Turkish J. Eng. Env. Sci.
29 (2005) , 271 – 278.

Performance Analysis of an Endoreversible Braysson Cycle Based on the Ecological Criterion

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Received 09.02.2005

Abstract

A performance analysis based on the ecological criterion was carried out for an endoreversible Braysson cycle model that includes finite rate heat transfer irreversibility. The ecological objective function is defined as the power output minus the loss power, which is equal to the product of the environmental temperature and the entropy production rate. The maximization of the ecological function was achieved for various design parameters and the obtained results are compared with those obtained using the maximum power criterion.

Key words: Braysson cycle, Finite-time thermodynamics, Ecological performance, Thermal efficiency.

Introduction

The thermal efficiency of the reversible Carnot cycle in classical thermodynamics has been used as a theoretical upper limit for heat engines. This upper limit can be reached by means of infinitely slow processes due to the requirements of thermodynamic equilibrium, which means infinite time. Therefore, it is not possible to obtain a finite power using heat exchangers with finite heat transfer areas. In order to obtain a finite amount of power, it is necessary to have heat exchangers with infinite heat transfer areas. Therefore, the reversible Carnot cycle cannot be considered a comparison standard for practical heat engines from the power and size perspective, although it has an upper thermal efficiency limit. In order to obtain a certain power with finite sizes, the reversible Carnot cycle can be extended to an endoreversible Carnot cycle by taking the irreversibility of finite time heat transfer into account, and the maximum power (MP) conditions investigated. As a result, for practical heat engines, the reversible Carnot cycle has a thermal efficiency upper limit, while the endoreversible Carnot cycle has a maximum power limit.

Many optimization studies based on various performance criteria have been carried out for endoreversible and irreversible heat engine models using finite time thermodynamics theory. Detailed information about these optimization works can be found in literature surveys written by Bejan (1996) and Chen et al. (1999). In these studies, the performance of heat engines has been analyzed for the power, thermal efficiency, specific power, power density, entropy generation, thermo-economics and ecological objectives by taking into account finite rate external heat transfer irreversibility and/or internal irreversibilities. Over the last decade some authors have studied the ecological performance of endoreversible and irreversible heat engines by considering finite time and finite size constraints (Angulo-Brown, 1991; Yan, 1993; Cheng and Chen, 1998; Cheng and Chen, 1999; Sahin et. al., 2002). The Braysson cycle as a new gas turbine cycle has been introduced firstly by Frost et al. (1997). They proposed a hybrid cycle of the conventional Brayton and Ericsson cycles, which are the simplest gas turbine cycles. The Braysson cycle has a high temperature heat addition process as a Brayton cycle and a low temperature heat rejection process as an Ericsson cycle. It thus incorporates the thermodynamic advantages of a combined gas and steam turbine cycle without the irreversibilities of the boiler and the ancillaries of the steam turbine/condenser plant. Thus the Brayson cycle is an alternative to a conventional combined gas and steam turbine power plant.

The First Law analysis of a Braysson cycle based on energy balance was carried out by Frost et al. (1997) and the Second Law analysis based on exergy balance was carried out by Zheng et al. (2001). The power and efficiency performance analysis of an endoreversible Braysson cycle was studied by Zheng et al. (2002).

A performance analysis of Braysson cycle using a finite-time ecological optimization technique does not appear to have been published. Application of the maximum ecological objective function to a Braysson cycle seemed interesting since the preservation of natural resources may be considered in the analysis as well as power output.

The Theoretical Model

Temperature-entropy and pressure-volume diagrams of an endoreversible Braysson cycle operating between two extreme temperatures (T$_H$ and T$_L$) are shown in Figure 1. If U$_H$ and U$_L$ are the heat conductances of the hot- and cold-side heat exchangers respectively and C$_{wf}$ is the thermal capacitance rate of working fluid then the heat flow rate $\dot{Q}_H$from the hot reservoir to the heat engine can be written as

$$\dot{Q}_H = C_{wf}\varepsilon_H(T_H - T_2). \tag{1}$$

The heat flow rate $\dot{Q}_L$from the heat engine to the cold reservoir can be written as

$$\dot{Q}_L = U_L(T_4 - T_L). \tag{2}$$

The effectiveness of the hot-side heat exchanger (ε_H) in Eq. (1) is defined as

$$\varepsilon_H = 1 - e^{-N_H} \tag{3}$$

where N$_H$ is the number of heat transfer units

$$N_H = \frac{U_H}{C_{wf}}. \tag{4}$$

From the 2-3 isobaric process

$$\dot{Q}_H = \dot{m}Cp(T_3 - T_2) = C_{wf}(T_3 - T_2) \tag{5}$$

where $\dot{m}$ and Cp are the mass flow rate and constant pressure specific heat of the working fluid, respectively.

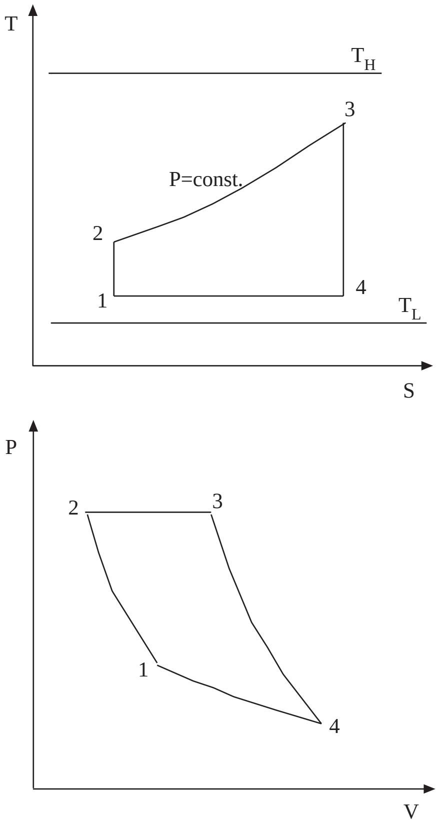

Figure 1. T-S and P-V diagrams of an endoreversible Braysson cycle.

The net power output ($\dot{W}$) of the Braysson cycle is

$$\dot{W} = \dot{Q}_H - \dot{Q}_L = C_{wf}\varepsilon_H(T_H - T_2) - U_L(T_4 - T_L) \tag{6}$$

and thermal efficiency (η) is

$$\eta = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = 1 - \frac{U_L(T_4 - T_L)}{C_{wf}\varepsilon_H(T_H - T_2)} \tag{7}$$

The objective function of ecological optimization, which was proposed by Angulo-Brown (1991) and modified by Yan (1993), is

$$E = \dot{W} - T_0\dot{S}_g \tag{8}$$

where T_0 is the environment temperature and $\dot{S}_g$ is the entropy generation rate. The entropy generation rate of the Braysson cycle is defined as

$$\dot{S}_g = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H}. \tag{9}$$

The optimization of the ecological function (Eq. (8)) represents the best compromise between power output $\dot{W}$ and power loss $T_0\dot{S}_g$, which is produced by entropy generation in the system and its surroundings. Substitution of Eqs. (6) and (9) into Eq. (8) gives

$$E = \left(1+\tfrac{1}{\tau}\right)\dot{Q}_H - 2\dot{Q}_L = \left(1+\tfrac{1}{\tau}\right)C_{wf}\varepsilon_H(T_H - T_2) - 2U_L(T_4 - T_L) \tag{10}$$

where $\tau = T_H/T_L$ is the extreme temperature ratio of the cycle and the reference temperature, T_0, is assumed to be equal to the cold-side heat reservoir temperature, T_L. The Second Law of Thermodynamics requires that

$$U_L(T_4 - T_L)/T_4 = \dot{m}Cp\ln(T_3/T_2) = C_{wf}\ln(T_3/T_2) \tag{11}$$

Defining a dimensionless working fluid temperature ratio x as given by Zheng et al. (2002)

$$x = U_L(T_4 - T_L)/C_{wf}T_4 = \ln(T_3/T_2) \tag{12}$$

From Eq. (12), T_4 and T_3 can be written as

$$T_4 = \frac{U_L T_L}{(U_L - C_{wf}x)} \tag{13}$$

$$T_3 = T_2 e^x \tag{14}$$

By using Eqs. (1) and (5), T_2 and T_3 can be derived as

$$T_2 = \varepsilon_H T_H/(e^x + \varepsilon_H - 1) \tag{15}$$

$$T_3 = \varepsilon_H T_H e^x/(e^x + \varepsilon_H - 1) \tag{16}$$

Substituting Eqs. (14), (15) and (16) into Eqs. (10), (6) and (7) yields the dimensionless ecologic function $\left[\bar{E} = E/(C_{wf}T_L)\right]$, dimensionless power output $\left[\bar{W} = \dot{W}/(C_{wf}T_L)\right]$ and thermal efficiency (η)

$$\bar{E} = \left[\left(1+\frac{1}{\tau}\right)\left(\frac{\varepsilon_H\tau(e^x - 1)}{(e^x + \varepsilon_H - 1)}\right) - \left(\frac{2U_L x}{U_L - xC_{wf}}\right)\right] \tag{17}$$

$$\bar{W} = \frac{\varepsilon_H\tau(e^x - 1)}{(e^x + \varepsilon_H - 1)} - \frac{xU_L}{U_L - C_{wf}x} \tag{18}$$

$$\eta = 1 - \frac{xU_L(e^x + \varepsilon_H - 1)}{\varepsilon_H\tau(U_L - C_{wf}x)(e^x - 1)} \tag{19}$$

Equation (17) is the most important result of this study and it gives the relationship between the dimensionless ecologic function and dimensionless working fluid temperature ratio. It is possible to find the optimum dimensionless working fluid temperature ratio (x_{opt}) by taking the derivative of $\bar{E}$ with respect to x and setting it equal to zero ($d\bar{E}/dx = 0$) as

$$\frac{\varepsilon_H^2(1+\tau)e^{x_{opt}}}{(e^{x_{opt}} + \varepsilon_H - 1)^2} = \frac{2U_L^2}{(U_L - x_{opt}C_{wf})^2} \tag{20}$$

The maximum ecologic function can be found by substituting x_{opt} into Eq. (17), i.e.

$$(\bar{E})_{\max} = \left[\left(1+\frac{1}{\tau}\right)\left(\frac{\varepsilon_H\tau(e^{x_{opt}} - 1)}{(e^{x_{opt}} + \varepsilon_H - 1)}\right) - \left(\frac{2U_L x_{opt}}{U_L - x_{opt}C_{wf}}\right)\right] \tag{21}$$

The thermal efficiency at maximum ecology can also be found as

$$\eta_{me} = 1 - \frac{x_{opt}U_L(e^{x_{opt}} + \varepsilon_H - 1)}{\varepsilon_H\tau(U_L - C_{wf}x_{opt})(e^{x_{opt}} - 1)} \tag{22}$$

Results and Discussion

Numerical examples are provided in order to show the advantages and disadvantages of the design at maximum ecological function conditions. The variation of the dimensionless ecological function ($\bar{E}$) with respect to the dimensionless power output ($\bar{W}$) is shown in Figure 2. The effects of extreme temperature ratio (τ), hot-side heat exchanger effectiveness (ε_H) and cold-side heat exchanger heat conductance (U_L) on the ecological performance are shown in Figures 2(a), (b) and (c), respectively. It is observed that both the maximum ecological function point and the maximum power output point exist. The power output at maximum ecological conditions is slightly lower than the maximum power output conditions as shown, in Figure 2. It is clearly shown that an increase in τ, ε_H and U_L yields a significant increase in global and optimal performance. If the entropy generation rate is larger than the power output of the cycle then the dimensionless ecological function ($\bar{E}$) would have negative values, as seen from Figure 2.

Figure 3 shows the variation of the dimensionless ecological function with respect to thermal efficiency for different extreme temperature ratio, hot-side heat exchanger effectiveness and cold-side heat exchanger heat conductance values. The thermal efficiency at maximum ecological function (η_{me}) increases for increasing extreme temperature ratios but it stays approximately constant with ε_H and U_L. These results can be observed more clearly in Figure 7.

The dimensionless ecological function is plotted against the working fluid temperature ratio (x) of an endoreversible Braysson cycle in Figure 4. It can be seen that the optimum x value, corresponding to the maximum ecological performance condition, increases with increasing τ, ε_H and U_L.

The variations of the dimensionless power and dimensionless ecological function with respect to x and η can be seen in Figures 5 and 6, respectively. Figure 5 shows that x_{opt} is greater for maximum power condition than the maximum ecologic function conditions. On the other hand, Figure 6 shows that the thermal efficiency at maximum ecology (η_{me}) is greater than the thermal efficiency at maximum power (η_{mp}). The dimensionless power at maximum ecologic performance ($\bar{W}_{me}$) is lower than maximum power ($\bar{W}_{\max}$). If the design parameters are selected at the maximum ecology condition instead of maximum power conditions, thermal efficiency will be greater, but power output will be smaller.

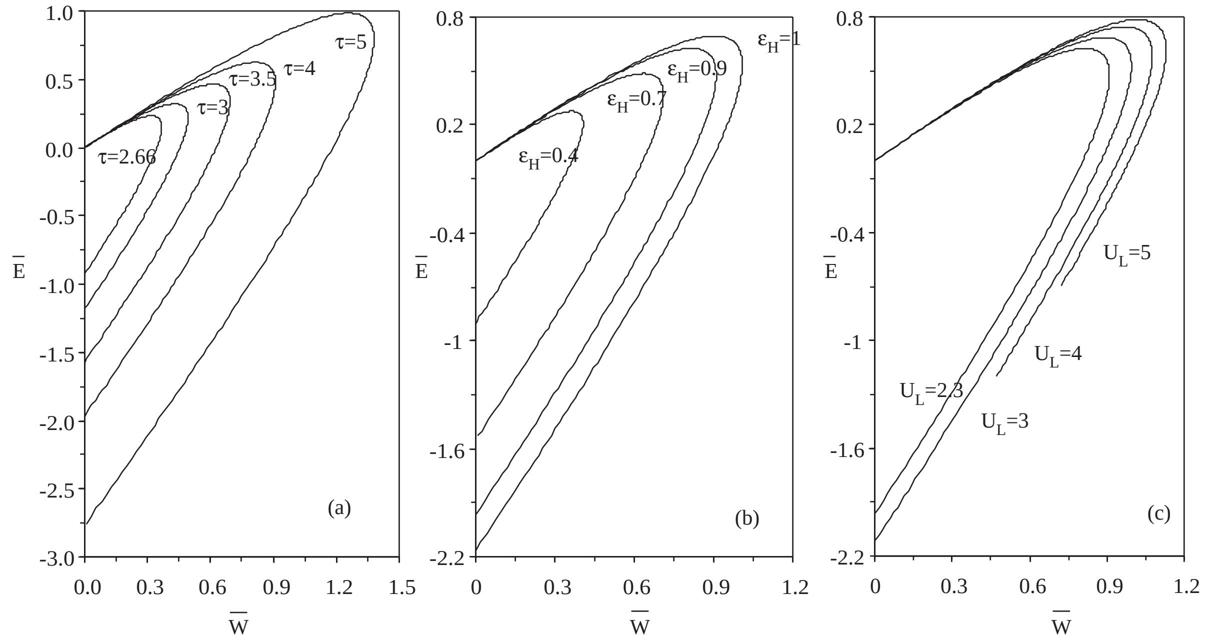

Figure 2. Dimensionless ecological function with respect to dimensionless power output (C_{wf} = 1 kW/K) for different extreme temperature ratio (ε_H = 0.9, U_L = 2.3 kW/K) (a); hot-side heat exchanger effectiveness (τ = 4, U_L = 2.3 kW/K) (b); and cold-side heat exchanger heat conductance (ε_H = 0.9, τ = 4) (c).

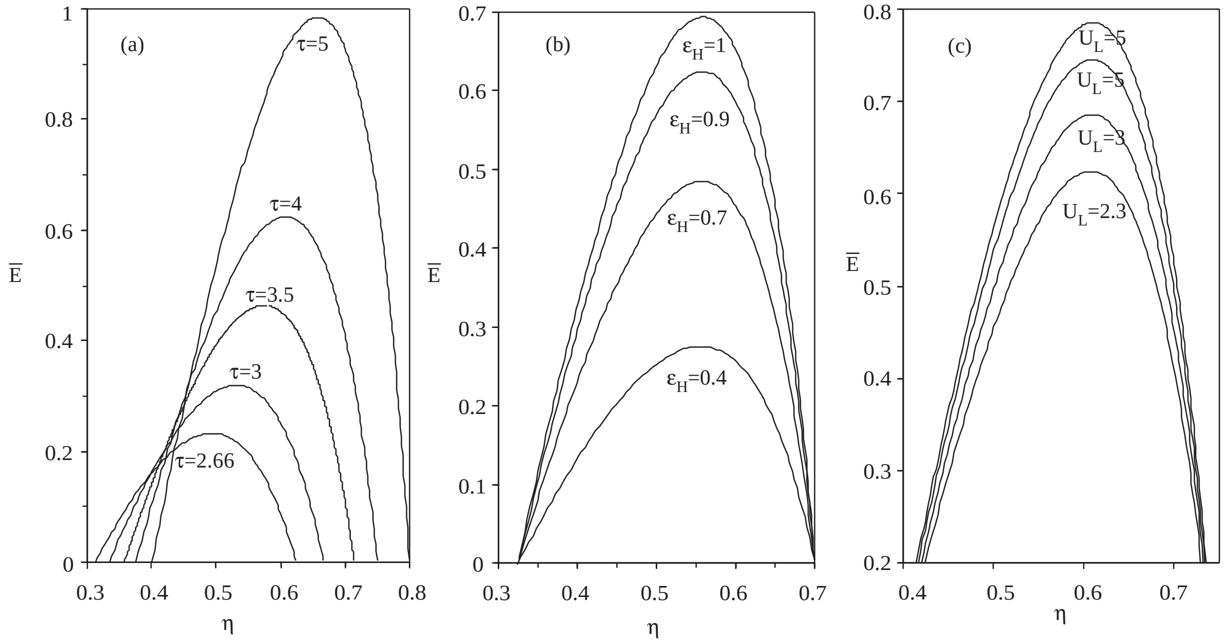

Figure 3. Dimensionless ecological function with respect to thermal efficiency (C_{wf} = 1 kW/K) for different extreme temperature ratio ($\varepsilon_H = 0.9$, U_L = 2.3 kW/K) (a); hot-side heat exchanger effectiveness ($\tau = 4$, U_L = 2.3 kW/K) (b); and cold-side heat exchanger heat conductance ($\varepsilon_H = 0.9$, $\tau = 4$) (c).

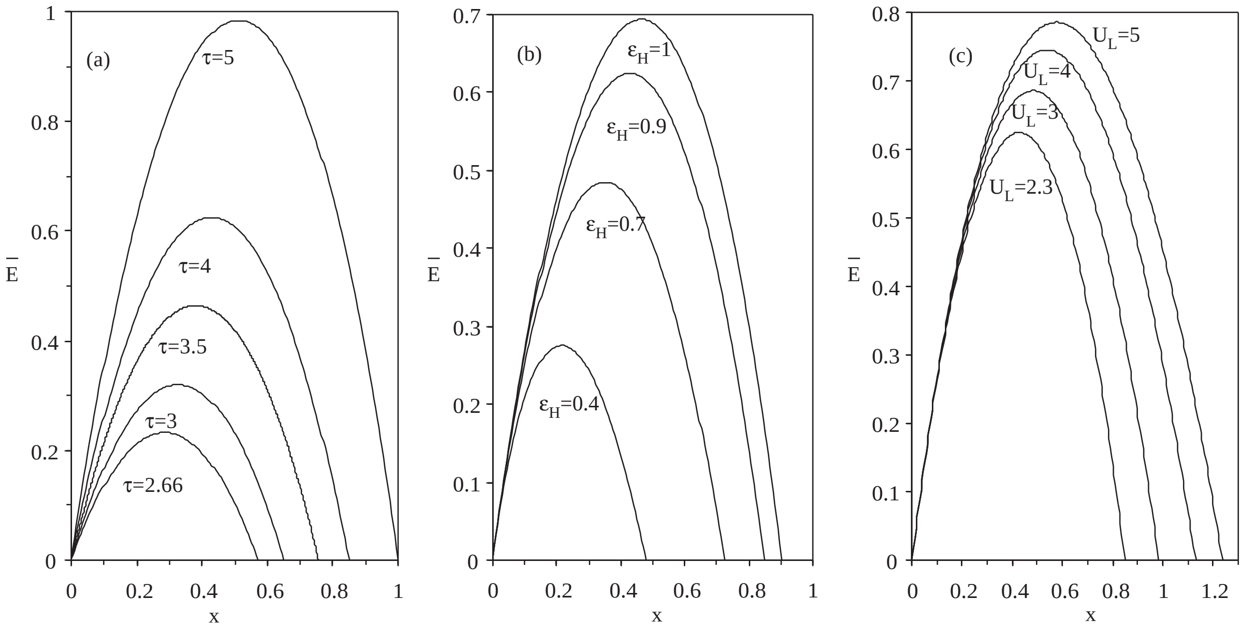

Figure 4. Dimensionless ecological function with respect to working fluid temperature ratio (C_{wf} = 1 kW/K) for different extreme temperature ratio ($\varepsilon_H = 0.9$, U_L = 2.3 kW/K) (a); hot-side heat exchanger effectiveness ($\tau = 4$, U_L = 2.3 kW/K) (b); and cold-side heat exchanger heat conductance ($\varepsilon_H = 0.9$, $\tau = 4$) (c).

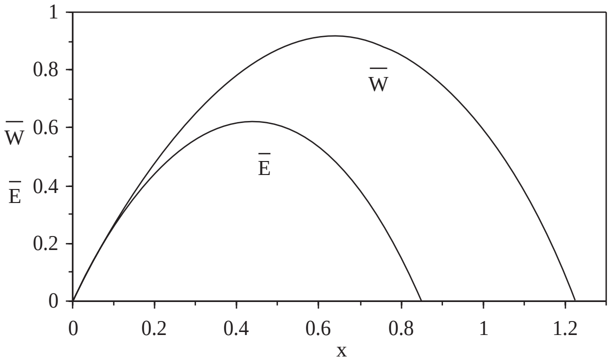

Figure 5. Variations of dimensionless ecological function and dimensionless power output with respect to working fluid temperature ratio (C_{wf} = 1 kW/K, $\tau = 4, \varepsilon_H = 0.9$ kW/K and U_L = 2.3 kW/K).

The variations of the optimal performances under maximum ecological function and maximum power conditions are plotted against extreme temperature ratio (τ), hot-side heat exchanger effectiveness (ε_H) and cold-side heat exchanger heat conductance (U_L) in Figures 7 (a), (b) and (c), respectively. The maximum ecological function ($\bar{E}_{\max}$) and maximum power output ($\bar{W}_{\max}$) rapidly increase with increasing τ and ε_H but gradually increase with increasing U_L. The thermal efficiency at maximum ecological function (η_{me}) and the thermal efficiency at maximum power output (η_{mp}) increase with increasing extreme temperature ratio but are not affected by hot-side heat exchanger effectiveness and cold-side heat exchanger heat conductance.

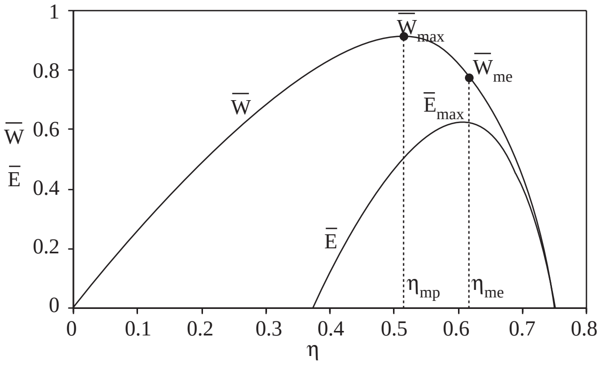

Figure 6. Variations of dimensionless ecological function, dimensionless power output and dimensionless power density with respect to thermal efficiency ($C_{wf} = 1$ kW/K, $\tau = 4$, $\varepsilon_H = 0.9$ kW/K and U_L = 2.3 kW/K).

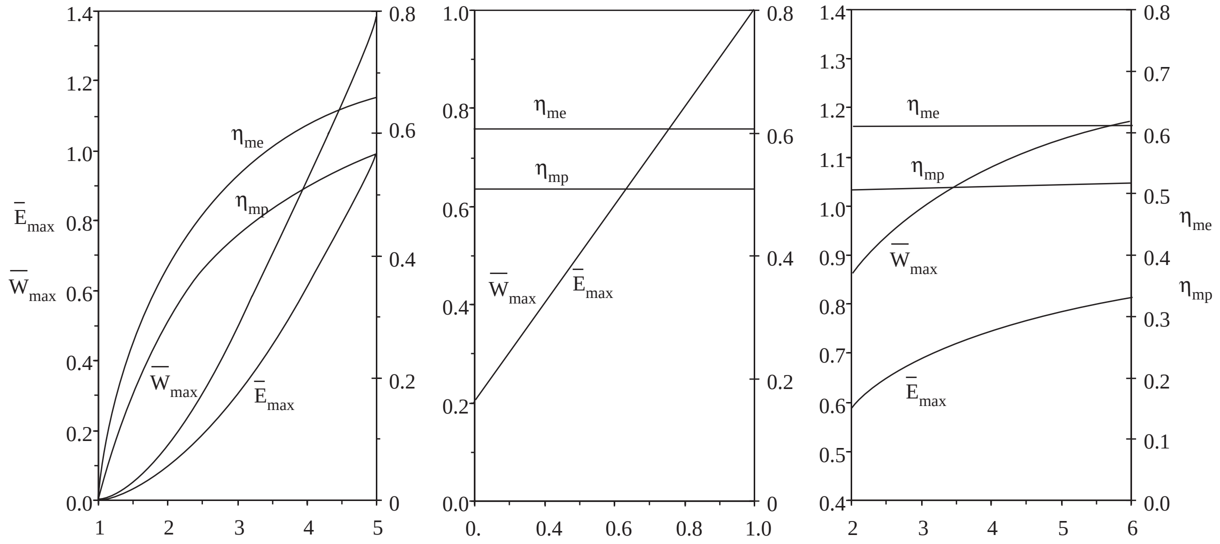

Figure 7. Dimensioless maximum ecological function, dimensionless maximum power output and thermal efficiencies at maximum ecology and at maximum power output (C_{wf} = 1 kW/K) with respect to extreme temperature ratio (ε_H = 0.9, U_L = 2.3 kW/K) (a); hot-side heat exchanger effectiveness (τ = 4, U_L = 2.3 kW/K) (b); and cold-side heat exchanger heat conductance (ε_H = 0.9, τ = 4) (c).

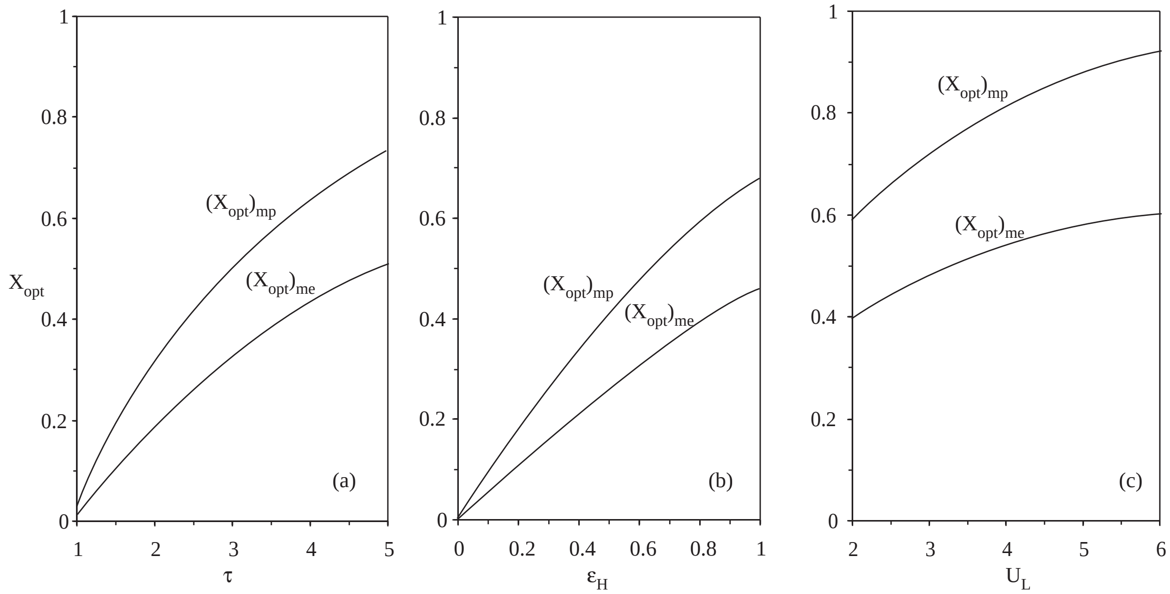

Figure 8. Optimum working fluid temperature ratios at maximum ecology and at maximum power output ($C_{wf} = 1$ kW/K) with respect to extreme temperature ratio ($\varepsilon_H = 0.9$, $U_L = 2.3$ kW/K) (a); hot-side heat exchanger effectiveness ($\tau = 4$, $U_L = 2.3$ kW/K) (b); and cold-side heat exchanger heat conductance ($\varepsilon_H = 0.9$, $\tau = 4$) (c).

The variation of the optimum x values for the maximum ecological performance condition $(x_{opt})_{me}$ and for the maximum power output condition $(x_{opt})_{mp}$ with respect to extreme temperature ratio, hot-side heat exchanger effectiveness and cold-side heat exchanger heat conductance are shown in Figure 8. τ and ε_H have a greater effect than U_L on the optimum working fluid temperature ratio for a Braysson heat engine.

Conclusion

A performance analysis was carried out for an endoreversible Braysson heat engine based on the maximum ecological criterion. The model includes finite rate heat transfer irreversibility. To see the effects of various design parameters (τ, ε_H, U_L etc.) on the ecologic performance and thermal efficiency of the endoreversible Braysson cycle some numerical calculations were performed. The results are compared with those obtained using the maximum power criterion. The analysis showed that with a little sacrifice in power output, we could achieve a substantial increase in thermal efficiency. The thermal efficiencies at maximum ecology and at maximum power output are only affected by the extreme temperature ratio and so in order to design a more effective Braysson heat engine, the extreme temperature ratio can be used as an important design parameter. This analysis may provide a basis for both the determination of optimal operating conditions and design parameters for real Braysson heat engines.

Nomenclature

Cwf	thermal capacitance rate of the working fluid
Cp	specific heat at constant pressure
E	ecological objective function
$\dot{m}$	mass flow rate of the working fluid
N	number of heat transfer units
P	pressure
S	entropy generation
$\dot{Q}$	heat rate
T	temperature
U	heat conductance of the heat exchanger
$\dot{W}$	power output
x	working fluid temperature ratio
ε	effectiveness of the heat exchanger
η	thermal efficiency
τ	extreme temperature ratio

Subscripts

H	hot-side
L	cold-side
max	maximum
me	maximum ecologic performance condition
mp	maximum power condition
opt	optimum

Superscript

—	dimensionless

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