocean is unaffected by the oceanic pressure field. In general, the oceanic pressure field drives a strong circulation at the outer slope, and this circulation weakens toward the coast. This contrasts with the coastal circulation resulting from an alongshore wind stress, which is strongest at the coast and weakens with distance offshore.

1. Introduction

Wind stress and vertical density structure have long been known to be important in driving nearshore and shelf currents. The idea that the open ocean may also play a significant role in driving "steady" continental shelf circulation appears to have become more accepted in recent years following the suggestion of Csanady (1976). In a barotropic depth-averaged model, Csanady (1978) found an analytical solution for coastal circulation resulting from a constant alongshore pressure gradient imposed by the open ocean at the outer edge of a continental shelf. His solution indicates that the pressure field is transmitted across the shelf, and this pressure field results in an associated velocity field.

More recently, Wang (1982) argued that the steep continental slope should also be included when investigating the effects of a constant open ocean pressure gradient on shelf circulation, and he concluded from a numerical study that a steep slope "insulates" the shelf from the open ocean, with steeper slopes providing more insulation. Taken to the limit of an infinitely steep slope, extensions of Wang's arguments suggest the open ocean and shelf become entirely decoupled, a result which is physically implausible. Wang concluded that open ocean pressure effects are ineffective at driving the circulation observed on the wide Mid-Atlantic Bight. Hsueh and Peng (1978), Chase (1979), Hopkins (1982) and Hopkins and Dieterle (1983) have further discussed oceanic pressure gradient forcing as a contributor to shelf circulation on the Mid-Atlantic Bight while Csanady and Shaw (1983) have argued that it is the insulating effect of the steep continental slope which is responsible for weak near bottom currents on the slope of the Bight.

To determine the relative importance of wind stress and open ocean influences on the narrow shelf of the west coast of the United States, Hickey and Pola (1983) used the long term mean steric height data of Reid and Mantyla (1976), in conjunction with relative monthly-mean adjusted sea level and wind stress. They concluded from this study that deep ocean currents contribute significantly toward the mean coastal elevation gradient south of San Francisco in the summer and fall, and that this gradient is effective in driving a coastal and shelf flow.

The role of the alongshore oceanic pressure gradient in driving coastal circulation therefore appears to be important on narrow continental margins but less important on wide margins. Even for the case of barotropic flow over idealised topography, the role of the width and shape of the continental margin in determining the transmission of the pressure field and the general circulation remains to be quantitatively explained. In this paper we answer some of these basic questions for the barotropic case.

The paper is organised as follows. Section 2 describes the formulation of the problem, while section 3 is concerned with the coastal circulation induced by an alongshore pressure gradient at the edge of a continental margin with linear depth profile. The case of a parabolic depth profile for the continental margin is dealt with in section 4 while section 5 discusses the results.

2. Formulation of the problem

In this section, we formulate the equations governing steady coastal circulation induced by an alongshore pressure gradient existing in the open ocean beyond the continental margin.

For a straight coastline, define axes such that x and y are distances offshore and alongshore, respectively, and assume that the depth h is a function of x only (Fig. 1). Bottom friction is assumed to be proportional to the depth-averaged velocity with friction coefficient r. This approximation is adequate for mean flow over much of the shelf (Csanady, 1976). As with the wind-driven circulation problem (Csanady, 1978; Middleton and Thomson, 1985), time dependent effects are unimportant in the alongshore momentum equation provided the frequency of oscillation $\omega \ll r/H$ where H is a scale depth for the coastal ocean. In addition,

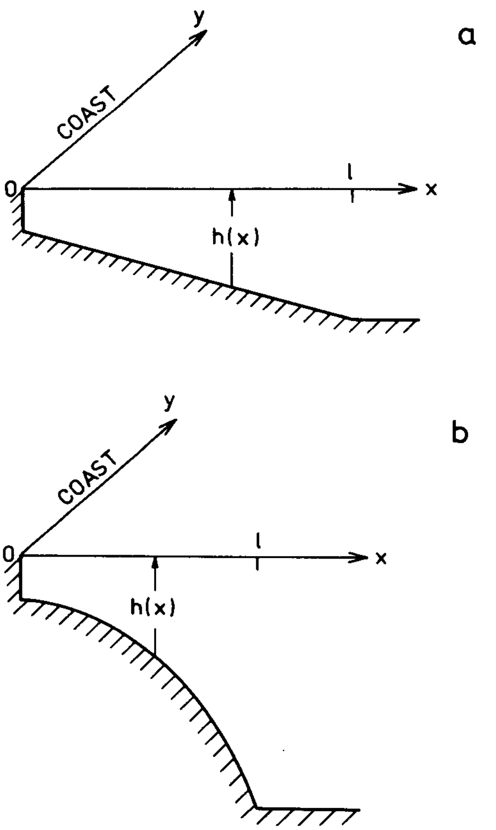

FIG. 1. Profiles of depth over the continental margin with (a) linearly increasing depth such that $h = h_0 + h_1 x$, and (b) parabolically increasing depth such that $h = h_0 + h_2 x^2$. The edge of the continental margin is at $x = l$, and the maximum depth $h(l)$. The continental margin includes the regions commonly known as the continental shelf and the continental slope.

time dependent and bottom friction terms may be neglected in the across-shelf momentum equation provided the length scale of the alongshore structure greatly exceeds the shelf width. With these assumptions the depth-averaged, linearized equations of motion for nondivergent flow on the continental margin may be written in terms of the across-shelf and alongshore velocities u and v, and the surface elevation ζ as

$$fv = g\zeta_x \quad (1)$$

$$fu = -g\zeta_y - rvh^{-1} \quad (2)$$

$$(uh)_x + (vh)_y = 0 \quad (3)$$

where the subscripts x and y denote partial differentiation and g is the acceleration due to gravity.

If variations in the Coriolis parameter f with alongshore distance y are small over distances comparable to the length scale of the alongshore structure, then f may be considered as a constant. Elimination of u and v from (1)–(3) results in a partial differential equation for the surface elevation ζ

$$r(fh_x)^{-1}\zeta_{xx} + \zeta_y = 0, \quad (4)$$

an equation identical to that found by Csanady (1978) for steady wind-driven circulation. One boundary condition is obtained by assuming that the depth $h \rightarrow h_0$ at the coast where $u = 0$, and from (1) and (2)

$$\zeta_x = -fh_0 r^{-1}\zeta_y, \quad x = 0. \quad (5)$$

If $h_0 = 0$ then (5) simplifies to $\zeta_x = 0$ implying zero alongshore velocity at the coast, while nonzero values of h_0 allow nonzero nearshore currents. The alongshore pressure gradient at the outer edge of the continental margin ($x = l$) enters the problem through the second boundary condition

$$\zeta(y) = F(y), \quad x = l. \quad (6)$$

We seek solutions to (4), for which "initial" conditions at $y = 0$ do not need specification, using the method of separation of variables. Assuming

$$\zeta = \mathrm{Re}\{Z(x)\phi(y)\} \quad (7)$$

the equation for the surface elevation structure becomes

$$\frac{r}{fh_x}\frac{Z_{xx}}{Z} = \frac{-\phi_y}{\phi} = \text{constant}. \quad (8)$$

For imaginary values of the separation constant $(= -ik$, k real), an alongshore structure, oscillatory in y such that

$$\phi = G\exp(iky), \quad (9)$$

allows matching to a sea level perturbation at the slope/ocean boundary of form

$$F(y) = G\cos(ky), \quad x = l \quad (10)$$

provided the across-shelf structure function obeys $Z(l) = 1$. For practical applications the depth $h(l)$ is sufficiently large that the frictional term in (2) becomes negligible and the flow is therefore essentially geostrophic over the outer slope. Assuming the flow is also geostrophic in the ocean $x > l$, the matching condition equating sea level at $x = l$ implies continuity in the across-shelf current (and across-shelf volume flux) at $x = l$ through the geostrophic relation $u = -gf^{-1}\partial\zeta/\partial y$. These are mandatory matching conditions required by the continuity equation, and both conditions are technically met by specifying $Z(l) = 1$.

Solutions for an arbitrary alongshore structure may be found by considering (9) and (10) as Fourier coefficients with amplitude G_k, then summing the contributions from each alongshore wavenumber k. Substantial insight into the role that the oceanic sea level plays in generating coastal circulation may therefore be obtained by considering an arbitrary coefficient.

Under separation of variables the across-shelf structure obeys

$$Z_{xx} + ikfh_x r^{-1} Z = 0 \qquad (11)$$

with boundary conditions

$$Z_x + ikfh_0 r^{-1} Z = 0, \quad x = 0$$

$$Z = 1, \quad x = l. \qquad (12)$$

The boundary conditions (12) are necessary and sufficient to solve (11). Any other boundary conditions (such as matching alongshore velocity components) would either overdetermine the problem if used in addition to (12) or, if used in place of one of (12), would render the solution physically unrealistic. For particular depth profiles, solutions are relatively easy to find, and particular cases are considered in the following sections.

3. Circulation on a continental margin with linear depth profile

For a continental margin having a constant gradient but with nonzero depth at the coastal wall as indicated in Fig. 1a,

$$h = h_0 + h_1 x. \qquad (13)$$

Some simplification of the equations is achieved by transforming to dimensionless variables X and Y, where

$$X = xL^{-1}, \quad Y = ky. \qquad (14)$$

Writing $\eta_0 = h_0(h_1 L)^{-1}$ the depth may be written as

$$h = h_1 L(\eta_0 + X). \qquad (15)$$

The structure equation resulting from imposed sea level $G \cos Y$ at $X = \delta$ $(= lL^{-1})$ may then be expressed as

$$\phi = G \exp(iY) \qquad (16)$$

$$\frac{d^2 Z}{dX^2} + \alpha^2 Z = 0 \qquad (17)$$

where the across-shelf structure obeys the boundary conditions

$$\frac{dZ}{dX} + \alpha^2 \eta_0 Z = 0, \quad X = 0 \qquad (18)$$

$$Z = 1, \quad X = \delta. \qquad (19)$$

The definition of L and the coefficient α depend upon the sign of f as follows

$$f > 0, \quad L^2 = \left(\frac{2r}{fkh_1}\right), \quad \alpha = (1 + i) \qquad (20)$$

$$f < 0, \quad L^2 = \left(\frac{-2r}{fkh_1}\right), \quad \alpha = (1 - i). \qquad (21)$$

To interpret the role played by δ, we write (for $f > 0$)

$$\delta = lL^{-1} = l\left(\frac{fkh_1}{2r}\right)^{1/2} = \left(\frac{fk}{r}\right)^{1/2} A^{1/2}$$

where $A = \frac{1}{2} l^2 h_1$ is the cross sectional area of flow from the coast $x = 0$ to $x = l$ below the depth h_0. Thus δ is linearly proportional to the scale length $A^{1/2}$. For constant continental margin width l, shelves with a larger gradient have larger $A^{1/2}$ and so larger δ, while for constant values of $h(l)$ shelves with a smaller gradient have larger $A^{1/2}$ and so larger δ. An alternative interpretation, pointed out by a reviewer, is found by writing

$$\delta^2 = \frac{1}{2}(h_1 l)(r/f)^{-1}(lk)$$

so that δ^2 is a ratio of mean depth $(h_1 l)$ to bottom frictional length (r/f) modified by the ratio of across-shelf to alongshore scales (lk).

The solution to (17) subject to (18) and (19) is

$$Z(X) = \frac{\cos(\alpha X + \theta)}{\cos(\alpha\delta + \theta)} \qquad (22)$$

where θ is a complex angle satisfying

$$\tan\theta = \frac{\alpha^2 \eta_0}{\alpha} = \alpha\eta_0. \qquad (23)$$

The circulation over the continental margin due to imposed alongshore sea level of form

$$\zeta = G \cos Y \quad \text{at} \quad X = \delta \qquad (24)$$

thus has sea level, alongshore velocity and transport streamfunction given by

$$\zeta = G\,\mathrm{Re}\{Z(X)e^{iY}\} \qquad (25)$$

$$v = \frac{gG}{fL}\mathrm{Re}\{V(X)e^{iY}\} \qquad (26)$$

$$\psi = \frac{gGh_1 L}{f}\mathrm{Re}\{P(X)e^{iY}\} \qquad (27)$$

where we have defined ψ by $v = h^{-1}\partial\psi/\partial x$, $u = -h^{-1}\partial\psi/\partial y$. The function $Z(X)$ is given by (22) while $V(X)$ and $P(X)$ are given by

$$V(X)=\frac{-\alpha\sin(\alpha X+\theta)}{\cos(\alpha\delta+\theta)} \quad (28)$$

$$P(X)=\frac{(\eta_0+X)\cos(\alpha X+\theta)-\alpha^{-1}\sin(\alpha X+\theta)}{\cos(\alpha\delta+\theta)}. \quad (29)$$

For selected nonzero values of η_0 and δ, and hence θ, the solutions (25)–(27) may be easily calculated.

a. Coastal depth $h_0 = 0$

Here $\eta_0 = 0$ and $\theta = 0$, and we first investigate the role played by the continental margin width δ.

Some general conclusions may be drawn from (25)–(27) near the coast where $x \to 0$, and the leading order terms give

$$\zeta \to \frac{G\cos(Y-\epsilon)}{|\cos(\alpha\delta)|}, \quad v \to \frac{qG}{|f|L}\frac{2X\sin(Y-\epsilon)}{|\cos(\alpha\delta)|}$$

$$\psi \to \frac{gGh_1L}{|f|}\frac{\frac{1}{3}X^3\sin(Y-\epsilon)}{|\cos(\alpha\delta)|}$$

where

$$|\cos\alpha\delta| = [\cos^2\delta\cosh^2\delta+\sin^2\delta\sinh^2\delta]^{1/2}$$

$$\tan\epsilon=\frac{\mathrm{Im}[\cos(\alpha\delta)]}{\mathrm{Re}[\cos(\alpha\delta)]}.$$

The magnitude of the sea level anomaly field at the coast is reduced by a factor of $|\cos(\alpha\delta)|^{-1}$ compared with that at $X = \delta$. Relatively narrow continental margins with $\delta \ll 1$ transmit imposed alongshore oceanic sea level fluctuations undiminished, while for wide continental margins with $\delta \gg 1$ the sea level perturbation is much reduced. The phase ϵ indicates that the maximum coastal elevation occurs at dimensional distance $k\epsilon$ alongshore compared with that at the shelf break. The alongshore velocity field is zero at the coast but increases linearly with distance offshore, and is maximum when the sea level gradient is maximum. The transport streamfunction increases with the cube of distance offshore.

The general features of the circulation are shown in Fig. 2 which shows contours of sea level and streamfunction for $\delta = 1$. The sea level perturbation is transmitted to the coast with some reduction, and the maximum sea level perturbation is offset in the alongshore direction. The streamfunction contours show the flow to be stagnant at the coast, but more energetic toward the outer slope. This is a characteristic feature of the flows considered here and occurs as a result of the larger effect of bottom friction on the depth averaged flow in shallower waters. By contrast, steady wind-driven currents are much stronger near the coast but become weaker in deeper water because a greater mass of water

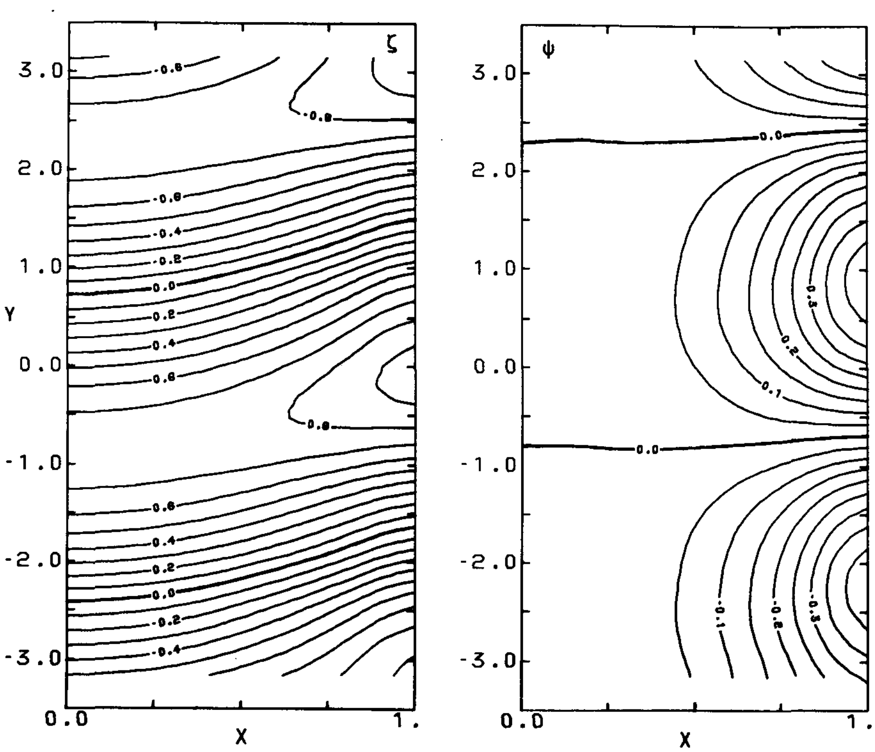

FIG. 2. Contours of sea level perturbation ζ and transport streamfunction ψ calculated for a dimensionless continental margin width of $\delta = 1$, and for zero coastal depth, for the linear depth profile.

is acted upon by the wind in deeper water (Csanady, 1978).

b. Coastal depth $h_0 \neq 0$

In the case $\eta_0 > 0$ and θ is nonzero. Given a value of δ, a measure of the transmission of the sea level field across the shelf is the value of Z at the coast, i.e., $|Z(0)|$. Figure 3a shows a plot of the coastal sea level $|Z(0)|$ as a function of η_0 and δ. For any value of η_0, increasing δ reduces the value of $|Z(0)|$ indicating a poorer transmission of the pressure field. For any given value of δ, transmission of the pressure field is substantially reduced as η_0 is increased.

The value of the streamfunction at $X = \delta$ is the total integrated alongshore transport from $X = \delta$ to the coast. The across-shelf component of the streamfunction $|P(\delta)|$ is plotted as a function of δ in Fig. 3b. The transport is larger for greater δ, and is also larger for larger values of the coastal depth η_0. Thus, even though the pressure field is more poorly transmitted for larger values of η_0, the total alongshore coastal transport is increased for larger values of η_0.

To illustrate the effects of the continental margin width δ and coastal depth η_0 on the velocity structure Fig. 4 shows the magnitude of the alongshore velocity $|V(X)|$ as a function of location X on the shelf for selected values of δ and η_0. Figure 4a has been calculated with $\eta_0 = 0$, i.e., zero coastal depth, and shows that the velocity structure increases linearly with distance from the coast at small values of X, with narrower continental margins (small δ) having substantially larger currents than wider continental margins (large δ). For the coastal depth $\eta_0 = 0.2$, Fig. 4b shows that coastal currents at $X = 0$ are larger for narrower continental margins, although as for Fig. 4a, it remains true that velocities increase monotonically with distance from the shore in all cases. For values of δ greater than about 1, the shelf edge currents at $X = \delta$ become smaller with increasing δ.

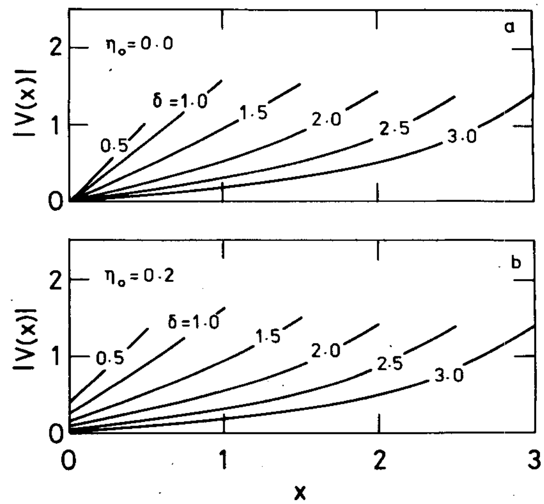

FIG. 4. The magnitude of the alongshore velocity structure function $|V(X)|$ plotted as a function of the dimensionless across-shelf position X for selected values of the dimensionless continental margin width δ. The continental shelf has a linear depth profile of form $h \propto \eta_0 + X$ with (a) $\eta_0 = 0.0$ and (b) $\eta_0 = 0.2$.

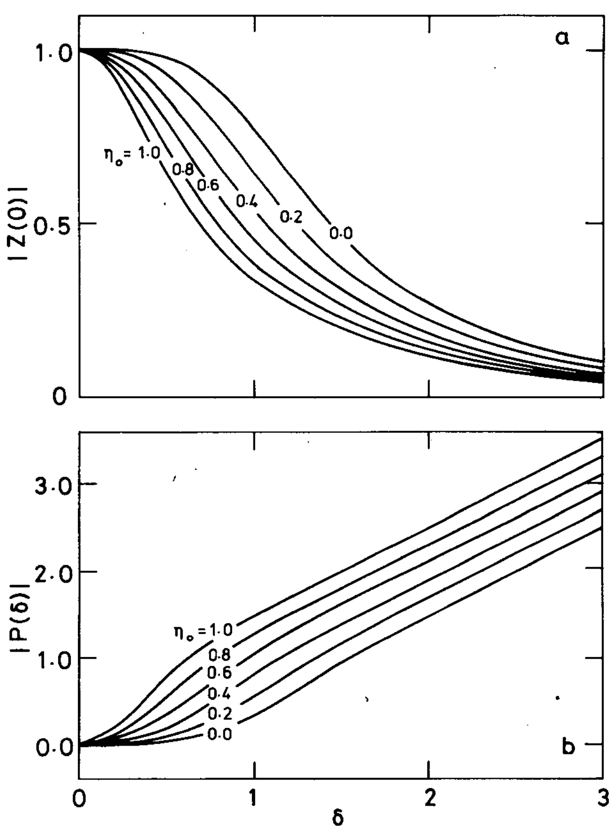

FIG. 3. Magnitudes of (a) coastal elevation $|Z(0)|$ and (b) total transport $|P(\delta)|$ plotted as a function of the dimensionless continental margin width δ for the linear depth profile with selected values of the dimensionless coastal depth η_0.

4. Circulation on a continental margin with parabolic depth profile

A parabolic depth profile has the form (Fig. 2b)

$$h = h_0 + h_2 x^2. \tag{30}$$

Transforming to dimensionless variables X and Y as in (14), the depth may be written in terms of $\eta_0 = h_0(h_2 L^2)^{-1}$ as

$$h = h_2 L^2(\eta_0 + X^2). \tag{31}$$

The structure equations resulting from imposed sea level $G \cos Y$ at $X = \delta$ are then

$$\phi = G \exp(iY) \tag{32}$$

$$\frac{d^2 Z}{dX^2} + 2\alpha^2 X Z = 0 \tag{33}$$

where the across-shelf structure obeys the boundary conditions

$$\frac{dZ}{dX}+\alpha^2\eta_0 Z=0, \quad X=0$$

$$Z=1, \quad X=\delta. \tag{34}$$

The definition of L and the coefficient α depend on the sign of f as follows

$$f>0, \quad L^3=\left(\frac{2r}{fkh_2}\right), \quad \alpha=1+i \tag{35}$$

$$f<0, \quad L^3=\left(\frac{-2r}{fkh_2}\right), \quad \alpha=1+i. \tag{36}$$

For this scaling the dimensionless continental margin width is (for $f>0$)

$$\delta=l\left(\frac{fkh_2}{2r}\right)^{1/3}=\left(\frac{3fk}{2r}\right)^{1/3}A^{1/3}$$

where $A=\frac{1}{3}h_2l^3$ is the cross sectional area of the coastal ocean from $x=0$ to $x=l$ below the depth h_0. For constant continental margin width l, a steeper gradient implies a larger A and hence larger δ; while for continental margins with a constant value of $h(l)$, a shallower gradient implies a larger A and hence larger δ.

Solutions to (33) are Airy functions of complex argument, or equivalently, Bessel functions of order ⅓ of complex argument (Abramowitz and Stegun, 1956, Ch. 10). However, these functions and their derivatives are not available on most computer systems and for the purposes of this application it is convenient to use the power series method for solution (Kreyszig, 1967, Ch. 3).

The general form of (33) for parabolic profiles is

$$\frac{d^2Z}{dX^2}+\alpha^2(\eta_1+2\eta_2X)Z=0 \tag{37}$$

and we seek solutions to (37) of form

$$Z=\sum_{m=0}^{\infty}C_mX^m. \tag{38}$$

Substituting (38) into (37) and equating coefficients of equal powers of X gives

$$C_2=-\frac{1}{2}\alpha^2\eta_1C_0 \tag{39}$$

and the general recursion relation

$$C_{m+3}=\frac{-\alpha^2(\eta_1C_{m+1}+2\eta_2C_m)}{(m+3)(m+2)}. \tag{40}$$

The boundary condition at the coast gives

$$C_1=-\alpha^2\eta_0C_0 \tag{41}$$

so that all coefficients may be found in terms of C_0, which is itself determined by the second boundary condition (34), i.e.,

$$Z(\delta)=\sum_{m=0}^{\infty}C_m\delta^m=1. \tag{42}$$

Solutions for the sea level, alongshore velocity and transport streamfunction are

$$\zeta=G\,\mathrm{Re}\{Z(X)e^{iY}\} \tag{43}$$

$$v=\frac{gG}{fL}\mathrm{Re}\{V(X)e^{iY}\} \tag{44}$$

$$\psi=\frac{Ggh_2L^2}{f}\mathrm{Re}\{P(X)e^{iY}\}. \tag{45}$$

In terms of the power series, the across-shelf structures are given by

$$Z(X)=\sum_{m=0}^{\infty}C_mX^m \tag{46}$$

$$V(X)=\sum_{m=1}^{\infty}mC_mX^{m-1} \tag{47}$$

$$P(X)=\eta_0C_1X+\left(\eta_0C_2+\frac{1}{2}\eta_1C_1\right)X^2 + \sum_{m=1}^{\infty}\left(\eta_0C_{m+2}+\eta_1\frac{m+1}{m+2}C_{m+1}+\eta_2\frac{m}{m+2}C_m\right)X^{m+2}. \tag{48}$$

Surface elevation and streamfunction contours for the parabolic depth profile with $\eta_0=0$, $\eta_1=0$, $\eta_2=1$ and $\delta=1$ shown in Fig. 5. In comparison with transport over the linear depth profile (Fig. 2), transport over the parabolic profile is somewhat reduced, except at the outer edge of the slope. The sea level perturbation is transmitted across the continental margin slightly more effectively, with the 0.8 contour now intercepting the coast. Quantitatively, however, there is little difference despite the different profile shape and different scaling.

Values of the coastal elevation $|Z(0)|$ and the total alongshore transport $|P(\delta)|$ are plotted in Fig. 6a, b as a function of δ for selected vaues of η_0. The depth profile is parabolic such that

$$h=h_0+h_2x^2=h_2L^2(\eta_0+X^2).$$

As in section 3, the coastal amplitude $|Z(0)|$ shows little reduction for small values of δ, implying that the pressure field is readily transmitted from the shelf edge to the coast. For larger values of δ the transmission becomes increasingly reduced. Increased values of the dimensionless coastal depth η_0 result in reduced values of $|Z(0)|$. The alongshore transport $|P(\delta)|$ shows increased transport as δ increases with transport increasing also as a function of coastal depth η_0.

The principal features displayed in Fig. 6 are thus

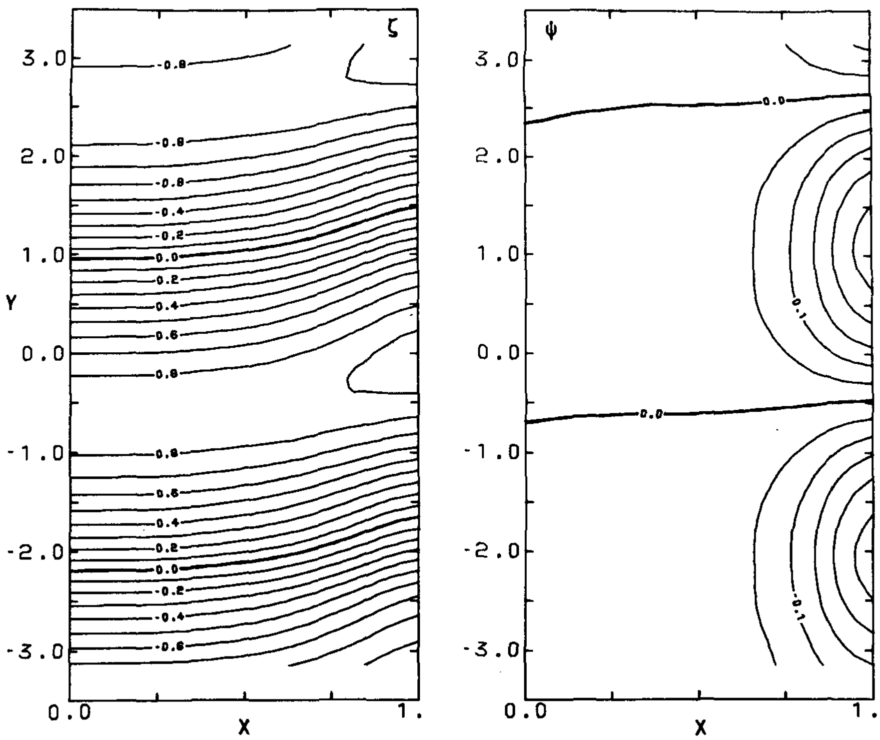

FIG. 5. Contours of sea level perturbation ζ and transport streamfunction ψ calculated for a dimensionless continental margin width of $\delta = 1$, and for zero coastal depth for the parabolic depth profile.

qualitatively similar to those of Fig. 3, despite the different scaling, although for the parabolic depth profile the coastal sea level $|Z(0)|$ generally decreases more rapidly with δ and the transport $P(\delta)$ generally increases more rapidly with δ than for the linear depth profile.

The across-shelf velocity structure $|V(X)|$ is plotted as a function of across-shelf location X in Fig. 7 for selected values of δ and for $\eta_0 = 0$, and $\eta_0 = 0.2$. In both cases currents increase monotonically with distance offshore, with stronger currents at $X = \delta$ occurring for larger δ in all cases. For midshelf locations, alongshore currents are, in general, larger for smaller values of δ. Thus oceanic sea level perturbations may generally be said to drive faster currents on narrower continental margins. Figure 7b shows the effect on currents of a non-zero coastal depth, with coastal currents being substantially stronger on narrower shelves.

5. Discussion

For barotropic flows on continental margins having linear or parabolic profiles, the effects of alongshore oceanic sea level perturbations on coastal circulation have been determined for both linear ($h = h_0 + h_1 x$) and parabolic ($h = h_0 + h_2 x^2$) depth profiles.

A key parameter is the dimensionless continental margin width δ, obtained by scaling the actual continental margin width l with the wavenumber k of the alongshore variability, the Coriolis parameter f, the linear friction coefficient r, and the shape constants h_1 and h_2 according to

$$\delta = \left(\frac{fk}{r}\right)^{1/2}\left[\frac{1}{2}h_1 l^2\right]^{1/2}, \quad \text{linear}$$

$$\delta = \left(\frac{3fk}{2r}\right)^{1/3}\left[\frac{1}{3}h_2 l^3\right]^{1/3}, \quad \text{parabolic.}$$

In each case the quantity in the square brackets relates to the geometry of the shelf, and is equal to the cross-sectional area A of the alongshore current below the level h_0, between $x = 0$ and $x = l$. Thus δ increases or decreases accordingly as A increases or decreases. For the linear depth profile, a scale length $L = (2r/fkh_1)^{1/2}$ can be defined such that $\delta = l/L$. This scale length is identical with that found by Csanady (1978) in his study of steady wind-driven circulation caused by an alongshore wind stress having wavenumber k. In general for a fixed continental margin width l, δ increases as the depth $h(l)$ increases, while for a fixed ocean depth $h(l)$, δ increases as l increases.

Calculations for both linear and parabolic depth profiles show that an alongshore open ocean pressure gradient caused through a sea level perturbation is most easily transmitted across narrow continental margins (small δ) with small coastal depths. The resulting alongshore coastal currents are largest on narrow continental margins with larger coastal depths while the total alongshore coastal transport is larger for both wider continental margins and larger coastal depths.

The phenomenon known as the "insulating effect of a steep continental slope" may now be put in perspective, since it is not the steepness of the slope which acts to insulate the shelf from the ocean, but the cross-sectional area A of flow over the continental margin. An appropriate measure of "insulation" is the dimensionless continental margin width δ.

As an idealised application to the west and east coasts of the United States, we choose l = 40 km, $h(l)$ = 2000 m for the west coast, and l = 200 km, $h(l)$ = 2000 m for the east coast, with h_0 = 0 in each case for the parabolic depth profile. Choosing $k = 10^{-6}$ m^{-1} and $r = 5 \times 10^{-4}$ m s^{-1} gives $\delta \sim 2$ (west coast) and $\delta \sim 3.5$ (east coast). Reference to Figs. 6 and 7 indicates the coastal sea level perturbation and alongshore coastal current to be practically non-existent for $\delta = 3.5$, while ~20% the pressure gradient is transferred to the coast for $\delta = 2$. Over the inner shelf, substantially greater currents are to be expected for $\delta = 2$ than $\delta = 3.5$. These results are consistent with the observations.

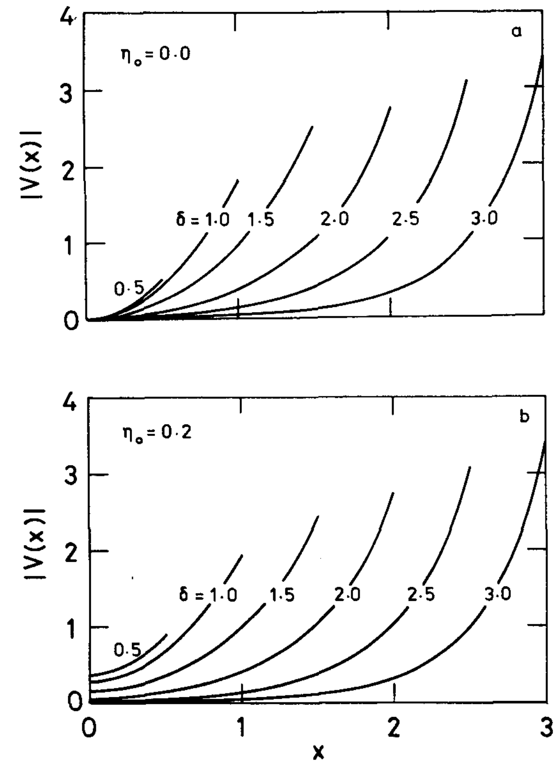

FIG. 7. The magnitude of the alongshore velocity structure function $|V(X)|$ plotted as a function of the dimensionless across-shelf position X for selected values of the dimensionless continental margin width δ. The continental shelf has a parabolic depth profile of form $h \propto \eta_0 + X^2$ with (a) η_0 = 0.0 and (b) η_0 = 0.2.

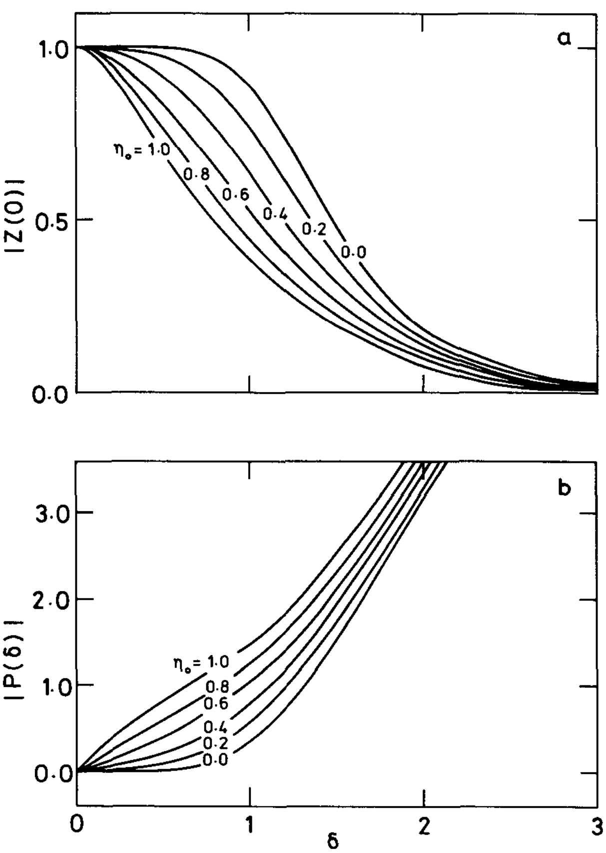

FIG. 6. Magnitudes of (a) coastal elevation $|Z(0)|$ and (b) total transport $|P(\delta)|$ plotted as a function of the dimensionless continental margin width δ for the parabolic depth profile with selected values of the dimensionless coastal depth η_0.

The present model is clearly quite restrictive in that it only considers depth-averaged, horizontal currents in a barotropic ocean. Calculations are only made here for linear and parabolic depth profiles, but the extension to profiles of higher polynomial form for actual applications is straightforward with the present method. Other numerical methods may also prove appropriate to calculate across-shelf structures if actual profiles are to be used. Another limitation is the neglect of any baroclinic effects. While it is often true that shallow coastal regions are well mixed, the open ocean is always stratified. In this context, the results presented here describe an upperbound for the strength of induced coastal circulation.

Within its scope, the present model appears to clearly elucidate the role that oceanic pressure gradients play in generating coastal circulation.

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