Unsteady flow of a conducting dusty fluid between two parallel plates

B.J.Gireesha, C.S.Bagewadi and P.Venkatesh

Abstract. The present problem deals with the laminar flow of an electrically conducting viscous incompressible fluid with embedded non-conducting identical spherical particles through a long channel under influence of a uniform magnetic field and taking the fluid and dust particles to be initially at rest. The changes in velocity profiles for fluid and dust particles have been determined and the effect of strength of magnetic field on velocity profiles at fixed time has been depicted graphically.

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Key words: Frenet frame field system, parallel plates, dusty fluid, velocity of dust phase and fluid phase, conducting dusty fluid, magnetic field.

§1. Introduction

The influence of dust particles on viscous flows has great importance in petroleum industry and in the purification of crude oil. Other important applications of dust particles in boundary layer, include soil erosion by natural winds and dust entrainment in a cloud during nuclear explosion. Also such flows have occur in a wide range of areas of technical importance like fludization, flow in rocket tubes, combustion, paint spraying and more recently blood flows in capillaries.

Considerable work has already been done on such models of dusty fluid flow. P.G.Saffman [12] formulated the basic equations for the flow of dusty fluid. Regarding the plate problems, Lokenath Debnath et al [9], Liu [8], Michael et al [10] have studied the flow produced by the motion of an infinite plane in a steady fluid occupying the semi-infinite space above it. Later, M.C.Baral [4] has discussed the plane parallel flow of conducting dusty gas. To investigate the kinematical properties of fluid flows in the field of fluid mechanics some researchers like Kanwal [7], Truesdell [13], Indrasena [6], Purushotham et al [11], Bagewadi et al [1, 2, 3] have applied differential geometry techniques. Further, recently the authors [2, 3] have studied two-dimensional dusty fluid flow in Frenet frame field system. The present paper considers the flow of a conducting viscous incompressible fluid with embedded nonconducting identical spherical particles between two infinite parallel plates in which one plate is moving

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with the uniform speed under the influence of constant magnetic field. Initially the fluid and dust particles are at rest. Finally the changes in the velocity profiles at of fluid and dust particles are shown graphically.

§2. Equations of motion

The equations of motion of unsteady viscous incompressible fluid with uniform distribution of dust particles are given by

For fluid phase

$$\nabla.\overrightarrow{u} = 0 \tag{2.2.1}$$

$$\frac{\partial\overrightarrow{u}}{\partial t} + (\overrightarrow{u}.\nabla)\overrightarrow{u} = -\rho^{-1}\nabla p + \upsilon\nabla^2\overrightarrow{u} + \frac{kN}{\rho}(\overrightarrow{v} - \overrightarrow{u}) + \frac{1}{\rho}(\overrightarrow{J} \times \overrightarrow{B})$$

For dust phase

$$\nabla.\overrightarrow{v} = 0 \tag{2.2.2}$$

$$\frac{\partial\overrightarrow{v}}{\partial t} + (\overrightarrow{v}.\nabla)\overrightarrow{v} = \frac{k}{m}(\overrightarrow{u} - \overrightarrow{v}) \tag{2.2.3}$$

We have following nomenclature:

$\overrightarrow{u}-$velocity of the fluid phase, $\overrightarrow{v}-$velocity density of dust phase, $\rho-$density of the gas, $p-$pressure of the fluid, $N-$number of density of dust particles, $\upsilon-$kinematic viscosity, $k = 6\pi a\mu-$Stoke's resistance (drag coefficient), $a-$spherical radius of dust particle, $m-$mass of the dust particle, $\mu-$the co-efficient of viscosity of fluid particles, $t-$time and $\overrightarrow{J}$ and $\overrightarrow{B}$ given by Maxwell's equations and Ohm's law, namely,

$$\nabla \times \overrightarrow{H} = 4\pi\overrightarrow{J},\ \nabla \times \overrightarrow{B} = 0,\ \nabla \times \overrightarrow{E} = 0,\ \overrightarrow{J} = \sigma[\overrightarrow{E} + \overrightarrow{u} \times \overrightarrow{B}]$$

Here $\overrightarrow{H}-$magnetic field, $\overrightarrow{J}-$current density, $\overrightarrow{B}-$magnetic flux, $\overrightarrow{E}$-electric field.

It is assumed that the effect of induced magnetic fields produced by the motion of the electrically conducting gas is negligible and no external electric field is applied. With those assumptions the magnetic field $\overrightarrow{J} \times \overrightarrow{B}$ of the body force in (2.2.2) reduces simply to $-\sigma B_0^2\overrightarrow{u}$.

Let $\overrightarrow{s}$, $\overrightarrow{n}$, $\overrightarrow{b}$ be triply orthogonal unit vectors tangent, principal normal, binormal respectively to the spatial curves of congruences formed by fluid phase velocity and dusty phase velocity lines respectively, Geometrical relations are given by Frenet formulae [5]

$$(2.2.4)\quad \begin{array}{ll} i) & \dfrac{\partial \overrightarrow{s}}{\partial s} = k_s \overrightarrow{n},\ \dfrac{\partial \overrightarrow{n}}{\partial s} = \tau_s \overrightarrow{b} - k_s \overrightarrow{s},\ \dfrac{\partial \overrightarrow{b}}{\partial s} = -\tau_s \overrightarrow{n} \\ ii) & \dfrac{\partial \overrightarrow{n}}{\partial n} = k'_n \overrightarrow{s},\ \dfrac{\partial \overrightarrow{b}}{\partial n} = -\sigma'_n \overrightarrow{s},\ \dfrac{\partial \overrightarrow{s}}{\partial n} = \sigma'_n \overrightarrow{b} - k'_n \overrightarrow{n} \\ iii) & \dfrac{\partial \overrightarrow{b}}{\partial b} = k''_b \overrightarrow{s},\ \dfrac{\partial \overrightarrow{n}}{\partial b} = -\sigma''_b \overrightarrow{s},\ \dfrac{\partial \overrightarrow{s}}{\partial b} = \sigma''_b \overrightarrow{n} - k''_b \overrightarrow{b} \\ iv) & \nabla.\overrightarrow{s} = \theta_{ns} + \theta_{bs};\ \nabla.\overrightarrow{n} = \theta_{bn} - k_s;\ \nabla.\overrightarrow{b} = \theta_{nb} \end{array}$$

where $\partial/\partial s$, $\partial/\partial n$ and $\partial/\partial b$ are the intrinsic differential operators along fluid phase velocity (or dust phase velocity) lines, principal normal and binormal. The functions (k_s, k'_n, k''_b) and $(\tau_s, \sigma'_n, \sigma''_b)$ are the curvatures and torsion of the above curves and θ_{ns} and θ_{bs} are normal deformations of these spatial curves along their principal normal and binormal respectively.

§3. Formulation and Solution of the Problem

The present discussion considers a viscous incompressible, dusty fluid bounded by two infinite flat plates in which one plate is moving with the constant speed u_0. Both the fluid and the dust particle clouds are supposed to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size. The number density of the particles is taken as a constant throughout the flow. It is assumed that the dust particles are electrically nonconducting and neutral. The motion of the dusty fluid is due to magnetic field of constant strength B_0. Under these assumptions the flow will be a parallel flow in which the streamlines are along the tangential direction and the velocities are varies along binormal direction and with time t, since we extended the fluid to infinity in the principal normal direction.

By virtue of system of equations (2.2.4) the intrinsic decomposition of equations (2.2.2) and (2.2.3) give the following forms;

$$(3.3.5)\quad \frac{\partial u_s}{\partial t} = \nu \left[\frac{\partial^2 u_s}{\partial b^2} - C_r u_s\right] + \frac{kN}{\rho}(v_s - u_s) - Du_s$$

$$(3.3.6)\quad 2u_s^2 k_s = \nu \left[2\sigma''_b \frac{\partial u_s}{\partial b} - u_s k_s^2\right]$$

$$(3.3.7)\quad 0 = \nu \left[u_s k_s \tau_s - 2k''_b \frac{\partial u_s}{\partial b}\right]$$

$$(3.3.8)\quad \frac{\partial v_s}{\partial t} = \frac{k}{m}(u_s - v_s)$$

$$(3.3.9)\quad 2v_s^2 k_s = 0$$

where $D = \frac{\sigma B_0^2}{\rho}$ and $C_r = (\sigma'^2_b + k'^2_n + k'^2_b + \sigma''^2_b)$ is called curvature number [3].

From equation (3.3.9) we see that $v_s^2 k_s = 0$ which implies either $v_s = 0$ or $k_s = 0$. The choice $v_s = 0$ is impossible, since if it happens then $u_s = 0$, which shows that the flow doesn't exist. Hence $k_s = 0$, it suggests that the curvature of the streamline along tangential direction is zero. Thus no radial flow exists.

Equation (3.3.5) and (3.3.8) are to be solved subject to the initial and boundary conditions;

$$\left\{\begin{array}{lll} \text{Initial condition;} & \text{at } t = 0; u_s = 0, v_s = 0 \\ \text{Boundary condition;} & \text{for } t > 0; u_s = 0, \text{ at } b = 0 \text{ and} \\ & u_s = u_0 \text{ at } b = h \end{array}\right\} \tag{3.3.10}$$

We define Laplace transformations of u_s and v_s as

$$U = \int_0^\infty e^{-st} u_s dt \quad \text{and} \quad V = \int_0^\infty e^{-st} v_s dt \tag{3.3.11}$$

Applying the Laplace transform to equations (3.3.5), (3.3.8) and to boundary conditions 3.3.10, then by using initial conditions one obtains

$$sU = \nu\left[\frac{\partial^2 U}{\partial b^2} - C_r U\right] + \frac{L}{\tau}(V - U) - DU \tag{3.3.12}$$

$$sV = \frac{1}{\tau}(U - V) \tag{3.3.13}$$

$$U = 0, \text{ at } b = 0 \quad U = \frac{u_0}{s} \text{ at } b = h \tag{3.3.14}$$

where $L = \frac{mN}{\rho}$ and $\tau = \frac{m}{k}$. Equation (3.3.13) implies

$$V = \frac{U}{1 + s\tau} \tag{3.3.15}$$

Eliminating V from (3.3.12) and (3.3.15) we obtain the following equation

$$\frac{d^2 U}{db^2} - Q^2 U = 0 \tag{3.3.16}$$

where $Q^2 = \left(C_r + \frac{s}{\nu} + \frac{D}{\nu} + \frac{sL}{\nu(1+s\tau)}\right)$.

The velocities of fluid and dust particle are obtained by solving the equation (3.3.16) subjected to the boundary conditions (3.3.14) as follows

$$U = \frac{u_0}{s}\left\{\frac{sinhQb}{sinhQh}\right\} .$$

Using U in (3.3.15) we obtain V as

$$V = \frac{u_0}{s(1 + s\tau)}\left\{\frac{sinhQb}{sinhQh}\right\} .$$

By taking inverse Laplace transform to U and V, one can obtain

$$\begin{aligned}
u_s &= u_0\frac{sinhyb}{sinhyh} + \frac{2u_0\pi\nu}{h}\sum_{n=0}^{\infty} n(-1)^{n+1} sin\left(\frac{n\pi}{h}b\right) \\
&\quad \times \left[\frac{e^{x_1 t}(1+x_1\tau)^2}{x_1\left[(1+x_1\tau)^2+L\right]} + \frac{e^{x_2 t}(1+x_2\tau)^2}{x_2\left[(1+x_2\tau)^2+L\right]}\right] \\
v_s &= u_0\frac{sinhyb}{sinhyh} - u_0 e^{-\frac{t}{\tau}} + \frac{2u_0\pi\nu}{h}\sum_{n=0}^{\infty} n(-1)^{n+1} sin\left(\frac{n\pi}{h}b\right) \\
&\quad \times \left[\frac{e^{x_1 t}(1+x_1\tau)}{x_1\left[(1+x_1\tau)^2+L\right]} + \frac{e^{x_2 t}(1+x_2\tau)}{x_2\left[(1+x_2\tau)^2+L\right]}\right]
\end{aligned}$$

where

$$\begin{aligned}
x_1 &= -\frac{1}{2\tau}\left(1+L+D\tau+\nu C_r\tau+\nu\tau\frac{n^2\pi^2}{h^2}\right) \\
&\quad +\frac{1}{2\tau}\sqrt{\left(1+L+D\tau+\nu C_r\tau+\nu\tau\frac{n^2\pi^2}{h^2}\right)^2 - 4\tau\left(C_r\nu+D+\nu\frac{n^2\pi^2}{h^2}\right)} \\
x_2 &= -\frac{1}{2\tau}\left(1+L+D\tau+\nu C_r\tau+\nu\tau\frac{n^2\pi^2}{h^2}\right) \\
&\quad -\frac{1}{2\tau}\sqrt{\left(1+L+D\tau+\nu C_r\tau+\nu\tau\frac{n^2\pi^2}{h^2}\right)^2 - 4\tau\left(C_r\nu+D+\nu\frac{n^2\pi^2}{h^2}\right)} \\
y &= \frac{C_r\nu+D}{\nu}
\end{aligned}$$

§4. Conclusions

The velocity profiles for the fluid and dust particles are drawn in figure 1 and 2 respectively, which are parabolic. According Frenet approximation of a curve in the osculating plane the path of the curve near origin is parabolic. Hence the results obtained here are analogous to the above [5]. It is concluded that velocity of fluid particles is parallel to velocity of dust particles. Also it is evident from the graphs that, as we increase the strength of the magnetic field, it has an appreciable effect on the velocities of fluid and dust particles. Further one can observe that if the magnetic field is zero then results are in agreement with the plane Couette flow.

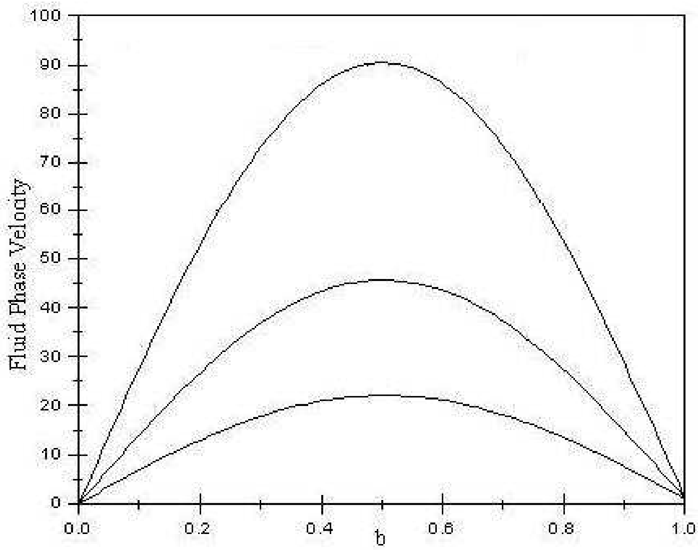

Figure-1: Variation of fluid velocity with b

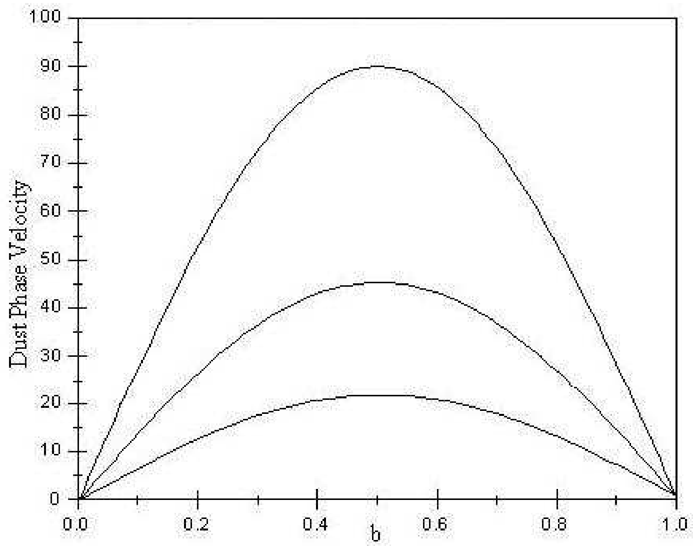

Figure-2: Variation of dust phase velocity with b

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