# The Response of Wave Directions to Changing Wind Directions

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#### **ABSTRACT**

From the premise that the net growth of wave energy induced by wind is centered around the wind direction, a relaxation model for the response of the main wave direction to changes in the wind direction for young sea states is derived. The time scale of this relaxation model is found to be equal to the time scale of the wave energy growth. A quantitative version of the model, based on universal growth rates of the waves under the local wind is found to be consistent with observations obtained in this study and with a published dataset.

#### 1. Introduction

The directional response of ocean waves to changes in wind direction is an important aspect of the numerical modeling of sea states. However, information on the wave directional response is scarce both as regards theory and observation. Present wave models consequently diverge widely in modelling this directional response as shown in the SWAMP study (SWAMP, 1985).

In the class of parametric wave hindcast models (e.g., Günther et al., 1979) and in statistical simulation models (e.g., Bern and Houmb, 1984) simple relaxation models can be used that relate changes in the local main wave direction to changes in the local wind direction. For instance, Günther et al. (1981) derived a relaxation model with a parameterization of the spectral energy balance of the waves:

$$\frac{\partial \theta_1}{\partial \tilde{t}} = \frac{1}{\tilde{\tau}_1} \sin(\theta_w - \theta_1),\tag{1}$$

in which the main wave direction  $\theta_1$  is the average wave momentum direction,  $\theta_w$  is the local wind direction and dimensionless time is denoted by  $\tilde{t} = gt/U$ , in which t is time, g is gravitational acceleration and U is wind speed at 10 m elevation. In this model the dimensionless time scale  $\tilde{\tau}_1$  is related to the dimensionless peak frequency  $\nu$  as

$$\tilde{\tau}_1 = \chi^{-1} \nu^{-2},\tag{2}$$

in which  $\tilde{\tau}_1 = g\tau_1/U$  and  $\nu = Uf_m/g$  with  $\tau_1$  and  $f_m$  as the relaxation time scale and the peak frequency, re-

spectively. From four observations of  $\tilde{\tau}_1$  Günther et al. (1981) found an average value of  $\chi = 0.21 \times 10^{-2}$ . A frequency-dependent version of this model had been suggested previously by Hasselmann et al. (1980) with the coefficients also estimated from observations. Allender et al. (1983) observed coefficients close to those of Hasselmann et al. (1980). However, the results obtained by Hasselmann et al. (1980) and Allender et al. (1983) deviate significantly (factor 3 to 4) from those obtained by Günther et al. (1981).

In this study we derive a directional relaxation model that is conceptually different from the above models in that in this model the time scale is estimated from the growth rate of the wave energy and not from observations of wave directions as in the above models. The results of our model are compared with pitch-and-roll buoy observations in the southern North Sea obtained in this study, and with the above results from the literature.

### 2. The model

Observations and theory both indicate that the spectrum of young sea states is forced into a universal shape by nonlinear wave-wave interactions and that spectral parameters are strongly correlated with the local wind (e.g., Hasselmann et al., 1973; Hasselmann et al., 1976). These characteristics of young sea states are exploited in parametric wave hindcast models. For these models the spectral energy balance equation of the waves is parameterized into a small number of prognostic

equations, one for each spectral parameter (for instance the JONSWAP parameters, Hasselmann et al., 1973, and the main wave direction, e.g., Günther and Rosenthal, 1985). Such a prognostic equation contains a source function which represents the wind-induced development of the spectral parameter. The relaxation model that is derived here may be considered as such a source function for the mean wave direction.

For the definition of wave direction it seems convenient to take one that is used in observations. The observations in the present study have been carried out with a pitch-and-roll buoy. The main wave direction  $\theta_0$  for these observations is defined as the frequency integrated mean wave direction in terms of Fourier coefficients of the two-dimensional spectrum (e.g., Longuet-Higgins et al., 1963):

$$\theta_0 = \arctan\left(\frac{b_1}{a_1}\right) \tag{3}$$

in which

$$a_1 = \int_0^{2\pi} \int_0^{\infty} \cos(\theta) E(f, \theta) df d\theta \tag{4}$$

$$b_1 = \int_0^{2\pi} \int_0^\infty \sin(\theta) E(f, \theta) df d\theta, \tag{5}$$

in which  $E(f, \theta)$  is the two-dimensional energy density of the waves as a function of frequency f and direction  $\theta$ . The local rate of change of the wave direction  $\theta_0$ , determined as the time derivative of Eq. (3) can be written, after some straightforward manipulations involving Eqs. (4) and (5) as

$$\frac{\partial \theta_0}{\partial t} = \frac{\frac{\partial}{\partial t} \int_0^{2\pi} \int_0^{\infty} \sin(\theta) E(f, \theta) df d\theta - \tan(\theta_0) \frac{\partial}{\partial t} \int_0^{2\pi} \int_0^{\infty} \cos(\theta) E(f, \theta) df d\theta}{1 + \tan^2(\theta_0) \int_0^{2\pi} \int_0^{\infty} \cos(\theta) E(f, \theta) df d\theta}.$$
 (6)

This can also be expressed in terms of the source function of the wave energy balance equation:

$$\frac{\partial}{\partial t}E(f,\theta) + \nabla c_g E(f,\theta) = S(f,\theta). \tag{7}$$

The speed of energy propagation is  $c_g$  and the sum total of all processes of wave generation and dissipation

is represented by the source function  $S(f, \theta)$ . For a homogeneous wave field Eq. (7) reduces to:

$$\frac{\partial}{\partial t}E(f,\theta) = S(f,\theta),\tag{8}$$

in which case the local rate of change of  $\theta_0$  in Eq. (6) can be written as

$$\frac{\partial \theta_0}{\partial t} = \frac{\int_0^{2\pi} \int_0^\infty \sin(\theta) S(f, \theta) df d\theta - \tan(\theta_0) \int_0^{2\pi} \int_0^\infty \cos(\theta) S(f, \theta) df d\theta}{1 + \tan^2(\theta_0) \int_0^{2\pi} \int_0^\infty \cos(\theta) E(f, \theta) df d\theta}.$$
 (9)

If we define the mean direction  $\theta_s$  of the source function  $S(f, \theta)$  similarly to  $\theta_0$ , i.e., replace  $E(f, \theta)$  by  $S(f, \theta)$  in (4) and (5), then Eq. (9) reduces to

$$\frac{\partial \theta_0}{\partial t} = \frac{\cos(\theta_0) \int_0^{2\pi} \int_0^{\infty} \cos(\theta) S(f, \theta) df d\theta}{\cos(\theta_s) \int_0^{2\pi} \int_0^{\infty} \cos(\theta) E(f, \theta) df d\theta} \sin(\theta_s - \theta_0).$$
 (10)

A simplification of this expression is not obvious in view of the rather complicated structure of the source function  $S(f, \theta)$  (e.g., Hasselmann, 1968). To arrive at a simple model, two rather crude assumptions are made here. The first is that the shapes of the directional distributions of  $E(f, \theta)$  and  $S(f, \theta)$  are frequency independent, equal to one another and symmetric around  $\theta_0$  and  $\theta_s$ , respectively. The second assumption is that the source function is centered around the wind direction

 $\theta_w$  such that  $\theta_s = \theta_w$ . Equation (10) then reduces to the following relaxation model:

$$\frac{\partial \theta_0}{\partial t} = \frac{\int_0^\infty S(f)df}{\int_0^\infty E(f)df} \sin(\theta_w - \theta_0),\tag{11}$$

in which E(f) and S(f) are the one-dimensional spectrum and the one-dimensional source function, respectively. Since the integral of S(f) over frequency equals the rate of change of the total wave energy (in the homogeneous situation considered here), the relaxation model can also be written as

$$\frac{\partial \theta_0}{\partial t} = \frac{1}{\epsilon} \frac{\partial \epsilon}{\partial t} \sin(\theta_w - \theta_0) \tag{12}$$

in which the total wave energy  $\epsilon$  is defined as

$$\epsilon = \int_0^\infty E(f)df. \tag{13}$$

In dimensionless form and with a time scale  $\tau$  the model reads:

$$\frac{\partial \theta_0}{\partial \tilde{t}} = \frac{1}{\tilde{\tau}} \sin(\theta_w - \theta_0) \tag{14}$$

with dimensionless time scale  $\tilde{\tau}$ :

$$\tilde{\tau} = \left(\frac{1}{\tilde{\epsilon}} \frac{\partial \tilde{\epsilon}}{\partial \tilde{t}}\right)^{-1},\tag{15}$$

in which  $\tilde{\epsilon} = \epsilon g^2/U^4$  and  $\tilde{\tau} = g\tau/U$ . We consider this relaxation model (14) with time scale (15) to be the main result of this study. It will be referred to in the following as model A.

To quantify the time scale  $\tau$ , we can use universal growth characteristics of waves in an ideal situation in which a homogeneous wind field starts to blow over a limitless ocean at time t=0. In such a situation the dimensionless wave energy  $\tilde{\epsilon}$  is a function of dimensionless time  $\tilde{t}$  and the normalized rate of wave growth can be expressed in terms of the dimensionless wave

energy  $\tilde{\epsilon}$ . The results of the SWAMP wave models give such a wave development (Fig. 1; SWAMP, 1985). Within the scatter of these results, the evolution of  $\tilde{\epsilon}$  can be well approximated with

$$\tilde{\epsilon} = a \tanh^d(b\tilde{t}^c). \tag{16}$$

We take the growth curve of the BMO model (British Meteorological Office) to be typical for the SWAMP growth curves since it is more or less an average of all these curves. It can be fairly well approximated with (16) and with the following values of the coefficients (Fig. 2):

$$a = 3.6 \times 10^{-3}$$

$$b = 2.1 \times 10^{-22}$$

$$c = 4.67$$

$$d = 0.3$$
(17)

The time derivative of  $\tilde{\epsilon}$ , expressed in terms of  $\tilde{\epsilon}$  itself, follows from Eq. (16):

$$\frac{\partial \tilde{\epsilon}}{\partial \tilde{t}} = abcd \left(\frac{\tilde{\epsilon}}{a}\right)^{(d-1)/d} \left[1 - \left(\frac{\tilde{\epsilon}}{a}\right)^{2/d}\right] \times \left\{\frac{1}{b} \operatorname{arctanh}\left[\left(\frac{\tilde{\epsilon}}{a}\right)^{1/d}\right]\right\}^{(c-1)/c}. \quad (18)$$

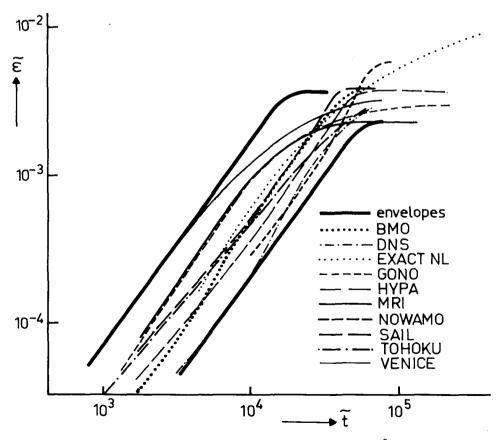


Fig. 1. The dimensionless wave energy  $\tilde{\epsilon}$  as a function of dimensionless time  $\tilde{t}$  in a homogeneous, stationary unbounded windfield for various models of the SWAMP study.

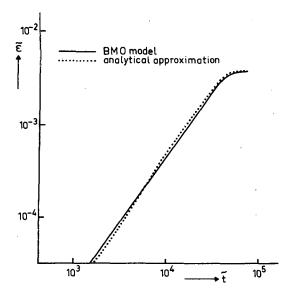


Fig. 2. The analytical approximation of the results of the British Meteorological Office model in the SWAMP study.

The corresponding dimensionless time scale  $\tilde{\tau}$  of the relaxation model (14) is given in Fig. 3 for the coefficients (17). This model, which is a quantitative version of model A, will be referred to as model B in the following.

To estimate the potential variation in the value of  $\tilde{\tau}$ , the coefficients of the upper- and lower envelopes of the SWAMP growth curves (see Fig. 1) have been used to find the corresponding envelopes of  $\tilde{\tau}$  in Fig. 3. Note that  $\tilde{\tau} \to \infty$  when the waves approach the fully developed state ( $\tilde{\epsilon} \to a$ ). This is not realistic since fully developed waves do change direction under a changing wind direction. This shortcoming is caused by the fact that Eq. (16) relates to an ideal situation where the wave direction is equal to the wind direction. It can be remedied by modifying the above growth rate of the

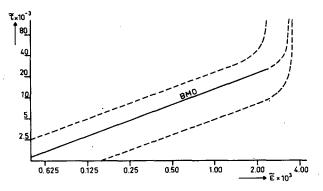


Fig. 3. The dimensionless time scale  $\tilde{\tau}$  of the directional relaxation model as a function of dimensionless wave energy  $\tilde{\epsilon}$  for the growth curve of the BMO model (solid line). Upper and lower envelopes of model time scales are obtained from lower and upper envelopes of the SWAMP growth curves (dashed line) (see Fig. 1).

waves to account for the difference between wind direction and wave direction, for instance, by relating the growth rate  $\partial \epsilon/\partial t$  in Eq. (18) to only a fraction of the wave energy. In analogy with Janssen et al. (1984), one could multiply  $\tilde{\epsilon}$  in the right-hand side of Eq. (18) by  $\{\pi - |\theta_w - \theta_0| - \sin|\theta_w - \theta_0|\}/\pi$ , which is that fraction of a  $\cos^2(\theta)$  directional distribution centered at  $\theta_0$  that overlaps another  $\cos^2(\theta)$  directional distribution centered at  $\theta_w$ . Such a modification reduces the value of  $\tilde{\tau}$  considerably for sea states close to or equal to the fully developed state and it slightly increases the time scale for young sea states which are considered here. This modification will be ignored in the following.

#### 3. Observations

#### a. Introduction

To compare our model with observations we have selected a number of pitch-and-roll buoy observations from a large dataset collected on a routine basis by the Ministry of Transport and Public Works of the Netherlands. The dataset contained about 850 observations (half-hour duration each at one-hour intervals) obtained in a period of 6 weeks in all kinds of weather. We selected from this data set observations of young sea states with as little influence of swell as possible.

## b. Geophysical conditions

The observations were taken at position 53°13′01″N, 03°13′12″E which is a location in the southern North Sea, approximately 90 km west of the Dutch islands (Fig. 4). The local water depth is about 30 m, which is

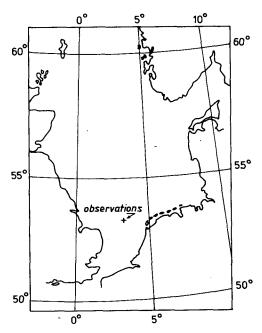


Fig. 4. Geographic location of the observations.

relatively deep for the observations finally selected for this study (peak frequency of selected wave spectra always  $\ge 0.16$  Hz). The observation period is the end of the yearly storm season in the southern North Sea. The wind speed (at 10 m elevation) varied from 0.8 to 19.5 m s<sup>-1</sup> and the significant wave height varied from 0.30 to 5.90 m (see Fig. 5). The peak frequency varied from 0.06 to 0.41 Hz.

The time scale in the above derived relaxation model applies only to young sea states since it is based on growth rates which are assumed to be universal for young sea states. We have therefore selected observations with respect to the absence of swell. Observations were therefore accepted only if the following three conditions were simultaneously fulfilled:

(i) the peak frequency  $f_m$  of the spectrum is higher than the direction-corrected Pierson-Moskowitz frequency (Pierson and Moskowitz, 1964).

$$f_m > f_{\rm PM}/\cos(\theta_w - \theta_0)$$

in which  $f_{PM} = 0.13 g/U$ .

(ii) The observed dimensionless wave energy  $\tilde{\epsilon}$  should be within a factor of 2 from the dimensionless wave energy obtained with the universal relationship suggested by Hasselmann et al. (1976, their Table 1):

$$\tilde{\epsilon} = 7.4 \times 10^{-6} \, \nu^{-3.05},$$
 (19)

with  $\nu$  as the observed dimensionless peak frequency and with a maximum value of the observations of  $\tilde{\epsilon} = 3.6 \times 10^{-3}$  (fully developed sea state, Pierson and Moskowitz, 1964).

(iii) The observed wave direction should change towards the observed wind direction.

#### c. Observations and analysis

A 2.50-m diameter WAVEC pitch-and-roll buoy was used for the wave observations (Van der Vlugt et al., 1981; Van der Vlugt, 1981). Records of heave, pitch and roll-signals with 30 minutes duration were available at 60-minute intervals from routine observations of the Ministry of Transport and Public Works in the Netherlands in the framework of other projects. The routine analysis of these data, which is based on Longuet-Higgins et al. (1963), Van der Vlugt et al. (1981) and Kuik and Van Vledder (1984), provided us with estimates

of the main wave direction  $\theta_0$  as defined in (3). The theoretically estimated statistical reliability of this direction is about 3° (rms error; Borgman et al., 1982; Kuik and Van Vledder, 1984). In view of additional effects (e.g., instrument noise) we estimate the rms measurement error to be about 5°.

The wind observations were also routine observations of the Ministry of Transport and Public Works carried out with conventional cup anemometers and wind vanes at 76 m elevation on a nearby gas production platform (within 1 km distance from the WAVEC buoy). These wind observations have been corrected to estimate free flow wind velocity at 10 m elevation with the results of wind tunnel experiments (Vermeulen et al., 1985) and with an assumed logarithmic wind profile with a drag coefficient  $c_{10} = 1.5 \times 10^{-3}$ . The wind observations were 10-min averages taken every 30 minutes. The wind direction was corrected by 5° to account for the Ekman effect in the atmospheric boundary layer. We estimate the rms error in the wind direction observations to be between 5° and 10°.

#### d. Analysis

The dimensionless time scale  $\tilde{\tau}$  is estimated from the wave direction observations with a central difference scheme:

$$\tilde{\tau}_{j} = \frac{2\Delta t}{\theta_{0,j+1} - \theta_{0,j-1}} \frac{g}{U_{j}} \sin(\theta_{w,j} - \theta_{0,j}), \tag{20}$$

in which  $\Delta t$  is the time interval between two successive wave observations (60 min) and j is the sequence number in the wave record.

The time scales thus observed will be compared with time scales estimated with the above derived model. Two versions of this model time scale are available: model A based on the observed wave energy and its rate of growth, Eq. (15), and model B based on the observed wave energy, Eqs. (15) and (18). The total observed wave energy  $\epsilon$  is estimated in accordance with its definition (13), and the observed value of  $\partial \epsilon/\partial t$  is determined with a central difference scheme:

$$\frac{\partial \epsilon}{\partial t} = \frac{\epsilon_{j+1} - \epsilon_{j-1}}{2\Delta t}.$$
 (21)

It should perhaps be noted that the selection criteria

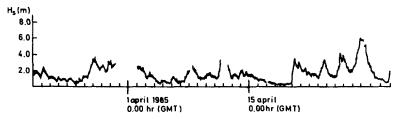


Fig. 5. Observed significant wave height  $(H_s = 4\sqrt{\epsilon})$  during the observation period.

(i) and (ii) of the previous section (3b) were applied to all data points appearing in the central difference schemes (20) and (21), i.e., at sequence numbers j-1, j and j+1.

Considering the measurement errors in the wind direction (rms error  $\approx 5^{\circ}$ -10°) and in the wave direction (rms error  $\approx 5^{\circ}$ ), observations have not been analyzed when the change in wave direction from sequence number j-1 to sequence number j+1 (two hour interval) is less than 10°; and simultaneously the difference between wave direction and wind direction at the central time point j is less than 10°. For similar reasons, the value of  $\partial \epsilon/\partial t$  has not been determined for observations for which the change in total wave energy from sequence number j-1 to sequence number j+1 (two hour interval) is less than 0.2 times the total wave energy at sequence number j itself. This condition implies that conditions of wave decay have not been considered.

To compare the observed time scales with the model of Hasselmann et al. (1980), the peak frequency of each selected wave observation is determined as the frequency where the maximum energy density in the observed spectrum occurred.

### e. Results

Of the original 850 observations only 8 observations passed the above indicated tests. A summary of the numerical value of these observations is given in Table 1. The observed dimensionless time scale for these observations is plotted against the observed dimensionless wave energy in Fig. 6.

#### 4. Discussion

The observed time scales scatter considerably when plotted as a function of the dimensionless wave energy (Fig. 6). This is undoubtedly in part due to shortcom-

ings in the present model. But it is also due to undesired effects in the observations such as the unavoidable inhomogeneity in the wave field and measurement errors. The scatter in the observations of  $\tilde{\tau}$  could in fact be expected if only because of the considerable variation between "universal" growth curves (see Fig. 6). Such a large scatter is not unusual for observations of directional relaxation time scales. Other published observations of wave direction relaxation contain also considerable scatter. In the observations of Günther et al. (1981), the value of the coefficient  $\chi$  (an average of only four observations, see Introduction) ranges from 0.10 to 0.35 ( $\times 10^{-2}$ ). Hasselmann et al. (1980) correlate observed values of the coefficient B (see below) with the dimensionless frequency  $f/f_m$  for a number of classes of U/c (where c is the phase velocity at frequency f). They report a correlation coefficient  $\gamma$  varying from 0.29 to 0.4. Allender et al. (1983) find a value for the same correlation coefficient of about 0.25. These are low values that indicate a high scatter in the observed time scales (since  $\gamma^2 \approx 0.1$ , only about 10% of the observed variance is explained by these models), not only as a function of  $f/f_m$  but also as a function of wave age  $U/c_m$  (where  $c_m$  is the phase velocity at the peak frequency  $f_m$ ; this wave age plays the same role as  $\tilde{\epsilon}$  in our model).

The trend of an increasing time scale with increasing wave energy, as indicated by our model B [Eqs. (15) and (18)], is not well detectable in the scatter of our observations. To compare these observations with model A, the response time scale has been determined with model A from the observed wave energy growth at each of the eight occasions where the observations were accepted. The values of these time scales are given in Table 1 together with the average value of the time scales and the root-mean-square error between model and observation. This has also been done for model B. Surprisingly, the model B time scale which is based on

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TABLE 1. Summary of observed wind and wave parameters of all accepted data points and the corresponding time scales. Directions are relative to true North (nautical convention). The significant wave height  $H_s = 4\sqrt{\epsilon}$ .

Date (1985) d-mo	Time (UTC)**	<i>H</i> <sub>s</sub> (m)	f <sub>m</sub> (Hz)	θ <sub>wave</sub> (°TN)	U <sub>10</sub> (m s <sup>-1</sup> )	θ <sub>wind</sub> (°TN)	$\begin{array}{c} \Delta\epsilon/\Delta t \\ (m^2 h^{-1}) \\ \times 10^3 \end{array}$	$\Delta \theta_{\text{wave}}/\Delta t$ (° h <sup>-1</sup> )	τ observed (h)	τ model A (h)	τ model B (h)	τ Hasselmann et al.* (h)
25-3	5:00	0.90	0.24	202	7.9	234	5.2	6.0	5.1	9.7	3.0	9.2
27-3	14:00	2.42	0.16	340	12.6	330	97.0	-13.0	0.8	3.8	5.3	13.8
29-3	16:00	2.42	0.17	235	15.0	215	82.5	-18.5	1.1	4.4	3.9	13.0
5-4	24:00	0.95	0.23	190	11.3	218	14.6	6.0	4.5	3.9	1.7	9.6
13-4	6:00	1.56	0.16	218	8.5	234	20.0	9.5	1.7	7.6	6.8	13.8
14-4	13:00	1.67	0.18	271	11.2	314	25.0	5.5	7.1	7.0	3.9	12.3
19-4	23:00	0.71	0.27	246	10.2	263	33.2	19.5	0.9	0.9	1.4	8.2
23-4	2:00	1.21	0.20	351	9.9	018	13.3	8.0	3.2	6.9	3.1	11.1
								average	3.0	5.5	3.6	11.4
								rms error	_	3.5	3.4	9.1

<sup>\*</sup> Hasselmann et al. (1981) applied to peak frequency

<sup>\*\*</sup> UTC = Universal Time Coordinated in lieu of GMT

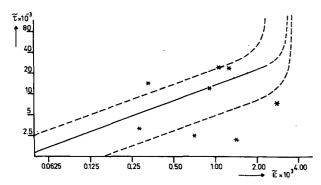


FIG. 6. Observed dimensionless time scales of directional wave relaxation as a function of dimensionless wave energy. Model time scale and envelopes of model time scale obtained from BMO curve and from upper and lower envelope of SWAMP growth curves (see Fig. 3).

an assumed universal wave growth rate agrees better with the observations (on average) than the model A time scale which is based on the observed growth rate.

Another comparison of the model with observations can be made with the model of Günther et al. (1981) which to a great extent is a parameterization of observations (see Introduction). A direct comparison is not possible because our time scale is expressed in terms of the dimensionless energy  $\tilde{\epsilon}$  whereas Günther et al. (1981) expressed their time scale in terms of the dimensionless peak frequency  $\nu$ . A simple transformation of  $\nu$  to  $\tilde{\epsilon}$  is readily made with the help of so-called universal relationships between dimensionless wave energy and dimensionless peak frequency, e.g., Eq. (19). The result of such a transformation, using Eq. (18) is given in Fig. 7. The agreement between this model and our model B based on the BMO growth curve is excellent. This is a happy coincidence considering 1) the con-

ceptual difference between the models and 2) the variations in the SWAMP wave growth curves. It must be noted that the BMO curve has been selected as a representative SWAMP growth curve; no attempt has been made to match our time scale to that of Günther et al. (1981) by selecting an "appropriate" wave growth curve.

Another comparison with observations is based on the empirical model of Hasselmann et al. (1980) who observed the response of the main wave direction as a function of frequency,

$$\frac{\partial \theta_3}{\partial \tilde{t}} = \frac{1}{\tilde{\tau}_3} \sin(\theta_w - \theta_3) \tag{22}$$

$$\tilde{\tau}_3(\nu_3) = \nu_3^{-1}/(2\pi B),$$
 (23)

in which  $v_3 = Uf/g$ , and  $\theta_3$  is the mean direction as a function of frequency [defined as in (3), (4) and (5) but with the integration over frequency omitted]. The dimensionless time scale  $\tilde{\tau}_3$  is defined analogously to  $\tilde{\tau}$ . Hasselmann et al. (1980) found from observations that  $B = 2.0 \times 10^{-5}$  (average b-value in their Table 5) and Allender et al. (1983) found, also from observations, a value close to this,  $B = 1.73 \times 10^{-5}$  (average B'-value in their Table 1). If we assume that the response of the main wave direction  $\theta_0$  is dominated by the response of the spectrum near the peak, which is reasonable for young sea states because the spectra are narrow and they are dominated by nonlinear wave-wave interactions near the peak, then Eq. (23) should roughly hold for the dimensionless peak frequency  $\nu$ . Using the same transformation from  $\nu$  to  $\tilde{\epsilon}$  as above, we find the results of Hasselmann et al. (1980) and Allender et al. (1983) as in Fig. 7. It is obvious from a comparison with Fig. 6 that these model time scales are considerably higher than the upper limit of the time scales observed in this study. In fact, these model time scales are three to four

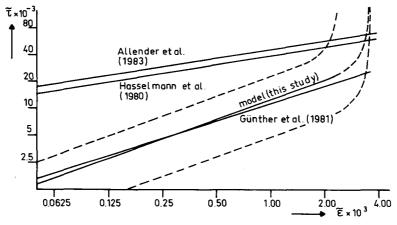


FIG. 7. Model time scales of directional wave relaxation as function of dimensionless wave energy. Dashed lines correspond to envelopes of SWAMP growth curves (see Fig. 3).

times larger than the average observed value in this study (see also Table 1).

It is obvious from Fig. 7 that there are two families of time scales which do not mutually agree: those of Hasselmann et al. (1980) and Allender et al. (1983) on the one hand and those of Günther et al. (1981) and this study on the other. This difference is not consistent with the scatter in the observed data of this study. One of the reviewers of the present paper suggested two possible reasons for the discrepancies: First, the intercomparison of the models would be inexact since the models of Hasselmann et al. (1980) and Allender et al. (1983) are frequency dependent whereas the models of Günther et al. (1981) and the present study are frequency integrated. However, this would explain only a small fraction of the observed discrepancy since  $\tilde{\tau}_3$ depends linearly on frequency and using, e.g., the mean frequency instead of the peak frequency in the above transformation from  $\tilde{\tau}_3$  to  $\tilde{\tau}$  would result in a change of only 10% to 20% in the value of  $\tilde{\tau}$ . Second is the possible existence of two distinct wave fields in a situation of a changing wind direction. During a rapid change in wind direction the spectrum may well develop bimodality, whereas during a slow change in wind direction the spectrum retains its unimodal character (e.g., Kuik and Holthuijsen, 1981). A corresponding difference in time scales for these two regimes of response is plausible and it may explain the observed discrepancies. However, to what degree the above datasets do actually differ with respect to the occurrence of unimodality or bimodality is uncertain (partly because pitch-and-roll buoys are not well suited to detect bimodality in the directional distributions).

#### 5. Conclusions

From the premise that the net wave growth is centered around the wind direction, we have derived a simple relaxation model for the directional response of waves in young sea states to changes in wind direction in homogeneous situations. The time scale of this response appears to be equal to the time scale of wave energy growth. For arbitrary situations this model needs to be supplemented with a wave propagation model.

The average of the model time scales based on assumed universal wave growth characteristics (model B of this study) agrees fairly well with the average of the observations of this study. The theoretically expected dependency of this time scale on dimensionless wave energy could not be detected within the scatter of these observations, but it is well confirmed by the observations of Günther et al. (1981). The observed data of this study and the model B of this study and the model of Günther et al. (1981) are mutually consistent. They do not agree with the models of Hasselmann et al. (1980) and Allender et al. (1983) (at least not as we interpret these). In fact, the time scales of Hasselmann

et al. (1980) and Allender et al. (1983) are a factor three to four larger than those of Günther et al. (1981) and this study. Possible reasons for this discrepancy are indicated.

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