Electronic properties of superconductor-semiconductor mesoscopic device

Attia.A. Awadalla

Abstract. In this paper we studied electronic properties through a semiconductor quantum dot which is coupled via tunnel barrier to two superconducting reservoirs. We derive an expression for the conductance and current density for a mesoscopic device. The tunneling probability is derived using the Bogoliubov-de Gennes equation (BdG) taking into consideration the effect of Coulomb blockade and the influence of magnetic field. The barrier height has been determined by Monte-Carlo simulation technique. The current density (current per unit energy) and conductance are calculated from the phase-coherent propagation of electron-like and hole-like excitation emitted by supercurrent reservoirs, together with electron and hole exciation from the semiconductor. The results show that an oscillatory behavior of both current density and conductance. These oscillations appears as random fluctuations in peak heights. This resonant behavior might be due to the multiple Andreev reflection at the interface and Coulomb oscillation due to Coulomb charging energy. Our results agree qualitatively with those in the literature.

M.S.C. 2000: 82D55, 74K30, 74F05, 82D99.
Key words: Electronic stub tuner, Chaotic Dynamics, fluctuations, Andreev reflections.

Introduction

In recent years our understanding of electron transport in mesoscpic conductors has greatly improved. It has become clear that at low temperature electrons can maintain their phase coherence over considerable distances. So it has now become possible to study devices through which the electron can travel ballistically, with outbeing scattered by impurities [17, 16, 6, 11]. It was predicted [26, 15] that the supercurrent in such a device, measured as a function of the width of the channel, should exhibits steps each time an additional one-dimensional channel is opened. In this paper we describe the supercurrent flow through S-Sm-S device with the use of transmission and reflection formalism. This formalism has already been applied for the description of electron transport in normal metals and semiconductor [7]-[21], the device geometry

Applied Sciences, Vol.8, 2006, pp. 23-30.

is illustrated in Fig.1. The length of the semiconductors is short compared to the elastic scattering length. The width may be larger or smaller than scattering length. At the interface between the semiconductor and superconductor elastic and Andreev scattering is present.

1 Method of Calculation

All elastic procrsses in the sample are represented by $2Nx2N$ transfer matrix S whose transmission T_{nm} ,T'_{nm} and reflection R_{nm} ,R'_{nm} coefficients can mix the left-hand side(LHS) and right-hand side(RHS) channels.T_{nm} (,T'_{nm}) is the probability of an electron traveling to the right (left) in the n^{th} channel on the LHS (RHS) to be transmitted into the n^{th} channel on the RHS(LHS), and R_{nm} ,(R'_{nm}) the probability of that electron to be back scattered into the n^{th} channel on the LHS(RHS) [13].

We consider the device geometry is illustrated in Fig.1, mesoscopic semiconductor between superconducting contacts. The transmission and reflection of the electron waves at junction formed by lead 1, and 2 can be described by S matrix [24] which relates the amplitudes of outgoing waves to the amplitudes of the incoming waves at junction, where S is expressed as

$$S = \begin{pmatrix} -(a+b) & \varepsilon^{1/2} & \varepsilon^{1/2} \\ \varepsilon^{1/2} & a & b \\ \varepsilon^{1/2} & b & a \end{pmatrix} \tag{1.1}$$

with $a=\frac{1}{2}[\sqrt{(1-2\varepsilon)}-1]$ and $b=\frac{1}{2}[\sqrt{(1-2\varepsilon)}+1]$. The parameter ε describes the strength of the coupling between the $1D$ channel and the reservoirs.

We use a well-known model [22, 8] where the pair potential $\Delta(x) = 0$ in semiconductor channel, and $\Delta = \Delta_o \exp(i\phi_{1,2})$ with $\phi_{1,2}$ *is* the Cooper pair phases for the two leads. The wave functions can be found from the time-independent Bogoliubov de Gennes equation [10]:

$$E \begin{pmatrix} u(x) \\ v(x) \end{pmatrix} = \begin{pmatrix} H - E_F & \Delta \\ \Delta^* & -(H^* - E_F) \end{pmatrix} \begin{pmatrix} u(x) \\ v(x) \end{pmatrix} \tag{1.2}$$

In the previous equation, $u(x)$ describrs the electron wave function, and $v(x)$ describes the hole wave function with an excitation energy E relative to the Fermi energy E_F. Their expressions in the corresponding regions are:

$$\begin{pmatrix} u(x) \\ v(x) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp(\pm iq^+ x), \qquad with \;\; \hbar q^+ = \sqrt{2m}\sqrt{E_F + E} \tag{1.3}$$

with $+(-)$ corresponding to electron excitation which move in positive (negative) x direction [29] and

$$\begin{pmatrix} u(x) \\ v(x) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \exp(\pm iq^- x), \qquad with \;\; \hbar q^- = \sqrt{2m}\sqrt{E_F - E} \tag{1.4}$$

and in the superconducting regions are [2]:

$$\begin{pmatrix} u(x) \\ v(x) \end{pmatrix} = \begin{pmatrix} u_o \exp(i\phi) \\ v_o \end{pmatrix} \exp(\pm\, ik_j^+ x) \tag{1.5}$$

where

$$k_j^+ = \left[\frac{2m}{\hbar^2}\left(E_F + (E^2 - \Delta_o^2)^{0.5} + V_o + V_b + \frac{U_C N^2}{2} + e\,\eta V_g N + (0.5\,\hbar\omega_c)\right) - \frac{j^2\pi^2}{W^2}\right]^{0.5}$$

with $+(-)$ corresponding to electronlike excitations which travel in positive or (negative) x direction. where V_o is the potential well depth of the semiconductor heterojunction quantum dot, E_F is the Fermi-energy,U_C is the charging energy and equals $e^2/2C$ in which C is the quantum dot capacitance and e is electron charge, ω_c is the cyclotron frequency and equals ,$eB(m)^{-1}, B$ is the magnetic field, N is the number of electrons in the quantum dot,η is the lever arm associated with the capacitance coupled to the gate, V_b is the Schottky barrier height, V_g is the gate voltage and m is the effective mass.

and

$$\begin{pmatrix} u(x) \\ v(x) \end{pmatrix} = \begin{pmatrix} u_o \exp(i\phi) \\ v_o \end{pmatrix} \exp(\pm\, ik_j^- x) \tag{1.6}$$

$$k_j^- = \left[\frac{2m}{\hbar^2}\left(E_F - (E^2 - \Delta_o^2)^{0.5} + V_o + V_b + \frac{U_C N^2}{2} + e\,\eta V_g N + (0.5\,\hbar\omega_c)\right) - \frac{j^2\pi^2}{W^2}\right]^{0.5}$$

with $+(-)$ corresponding to electronlike excitations which travel in negative or (positive) x direction.

The coherence factors are given by

$$u_o = \frac{1}{\sqrt{2}}\sqrt{1 + [1 - (\Delta_o/E)^2]^{0.5}} \tag{1.7}$$

$$v_o = \frac{1}{\sqrt{2}}\sqrt{1 - [1 - (\Delta_o/E)^2]^{0.5}} \tag{1.8}$$

The formalism we use is based on Buttiker description of phase-coherent of electron transport in multiprobe normal conductor [5]. The reflection probability $Rnm(\phi, E)$ is defined as current (in units of e) which flow back into the n^{th} reservoir as a result of the current which is emitted by the n^{th} channel at energy E. Similarly the transmission probability $Tnm(\phi, E)$is defined as the ratio of the current which flow into the n^{th}channel as a result of particle at energy E. Because the reservoirs can emit two types of particle, we add a superscript p, with $p = 1$ for electron (or electro-like) excitations and$p = 2$ for hole (or hole-like) excitations. The current density (current per unit energy) $J_n^{ex}(\phi, E)$ can be expressed as [19]:

$$J_n^{ex}(\phi, E) = \frac{2e}{h}\sum_{p=1.2} C_n^p(E) - R_{nn}^p(\phi, E) - \sum_{m\neq n} T_{nm}^p(\phi, E) \tag{1.9}$$

In equation (1.9), $C_n^p(E)$ indicates the ratio between the current and particle current carried by the excitations which are emitted by then^{th} channel, and $R_{nn}^p(\phi, E)$, $T_{nm}^p(\phi, E)$ are the transmission and reflection probapilities from different channels. The conductance for considered device could be expressed in terms of the transmission and reflection probabilitis of the various voltage probes [7, 4] :

$$G = \frac{e^2}{h}(\sum_{nm} T_{nm}(\phi, E)) \frac{2\sum_i v_i^{-1}}{\sum_i (1 + \sum_{nn} R_{nn}(\phi, E) - \sum_{nm} T_{nm}(\phi, E))v_i^{-1}} \tag{1.10}$$

Where v_i is the Fermi velocity in current direction in the i^{th} channel, defined by $(\hbar K_i/m)$ (K_i is the longitudinal Fermi wave vector of the i^{th} channel),ϕ is phase angle and T_{nm}, R_{nm} are the transmission and reflection probabilitis [12].

2 Results and Discussions

We investigate the dependence of the conductance and current density on the magnetic field, gate voltage and phase angle (see Figs 2,3 and 4 and also Fig.s 5,6 and 7). So, numerical calculations have been pereformed as follow:

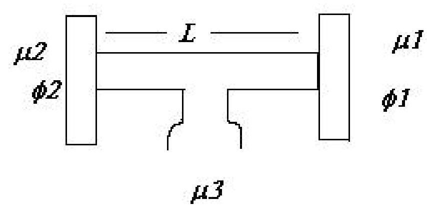

Fig. 1. Schematic represention of the model.

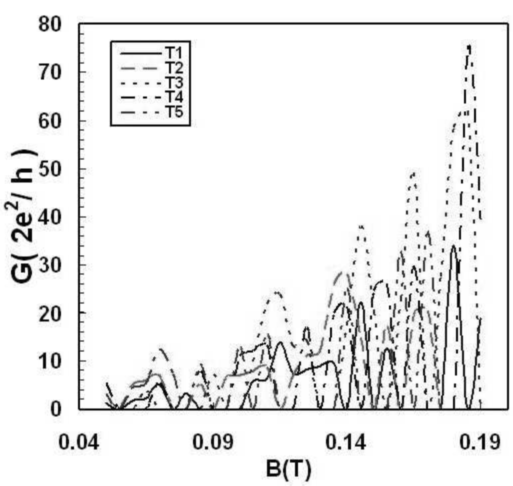

Fig. 2. The conductance-magnetic field dependence for different values of temperature T.

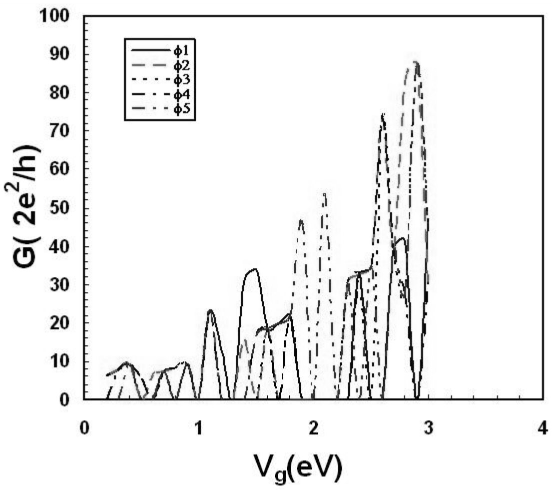

Fig. 3. The conductance-gate voltage dependence for different values of phase angle ϕ.

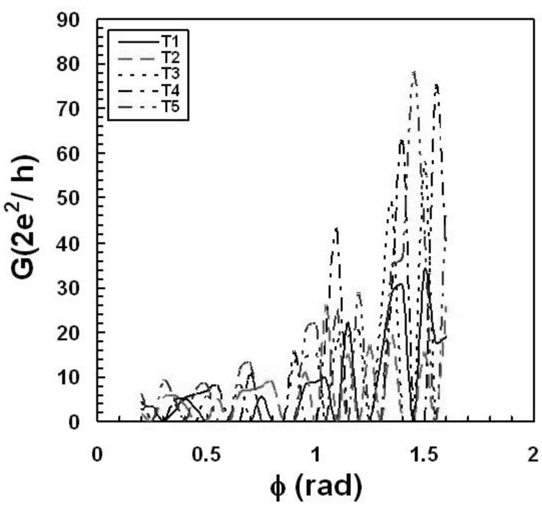

Fig. 4. The conductance-phase angle dependence for different values of temperature T.

The semiconducor heterostructure material was taken as $GaAs/AlGaAs$ and superconducting leads are niobium (Nb). In our calculations we consider the values of both mean free path and coherence length of quasiparticles are larger than the dimensions of the present mesoscopic device, so the transport will be treated as ballistic one. The electron transport through the device is treated as stochastic process, so that the tunneled electron energy has been taken as random number relative to the superconducting energy gap,Δ_o. Also, the Schottky barrier height V_b, is determined by using the Monte-Carlo-simulation technique and equals $0.47eV$. This value is in agreement with those found previously [2]-[4], [14]-[27].

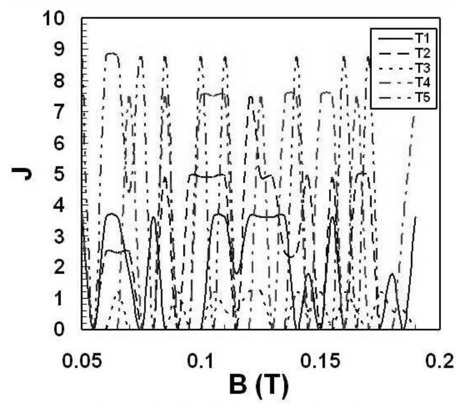

Fig. 5. The current density-magnetic field dependence for different values of temperature T.

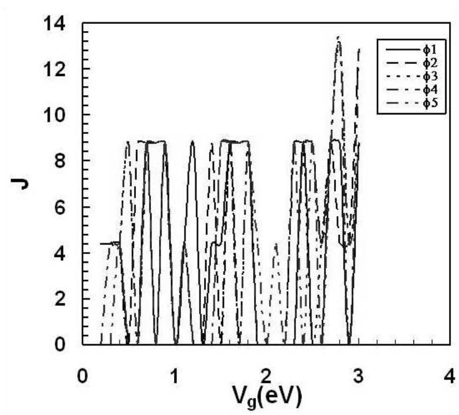

Fig. 6. The current density-gate voltage dependence for different values of phase angle ϕ.

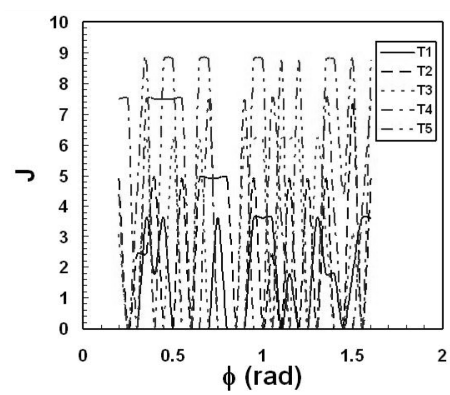

Fig. 7. The current density-phase angle dependence for different values of temperature T.

Our results show the following features:

1. As shown from (Figs 2,3 and 4), that the conductance G, oscillates as a functions of magnetic field B, gate voltage Vg, and phase angleϕ, and the oscillation peak, increases with increasing the above parameters(B, Vg,ϕ). These oscillations are due

to the Coulomb blockade effect [9] and the quantum interference of quasiparticles due to Andreev reflections processes at the semiconductor-superconductors interface [30].

2. Also from the Figs 5,6 and 7, the current density J, oscillates as functions of B, Vg, ϕ, but the oscillations in this case is random oscillation. The behavior may be explained according to resonant tunneling through the bound state formed within the quantum dot belonging to different Landau levels [31]-[3].

References

[1] A.F.Andreev,*Superconducting phase transitions in quantum dots*, JETP Lett. 64 (1996), 664-669.

[2] Arafa H. Aly and Adel H. Phillips, *Peltier effect of superconductor-semiconductor mesoscopic device*, Applied Sciences 7 (2005), 10-15.

[3] Attia A. Awad Alla and Adel H. Phillips, *Multiple Andreev reflection in quantum dot coupled weakly to superconducting reservoirs*, Egypt. J. Sol. 36 (2004), 129-138.

[4] Arafa. H. Aly and Adel H. Phillips, *Quantum transport in a superconductor-semiconductor mesoscopic system*, Phys. Stat. Sol.(b) , 232 (2002), 283-287.

[5] Attia A. Awad Alla , A. M. Hegazy, Adel H. Phillips and R.K. Wassef, *Transport propeerties of quantum interference device. The role of the Andreev states*", Egypt. J.Sol. 31, 3 (2000), 250-261.

[6] D.V. Averin,*Theory of single-electron charging of quantum dots*, Phys. Rev. B 44 (1991), 6199-6211.

[7] M. Buttiker, *Small normal-metal loop coupled to an electron reservoir* ,Phys. Rev. B 32 (1985), 1846-1849.

[8] M. Buttiker and T. M. Klapwijk, *Flux sensitivity of a piecewise normal and superconducting metal loop*, Phys. Rev. B 33 (1986), 5114-5117.

[9] S. De Franceschi, J. A. van Dam, E. P. A. M. Bakkers, L.F. Feiner, L. Gurevich, and L. P. Kouwenhoven, *Single-electron tunneling in InP nanowires*, Appl. Phys. Lett. 83 (2003), 344-346.

[10] P. G. De Gennes, *Superconductivity of Metals and Alloys*, Benjamin, New York, 1966.

[11] D. K. Ferry in *Transport in Nanostructure*, Cambridge University Press, Cambridge, 1997.

[12] A. Furusaki, H. Takayanagi and M. Tsukada, *Josepson effect of the superconducting quantum point contact*, Phys. Rev. B 45 (1992), 10563-10575.

[13] Y.Gefen, Y.Imry, and M.Ya Azbel, *Quantum oscillations in one-dimensional normal-metal rings*, Phys. Rev. A 30 (1984), 1982-1989.

[14] K. Held, E. Eisenberg, and B. L. Altshuler, *Random matrix theory for closed quantum dots with weak spin-orbit coupling*, Phys. Rev. Lett. 90 (2003), 106802-107000.

[15] H. van Houten, *Three-terminal quantum box resonant tunneling Josephson field-effect switch*, Appl. Phys. Lett. 58 (1991), 1326-1328.

[16] K. M. Lewis , C. Kurdak, S. Krishna and P. Bhattacharya , *Charge transformer to enhance noise performance of single-electron transistor amplifiers in high-capacitance applications*, Applied Physics Letters 80 (2002), 142-144.

[17] K. K. Likkarev," *Single-electron devices and their applications*", Proc. IEEE, 87 (1999), 606-632.

[18] J. B. Majer, F. G. Paauw, A. C. J. ter Haar, C. J. P. M. Harmans, and J. E. Mooij, *Spectroscopy on two coupled superconducting flux qubits*, Phys. Rev. Lett. 44 (2005), 090501.

[19] A. N. Mina, Adel H. Phillips, F. Shahin and N. S. AbdelGawad, *Quantum chaos in the conductance of mesoscopic system. Theory and Simulation approsch*, Physica C, 341-348, (2000), 301-303. Proc. IEEE, 87 (1999), 606-632.

[20] Y. Nakamura, Y. A. Pashkin, and J. S. Tasi, *Coherent control of macroscopic quantum states in a single-Cooper-pair box*, Nature 398 (1999), 786-788.

[21] M. A. Nielsen and I. L. Chuange, in *Quantum Computation and Quantum Information*, Cambridge, 2000.

[22] J. U. Nockel, *Conductance quantization and backscattering* , Phys. Rev. B 45 (1992), 14225-14230.

[23] U. Sivan and Y. Imry, *Multichannel Landauer formula for thermoelectric transport with application to thermopower near the mobility edge*, Phys. Rev. B 33 (1985), 551-558.

[24] C. G. Smith, *Low-dimensional quantum devices*, Rep. Prog. Phys. 59 (1996), 235-282.

[25] J. Tworzydlo, A.Tajic, and C. W. J. Beenakker, *Quantum-to-classical crossover of mesoscopic conductance fluctuations*, Phys. Rev. B 69, (2004), 165318-165323.

[26] R. Ugajin, *Quasiresonance due to electron-electron interaction in coupled quantum dots*, Applied Physics Letters 68 (1996), 2657-2659.

[27] L. M.K. Vandersypen, J. M. Elzeman, R. N. Schouten, L. H. Willem , van Beveren, R. Hanson and L. P. Kouwenhoven, *Real-time detection of single-electron tunneling using a quantum point contact*, Appl. Phys. Lett. 85 (2004), 4394-4396.

[28] B. J. van Wees , L. P. Kouwenhoven , C. J. P. M. Harmans, J. G. Williamson , C. E. Timmering , M. E. I. Broekaart , C.T. Foxon and J. J. Harris, *Observation of zero-Dimensional state in a one-Dimensional electron interferometer*, Phys. Rev. Lett. 62 (1989), 2523-2526.

[29] B. J. van Wees, K. M. H. Lenssen , and C. J. P. M. Harmans, *Transmission formalism for supercurrent flow in multiprobe superconductor-semiconductor-superconductor devices*", Phys. Rev. B 44 (1991), 470-473.

[30] W. G. van der Wiels, S. De Franceshi, J. M. Elzerman, T. Fujisawa, S.Tarucha, and L. P. Kouwenhoven, *Electron transport through double quantum dots*, Rev. Mod. Phys. 75 (2003), 1-22.

[31] F. K.Wilhelm and A. A. Golubov, *Mesoscopic proximity effect probed through superconducting tunneling contact*s, Phys. Rev. B 62 (2000), 5353-5356.

Author's address:

Attia.A.Awadalla
Physics Department, Faculty of Sciences,
Beni-Suef University, Egypt.
email: attiamd2005@yahoo.com