

# Anterior arch circumference adjustment - how much?

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In clinical practice, when treatment has progressed to the stage where an Angle Class 1 buccal occlusion has been reached or nearly reached, it sometimes becomes obvious that normal incisor overjet will not be possible without some spacing of the maxillary or mandibular incisors. In such cases, if the curve of Spee and the interincisal angle are acceptable, it may be concluded that a Bolton<sup>1</sup> discrepancy is present. Interproximal stripping in the arch that demonstrates excessive tooth material is then often prescribed. In other cases, space must be created for a missing lateral incisor or for a peg-shaped lateral that must be restored to its correct mesiodistal dimensions.

When constructing a Visual Treatment Objective (VTO),<sup>2,3</sup> the question that sometimes arises is how much sagittal displacement of the incisors will take place during the elimination of anterior crowding, spacing, or overjet reduction. Experiments led one of the authors to the real-

ization that the simple formula most clinicians use when constructing a VTO, which states that 1 mm sagittal change will need 2 mm of arch circumference compensation,<sup>4</sup> does not hold true for the anterior teeth. This is because the incisors are arranged across the apex of a curved archform. Posterior to the canines, this two-to-one formula is of acceptable accuracy for the average archform, because the buccal teeth of the two sides are nearly parallel to one another.

Showfety and Baker<sup>5</sup> presented tables that allowed the practitioner to determine the effect of incisor and canine movement on arch circumference. Their calculations were based on a circular anterior arch form. However, a more recent study by Jones and Richmond<sup>6</sup> revealed that a parabolic shape fits the natural dental arch best. Based on the assumption of a parabolic anterior arch form, it was decided to generate a reference table whereby changes in arch length and arch circumference could be accurately assessed.

## Abstract

A table is presented by means of which changes in arch length and arch circumference can be more accurately predicted, thereby simplifying treatment planning for cases with special problems, e.g. a Bolton discrepancy. A fundamental condition for the use of this table is that the anterior arch from canine to canine forms a parabolic curve.

## Keywords

Space analysis • Interproximal stripping • Overjet correction • Bolton discrepancy • Missing teeth

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**Table 1**  
**Arch length and arch circumference changes at intercanine widths of 18 - 46 mm (2 mm increments)**

Space shortage in k (mm)										Spacing in k (mm)											
b	a	-8	-7	-6	-5	-4	-3	-2	-1	b	a	+1	+2	+3	+4	+5	+6	+7	+8		
18	4	6.0	5.3	4.7	4.0	3.3	2.6	1.8	0.9	18	4	-1.1	-3.0								
	6	5.4	4.8	4.2	3.5	2.9	2.2	1.5	0.8		6	-0.8	-1.7	-2.8	-4.3	-6.0					
	8	5.0	4.4	3.8	3.2	2.6	2.0	1.3	0.7		8	-0.7	-1.4	-2.2	-3.1	-4.0	-5.2	-7.0			
	10	4.8	4.2	3.6	3.1	2.5	1.9	1.3	0.6		10	-0.6	-1.3	-2.0	-2.7	-3.4	-4.2	-5.0	-6.0		
	12	4.6	4.1	3.5	2.9	2.4	1.8	1.2	0.6		12	-0.6	-1.2	-1.9	-2.5	-3.2	-3.8	-4.5	-5.2		
	14	4.5	4.0	3.4	2.9	2.3	1.7	1.2	0.6		14	-0.6	-1.2	-1.8	-2.4	-3.0	-3.6	-4.3	-4.9		
	16	4.4	3.9	3.3	2.8	2.2	1.7	1.1	0.6		6	-0.6	-1.1	-1.7	-2.3	-2.9	-3.5	-4.1	-4.7		
18	4.4	3.8	3.3	2.7	2.2	1.7	1.1	0.6	8	-0.6	-1.1	-1.7	-2.2	-2.8	-3.4	-4.0	-4.6				
20	4.3	3.8	3.3	2.7	2.2	1.6	1.1	0.5	0	-0.5	-1.1	-1.7	-2.2	-2.8	-3.3	-3.9	-4.5				
20	4	6.3	5.6	4.9	4.2	3.5	2.7	1.9	1.0	20	4	-1.3									
	6	5.6	5.0	4.3	3.7	3.0	2.3	1.6	0.8		6	-0.9	-1.9	-3.1	-5.2	-6.0					
	8	5.2	4.6	4.0	3.4	2.7	2.1	1.4	0.7		8	-0.7	-1.5	-2.4	-3.3	-4.3	-5.7	-8.0			
	10	4.9	4.3	3.8	3.1	2.5	1.9	1.3	0.7		10	-0.7	-1.4	-2.1	-2.8	-3.6	-4.4	-5.4	-6.4		
	12	4.8	4.2	3.6	3.0	2.4	1.8	1.2	0.6		12	-0.6	-1.3	-1.9	-2.6	-3.3	-4.0	-4.7	-5.5		
	14	4.6	4.1	3.5	2.9	2.3	1.8	1.2	0.6		14	-0.6	-1.2	-1.8	-2.4	-3.1	-3.7	-4.4	-5.1		
	16	4.5	4.0	3.4	2.8	2.3	1.7	1.1	0.6		16	-0.6	-1.2	-1.8	-2.4	-3.0	-3.6	-4.2	-4.8		
18	4.4	3.9	3.3	2.8	2.2	1.7	1.1	0.6	18	-0.6	-1.1	-1.7	-2.3	-2.9	-3.5	-4.1	-4.7				
20	4.4	3.8	3.3	2.8	2.2	1.7	1.1	0.6	20	-0.6	-1.1	-1.7	-2.2	-2.8	-3.4	-4.0	-4.5				
22	4	6.5	5.8	5.1	4.4	3.7	2.9	2.0	1.1	22	4	-1.4									
	6	5.8	5.2	4.5	3.8	3.1	2.4	1.6	0.9		6	-0.9	-2.0	-3.4	-6.0						
	8	5.4	4.8	4.1	3.5	2.8	2.1	1.5	0.7		8	-0.8	-1.6	-2.5	-3.5	-4.7	-6.5	-8.0			
	10	5.1	4.5	3.9	3.2	2.6	2.0	1.3	0.7		10	-0.7	-1.4	-2.2	-2.9	-3.8	-4.7	-5.7	-7.0		
	12	4.9	4.3	3.7	3.1	2.5	1.9	1.3	0.6		12	-0.6	-1.3	-2.0	-2.7	-3.4	-4.1	-4.9	-5.8		
	14	4.7	4.1	3.6	3.0	2.4	1.8	1.2	0.6		14	-0.6	-1.2	-1.9	-2.5	-3.2	-3.8	-4.5	-5.2		
	16	4.6	4.0	3.5	2.9	2.3	1.8	1.2	0.6		16	-0.6	-1.2	-1.8	-2.4	-3.0	-3.7	-4.3	-5.0		
18	4.5	4.0	3.4	2.8	2.3	1.7	1.1	0.6	18	-0.6	-1.2	-1.7	-2.3	-2.9	-3.5	-4.1	-4.8				
20	4.4	3.9	3.3	2.8	2.2	1.7	1.1	0.6	20	-0.6	-1.1	-1.7	-2.3	-2.9	-3.4	-4.0	-4.6				
24	4	6.8	6.1	5.4	4.6	3.8	3.0	2.1	1.1	24	4	-1.5	-4.0								
	6	6.0	5.4	4.7	4.0	3.3	2.5	1.7	0.9		6	-1.0	-2.2	-3.8	-6.0						
	8	5.5	4.9	4.3	3.6	2.9	2.2	1.5	0.8		8	-0.8	-1.7	-2.6	-3.7	-5.1	-8.0				
	10	5.2	4.6	4.0	3.3	2.7	2.0	1.4	0.7		10	-0.7	-1.5	-2.3	-3.1	-4.0	-5.0	-6.1	-7.7		
	12	5.0	4.4	3.8	3.2	2.6	1.9	1.3	0.7		12	-0.7	-1.4	-2.1	-2.8	-3.5	-4.3	-5.1	-6.0		
	14	4.8	4.2	3.6	3.0	2.4	1.8	1.2	0.6		14	-0.6	-1.3	-1.9	-2.6	-3.3	-4.0	-4.7	-5.4		
	16	4.7	4.1	3.5	3.0	2.4	1.8	1.2	0.6		16	-0.6	-1.2	-1.8	-2.5	-3.1	-3.8	-4.4	-5.1		
18	4.6	4.0	3.5	2.9	2.3	1.7	1.2	0.6	18	-0.6	-1.2	-1.8	-2.4	-3.0	-3.6	-4.2	-4.9				
20	4.5	3.9	3.4	2.8	2.3	1.7	1.1	0.6	20	-0.6	-1.2	-1.7	-2.3	-2.9	-3.5	-4.1	-4.7				
26	4	7.0	6.3	5.6	4.8	4.0	3.2	2.2	1.2	26	4	-1.6									
	6	6.2	5.6	4.9	4.1	3.4	2.6	1.8	0.9		6	-1.1	-2.3	-4.2							
	8	5.7	5.1	4.4	3.7	3.0	2.3	1.6	0.8		8	-0.9	-1.8	-2.8	-4.0	-5.7					
	10	5.4	4.7	4.1	3.4	2.8	2.1	1.4	0.7		10	-0.8	-1.5	-2.4	-3.2	-4.2	-5.3	-6.6	-8.9		
	12	5.1	4.5	3.9	3.3	2.6	2.0	1.3	0.7		12	-0.7	-1.4	-2.1	-2.9	-3.7	-4.5	-5.4	-6.4		
	14	4.9	4.3	3.7	3.1	2.5	1.9	1.3	0.6		14	-0.6	-1.3	-2.0	-2.7	-3.4	-4.1	-4.9	-5.6		
	16	4.8	4.2	3.6	3.0	2.4	1.8	1.2	0.6		16	-0.6	-1.2	-1.9	-2.5	-3.2	-3.9	-4.5	-5.3		
18	4.7	4.1	3.5	2.9	2.4	1.8	1.2	0.6	18	-0.6	-1.2	-1.8	-2.4	-3.1	-3.7	-4.3	-5.0				
20	4.6	4.0	3.4	2.9	2.3	1.7	1.2	0.6	20	-0.6	-1.2	-1.8	-2.4	-3.0	-3.6	-4.2	-4.8				
28	4	7.3	6.5	5.8	5.0	4.2	3.3	2.3	1.3	28	4	-1.8	-4.0								
	6	6.5	5.8	5.0	4.3	3.5	2.7	1.9	1.0		6	-1.1	-2.5	-4.9							
	8	5.9	5.2	4.5	3.8	3.1	2.4	1.6	0.8		8	-0.9	-1.9	-3.0	-4.3	-6.4	-8.0				
	10	5.5	4.9	4.2	3.5	2.9	2.2	1.5	0.7		10	-0.8	-1.6	-2.5	-3.4	-4.4	-5.6	-7.2			
	12	5.2	4.6	4.0	3.3	2.7	2.0	1.4	0.7		12	-0.7	-1.4	-2.2	-3.0	-3.8	-4.7	-5.6	-6.7		
	14	5.0	4.4	3.8	3.2	2.6	1.9	1.3	0.7		14	-0.7	-1.3	-2.0	-2.8	-3.5	-4.2	-5.0	-5.9		
	16	4.9	4.3	3.7	3.1	2.5	1.9	1.2	0.6		16	-0.6	-1.3	-1.9	-2.6	-3.3	-4.0	-4.7	-5.4		
18	4.7	4.2	3.6	3.0	2.4	1.8	1.2	0.6	18	-0.6	-1.2	-1.9	-2.5	-3.1	-3.8	-4.4	-5.1				
20	4.6	4.1	3.5	2.9	2.3	1.8	1.2	0.6	20	-0.6	-1.2	-1.8	-2.4	-3.0	-3.7	-4.3	-4.9				
30	4	7.5	6.8	6.0	5.2	4.3	3.4	2.4	1.3	30	4	-2.0	-4.0								
	6	6.7	6.0	5.2	4.5	3.7	2.8	2.0	1.0		6	-1.2	-2.7	-6.0							
	8	6.1	5.4	4.7	4.0	3.2	2.5	1.7	0.9		8	-0.9	-2.0	-3.1	-4.6	-8.0					
	10	5.6	5.0	4.3	3.7	3.0	2.3	1.5	0.8		10	-0.8	-1.7	-2.6	-3.6	-4.7	-6.0	-8.0			
	12	5.3	4.7	4.1	3.4	2.8	2.1	1.4	0.7		12	-0.7	-1.5	-2.3	-3.1	-4.0	-4.9	-5.9	-7.1		
	14	5.1	4.5	3.9	3.3	2.6	2.0	1.3	0.7		14	-0.7	-1.4	-2.1	-2.8	-3.6	-4.4	-5.2	-6.1		
	16	4.9	4.3	3.7	3.1	2.5	1.9	1.3	0.6		16	-0.7	-1.3	-2.0	-2.7	-3.4	-4.1	-4.8	-5.6		
18	4.8	4.2	3.6	3.0	2.4	1.8	1.2	0.6	18	-0.6	-1.3	-1.9	-2.5	-3.2	-3.9	-4.6	-5.3				
20	4.7	4.1	3.5	3.0	2.4	1.8	1.2	0.6	20	-0.6	-1.2	-1.8	-2.5	-3.1	-3.7	-4.4	-5.0				

k =arch circumference (mm); a=arch length (mm); b=intercanine width (mm)

**Table 1, continued**  
**Arch length and arch circumference changes at intercanine widths of 18 - 46 mm (2 mm increments)**

Space shortage in k (mm)										Spacing in k (mm)											
b	a	-8	-7	-6	-5	-4	-3	-2	-1	b	a	+1	+2	+3	+4	+5	+6	+7	+8		
32	4	7.8	7.0	6.2	5.4	4.5	3.6	2.6	1.4	32	4	-2.1									
	6	6.9	6.1	5.4	4.6	3.8	3.0	2.1	1.1		6	-1.3	-2.9								
	8	6.2	5.6	4.8	4.1	3.4	2.6	1.8	0.9		8	-1.0	-2.1	-3.3	-5.0						
	10	5.8	5.1	4.4	3.8	3.0	2.3	1.6	0.8		10	-0.8	-1.7	-2.7	-3.7	-4.9	-6.4	-9.5			
	12	5.5	4.8	4.2	3.5	2.8	2.2	1.5	0.7		12	-0.8	-1.5	-2.4	-3.2	-4.1	-5.1	-6.2	-7.5		
	14	5.2	4.6	4.0	3.3	2.7	2.0	1.4	0.7		14	-0.7	-1.4	-2.2	-2.9	-3.7	-4.5	-5.4	-6.3		
	16	5.0	4.4	3.8	3.2	2.6	1.9	1.3	0.7		16	-0.7	-1.3	-2.0	-2.7	-3.5	-4.2	-5.0	-5.8		
18	4.9	4.3	3.7	3.1	2.5	1.9	1.3	0.6	18	-0.6	-1.3	-1.9	-2.6	-3.3	-4.0	-4.7	-5.4				
20	4.8	4.2	3.6	3.0	2.4	1.8	1.2	0.6	20	-0.6	-1.2	-1.9	-2.5	-3.2	-3.8	-4.5	-5.1				
34	4	8.0	7.2	6.4	5.6	4.7	3.7	2.7	1.5	34	4	-2.3									
	6	7.1	6.3	5.6	4.8	3.9	3.1	2.1	1.1		6	-1.3	-3.1								
	8	6.4	5.7	5.0	4.2	3.5	2.7	1.8	0.9		8	-1.0	-2.2	-3.5	-5.5	-8.0					
	10	5.9	5.3	4.6	3.9	3.1	2.4	1.6	0.8		10	-0.9	-1.8	-2.8	-3.9	-5.2	-7.0				
	12	5.6	4.9	4.3	3.6	2.9	2.2	1.5	0.8		12	-0.8	-1.6	-2.4	-3.3	-4.3	-5.4	-6.6	-8.0		
	14	5.3	4.7	4.1	3.4	2.7	2.1	1.4	0.7		14	-0.7	-1.5	-2.2	-3.0	-3.8	-4.7	-5.6	-6.6		
	16	5.1	4.5	3.9	3.3	2.6	2.0	1.3	0.7		16	-0.7	-1.4	-2.1	-2.8	-3.6	-4.3	-5.1	-5.9		
18	5.0	4.4	3.8	3.2	2.5	1.9	1.3	0.6	18	-0.7	-1.3	-2.0	-2.7	-3.4	-4.1	-4.8	-5.5				
20	4.9	4.3	3.7	3.1	2.5	1.9	1.2	0.6	20	-0.6	-1.3	-1.9	-2.6	-3.2	-3.9	-4.6	-5.3				
36	4	8.3	7.5	6.6	5.8	4.8	3.9	2.8	1.5	36	4	-2.6	-4.0								
	6	7.3	6.5	5.7	4.9	4.1	3.2	2.2	1.2		6	-1.4	-3.4	-6.0							
	8	6.6	5.9	5.1	4.4	3.6	2.7	1.9	1.0		8	-1.1	-2.3	-3.8	-6.0	-8.0					
	10	6.1	5.4	4.7	4.0	3.2	2.5	1.7	0.9		10	-0.9	-1.9	-2.9	-4.1	-5.5	-7.6				
	12	5.7	5.1	4.4	3.7	3.0	2.3	1.5	0.8		12	-0.8	-1.6	-2.5	-3.5	-4.5	-5.6	-6.9	-8.6		
	14	5.4	4.8	4.1	3.5	2.8	2.1	1.4	0.7		14	-0.7	-1.5	-2.3	-3.1	-4.0	-4.9	-5.8	-6.9		
	16	5.2	4.6	4.0	3.3	2.7	2.0	1.4	0.7		16	-0.7	-1.4	-2.1	-2.9	-3.6	-4.4	-5.3	-6.1		
18	5.1	4.4	3.8	3.2	2.6	1.9	1.3	0.7	18	-0.7	-1.3	-2.0	-2.7	-3.4	-4.2	-4.9	-5.7				
20	4.9	4.3	3.7	3.1	2.5	1.9	1.3	0.6	20	-0.6	-1.3	-1.9	-2.6	-3.3	-4.0	-4.7	-5.4				
38	4	8.5	7.7	6.8	5.9	5.0	4.0	2.9	1.6	38	4	-2.8	-4.0								
	6	7.5	6.7	5.9	5.1	4.2	3.3	2.3	1.2		6	-1.5	-3.6								
	8	6.8	6.0	5.3	4.5	3.7	2.8	2.0	1.0		8	-1.1	-2.4	-4.0	-6.8						
	10	6.3	5.5	4.8	4.1	3.3	2.5	1.7	0.9		10	-0.9	-1.9	-3.1	-4.3	-5.9	-8.5				
	12	5.9	5.2	4.5	3.8	3.1	2.3	1.6	0.8		12	-0.8	-1.7	-2.6	-3.6	-4.7	-5.9	-7.3	-9.3		
	14	5.6	4.9	4.2	3.6	2.9	2.2	1.5	0.7		14	-0.8	-1.5	-2.4	-3.2	-4.1	-5.0	-6.1	-7.2		
	16	5.3	4.7	4.0	3.4	2.7	2.1	1.4	0.7		16	-0.7	-1.4	-2.2	-3.0	-3.8	-4.6	-5.4	-6.3		
18	5.1	4.5	3.9	3.3	2.6	2.0	1.3	0.7	18	-0.7	-1.4	-2.1	-2.8	-3.5	-4.3	-5.0	-5.8				
20	5.0	4.4	3.8	3.2	2.5	1.9	1.3	0.6	20	-0.7	-1.3	-2.0	-2.7	-3.3	-4.1	-4.8	-5.5				
40	4	8.7	7.9	7.0	6.1	5.2	4.1	3.0	1.7	40	4	-3.2	-4.0								
	6	7.7	6.9	6.1	5.2	4.3	3.4	2.4	1.3		6	-1.5	-3.9	-6.0							
	8	7.0	6.2	5.4	4.6	3.8	2.9	2.0	1.0		8	-1.2	-2.5	-4.2							
	10	6.4	5.7	4.9	4.2	3.4	2.6	1.8	0.9		10	-1.0	-2.0	-3.2	-4.5	-6.3	-10				
	12	6.0	5.3	4.6	3.9	3.1	2.4	1.6	0.8		12	-0.9	-1.8	-2.7	-3.7	-4.9	-6.2	-7.7	-10.4		
	14	5.7	5.0	4.3	3.6	2.9	2.2	1.5	0.8		14	-0.8	-1.6	-2.4	-3.3	-4.2	-5.2	-6.3	-7.5		
	16	5.4	4.8	4.1	3.5	2.8	2.1	1.4	0.7		16	-0.7	-1.5	-2.2	-3.0	-3.9	-4.7	-5.6	-6.5		
18	5.2	4.6	4.0	3.3	2.7	2.0	1.4	0.7	18	-0.7	-1.4	-2.1	-2.9	-3.6	-4.4	-5.2	-6.0				
20	5.1	4.5	3.8	3.2	2.6	1.9	1.3	0.7	20	-0.7	-1.3	-2.0	-2.7	-3.4	-4.1	-4.9	-5.6				
42	4	8.9	8.1	7.2	6.3	5.3	4.3	3.1	1.7	42	4	-4.0									
	6	7.9	7.1	6.3	5.4	4.5	3.5	2.5	1.3		6	-1.6	-4.3								
	8	7.1	6.4	5.6	4.8	3.9	3.0	2.1	1.1		8	-1.2	-2.6	-4.5	-8.0						
	10	6.6	5.8	5.1	4.3	3.5	2.7	1.8	0.9		10	-1.0	-2.1	-3.3	-4.8	-6.7					
	12	6.1	5.4	4.7	4.0	3.2	2.5	1.7	0.8		12	-0.9	-1.8	-2.8	-3.9	-5.1	-6.5	-8.2	-12.0		
	14	5.8	5.1	4.4	3.7	3.0	2.3	1.5	0.8		14	-0.8	-1.6	-2.5	-3.4	-4.4	-5.4	-6.5	-7.8		
	16	5.5	4.9	4.2	3.5	2.8	2.2	1.4	0.7		16	-0.7	-1.5	-2.3	-3.1	-4.0	-4.8	-5.8	-6.8		
18	5.3	4.7	4.0	3.4	2.7	2.1	1.4	0.7	18	-0.7	-1.4	-2.2	-2.9	-3.7	-4.5	-5.3	-6.2				
20	5.2	4.5	3.9	3.3	2.6	2.0	1.3	0.7	20	-0.7	-1.4	-2.1	-2.8	-3.5	-4.2	-5.0	-5.8				
44	4	9.2	8.3	7.4	6.5	5.5	4.4	3.2	1.8	44	4	-4.0									
	6	8.1	7.3	6.4	5.5	4.6	3.6	2.5	1.4		6	-1.7	-4.8								
	8	7.3	6.5	5.7	4.9	4.0	3.1	2.1	1.1		8	-1.3	-2.7	-4.8							
	10	6.7	6.0	5.2	4.4	3.6	2.8	1.9	1.0		10	-1.0	-2.2	-3.5	-5.0	-7.2	-10.0				
	12	6.3	5.5	4.8	4.1	3.3	2.5	1.7	0.9		12	-0.9	-1.9	-2.9	-4.0	-5.3	-6.8	-8.8			
	14	5.9	5.2	4.5	3.8	3.1	2.3	1.6	0.8		14	-0.8	-1.7	-2.6	-3.5	-4.5	-5.6	-6.8	-8.2		
	16	5.6	5.0	4.3	3.6	2.9	2.2	1.5	0.7		16	-0.8	-1.6	-2.4	-3.2	-4.1	-5.0	-6.0	-7.0		
18	5.4	4.8	4.1	3.4	2.8	2.1	1.4	0.7	18	-0.7	-1.5	-2.2	-3.0	-3.8	-4.6	-5.4	-6.3				
20	5.2	4.6	4.0	3.3	2.7	2.0	1.4	0.7	20	-0.7	-1.4	-2.1	-2.8	-3.6	-4.3	-5.1	-5.9				
46	4	9.4	8.5	7.6	6.6	5.6	4.5	3.3	1.8	46	4										
	6	8.3	7.5	6.6	5.7	4.7	3.7	2.6	1.4		6	-1.8	-5.6								
	8	7.5	6.7	5.9	5.0	4.1	3.2	2.2	1.2		8	-1.3	-2.9	-5.1	-8						
	10	6.9	6.1	5.3	4.5	3.7	2.8	1.9	1.0		10	-1.1	-2.3	-3.6	-5.3	-7.8					
	12	6.4	5.7	4.9	4.2	3.4	2.6	1.7	0.9		12	-0.9	-1.9	-3.0	-4.2	-5.5	-7.1	-9.6			
	14	6.0	5.3	4.6	3.9	3.1	2.4	1.6	0.8		14	-0.8	-1.7	-2.6	-3.6	-4.7	-5.8	-7.1	-8.6		
	16	5.7	5.1	4.4	3.7	3.0	2.2	1.5	0.8		16	-0.8	-1.6	-2.4	-3.3	-4.2	-5.1	-6.1	-7.2		
18	5.5	4.8	4.2	3.5	2.8	2.1	1.4	0.7	18	-0.7	-1.5	-2.3	-3.1	-3.9	-4.7	-5.6	-6.5				
20	5.3	4.7	4.0	3.4	2.7	2.1	1.4	0.7	20	-0.7	-1.4	-2.1	-2.9	-3.6	-4.4	-5.2	-6.0				

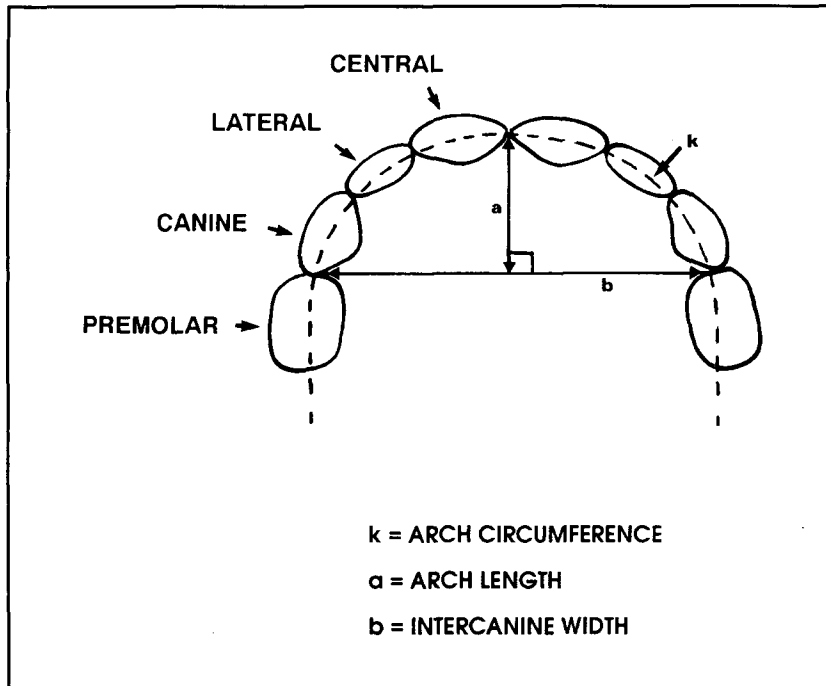


Figure 1

**Method**

One of the authors adapted a fundamental mathematic equation<sup>7</sup> of the parabolic shape for use with a computer. The resulting computer software was used to calculate the figures presented in Table 1. The complete mathematical derivation and the computer program used are presented in the Addendum. The following formula was used:

$$c = b \cdot b / 2 / a$$

$$r = \text{sqr} (c \cdot c + b \cdot b)$$

$$k = [ b \cdot r / 2 + c \cdot c / 2 \cdot \log (b + r) / c ] \cdot 2 / c$$

where a = arch length in millimeters

b = intercanine width in millimeters

k = arch circumference in millimeters

Figure 1 illustrates these parameters. Note that the arch circumference (k) was chosen to connect the interproximal contact points.

**Discussion**

By consulting Table 1 the change in arch length (a) following a change in arch circumference (k) can be predicted, and vice versa. This means that it is possible to predict the exact arch length change necessary in order to create the space needed for a missing lateral incisor, or the amount of stripping that would be necessary to correct a Bolton discrepancy. The tables may also be used to construct a more accurate VTO during treatment planning, especially where anterior crowding or spacing is present.

Note that the shorter the arch length, the more critical the arch circumference adjustments become. This means that for every millimeter change in arch circumference, the overjet will be affected to a larger extent in the case with short arch length than in the case with greater arch length. This can be demonstrated by consulting Table 1 in the section for an intercanine width (b) = 46 mm, an arch length (a) = 4 mm, and space shortage in arch circumference (k) = - 1 mm, where (a) will have to be increased by 1.8 mm in order to accommodate the teeth. In comparison, with (b) = 46 mm and (a) = 20 mm, an increase in arch length of only 0.7 mm would be needed in order to accommodate the same amount of space shortage.

It is generally erroneously assumed that for every 1 millimeter change in arch circumference of the anterior segment of the arch there will be a .5 millimeter change in arch length. It is, in fact, possible to effect a sagittal change of more than 1 mm by changing anterior arch circumference by 1 mm. The above example also illustrates this point.

The following hypothetical examples further illustrate the usefulness of the accompanying table:

1. A Class 1 buccal and canine occlusion has been achieved and the intercanine distance should be maintained. There remains an overjet of 5 mm, due to a Bolton discrepancy, which should be reduced by 4 mm. Measurements show that the intercanine distance (b) is 36 mm and the arch length (a) is 16 mm.

Referring to the  $b = 36$  section and along the line  $a = 16$  we find that the -4 position lies half-way between the -3.6 and -4.4 stations. Referring vertically we find that a space of between 5 and 6 mm is needed, which means that the anterior segment should be stripped by 5.5 mm in order to reduce the overjet by 4 mm.

According to usual VTO practice, however, 8 mm of space in arch circumference would have been the amount assumed necessary to retract the incisors by 4 mm.

2. This case has a mandibular (b) = 18 mm, (a) = 4 mm, and crowding of the mandibular incisors of 5. Usual VTO practice dictates that the mandibular incisors will be displaced anteriorly by 2.5 mm during alignment, if intercanine width is maintained. The table, however, shows that the mandibular incisors will be advanced by 4 mm.

3. With an upper (b) = 30 mm, an (a) = 12 mm and spacing of the maxillary incisors of 5 mm, one would expect retraction of the maxillary incisors to reduce the overjet by 2.5 mm. The table predicts a reduction of 3.4 mm.

4. To accommodate a missing maxillary lateral incisor, a space of 6 mm must be created in this example. The (b) = 34 mm and the (a) = 8 mm. The table predicts that this can be achieved by increasing the overjet by 5 mm if the intercanine width is not altered. With this information it is possible to create an accurate VTO for this case.

### Conclusion

These tables relating changes in arch length and arch circumference can be of use to the clinician.

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## Addendum 1: Mathematical derivations

$$y = a\left(1 - \frac{x^2}{b^2}\right)$$

$$\frac{dy}{dx} = -\frac{2ax}{b^2}$$

$$c = \frac{b^2}{2a}$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$= \sqrt{1 + \frac{4a^2 x^2}{b^4}} dx$$

$$ds = \frac{2a}{b^2} \sqrt{\left(\frac{b^2}{2a}\right)^2 + x^2} dx$$

$$ds = \frac{1}{c} \sqrt{c^2 + x^2} dx$$

$$\text{let } z = (x^2 + c^2)^{1/2}$$

$$r = (b^2 + c^2)^{1/2}$$

$$cs = \int_0^b \sqrt{c^2 + x^2} dx$$

$$\int z dx = \frac{xz}{2} + \frac{c^2}{2} \log(x+z)^*$$

$$cs = \left[ \frac{x}{2} (x^2 + c^2)^{1/2} + \frac{c^2}{2} \log(x + (x^2 + c^2)^{1/2}) \right]$$

$$= \left[ \frac{b}{2} \frac{(b^2 + c^2)^{1/2}}{r} + \frac{c^2}{2} \log\left(b + \frac{(b^2 + c^2)^{1/2}}{r}\right) - \frac{c^2}{2} \log c \right]$$

$$S = \frac{1}{c} \left[ \frac{br}{2} + \frac{c^2}{2} \log(b+r) - \frac{c^2}{2} \log c \right]$$

$$S = \frac{br}{2c} + \frac{c}{2} [\log(b+r) - \log c]$$

$$2S = \frac{br}{c} + c[\log(b+r) - \log c]$$

\*Dwight: Tables of integrals and other mathematical data. McWilliam, 1965, p 56