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An Efficient Seismic Analysis Procedure for Torsionally Coupled Multistory Buildings Including Soil-Structure Interaction

Erkan ÇELEBİ
Sakarya University, Department of Civil Engineering
Sakarya-TURKEY
e-mail: ecelebi@sakarya.edu.tr
A. Necmettin GÜNDÜZ
İstanbul Technical University, Department of Civil Engineering
İstanbul-TURKEY

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Abstract

In this paper, a simplified methodology of analysis for the seismic response of 3-dimensional irregular high-rise buildings on a rigid footing resting on the surface of a linear elastic half-space is formulated. An efficient method using modal decomposition and carried out in the frequency domain by using the fast Fourier transform to obtain the structural response of torsionally asymmetric buildings, including soil-structure interaction effects, is presented. Applying this algorithm, full advantage is taken of classical normal mode approximation, and the interaction problem is solved easily and effectively within the framework of the Fourier-transformed frequency domain analysis for a fixed-base structure. The matrix formulation of this method produces accurate approximation with less computational effort, in spite of using the frequency dependent impedance functions.

Key words: Torsionally coupled multistory buildings, Soil-structure interaction, Frequency domain analysis, Impedance functions, Modal decomposition.

Introduction

The comprehensive studies (Chandler and Hutchinson, 1986; Cruz and Chopra, 1986b; Hejal and Chopra, 1989) conducted by a number of researchers in the past few decades and investigations of the effects of past earthquakes have shown that in buildings with non-coincident centers of mass and resistance, significant coupling may occur between the translational and the torsional displacements of the floor diaphragms even when the earthquake induces uniform rigid base translations. However, these studies assumed that the asymmetric structure is supported by a rigid foundation. The elastic response of structures to earthquake ground excitations is influenced by deformability of the foundation medium. Therefore, the rigid foundation assumption represents an approximation to real conditions. It is widely recognized that the dynamic response of a structure supported on soft soil may be different from the response of a similarly excited, identical structure supported on firm ground. The effects of soil-structure interaction (SSI) on the dynamic response of building systems have been the subject of numerous investigations in recent years. Most of the previous SSI studies (Chopra and Guiterrez, 1974; Wu and Smith, 1995; Wu, 1997) on buildings, however, were restricted to 2-dimensional (planar) multistory frames due to the fact that the governing equations of motion for structures with many degrees of freedom, as well as the methods of solving these equations, are relatively complicated and involve a considerable computational costs.

Various analytical and numerical techniques were also proposed and developed to efficiently simplify SSI analysis, such as transmitting boundaries of different kinds, boundary elements, and infinite elements and their coupling procedures for the modelling of unbounded media (Meek and Wolf, 1992; Aydınoğlu, 1993b; Wolf and Song, 1996). However, the complicated formulation and intensive computation necessary to obtain the exact solution to this problem have so far restricted its common application to traditional engineering practice. The foundation on flexible soil may be idealized in structural dynamics as a simple spring-dashpot-mass model and the solution can be performed in the frequency domain. Since most of the complication in the solution of the equations of motion results from the frequency dependence of the dynamic soil stiffness, many studies (Ghaffer and Chapel, 1983; Wolf, 1997) have been performed to simulate the SSI phenomenon by representing the soil with frequency-independent impedance functions by constant parameters. However, this proposed approximation may not be valid when the soil surface is shallow relative to the base dimensions of the structure (Tsai *et al.*, 1974). Additional research on SSI proposed by Sivakumaran (1990) and Chandler and Sikaroudi (1992) has been dedicated to the application of modal analysis techniques. Substitution of structural deformations, in combination with the dynamic SSI force-displacement relationships, into the governing equations of the whole system results in integro-differential equations for footing displacements, which are solved numerically by step by step integration in time domain. Such a procedure, given by Sivakumaran *et al.* (1992), requires a large computational effort, particularly for 3-dimensional asymmetric multistory buildings.

In the present paper, an efficient method using modal decomposition and performed in the frequency domain by using the fast Fourier transform algorithm to obtain the dynamic response of torsionally asymmetric buildings including the SSI effect is presented. The effects of the foundation medium on the structural response are simulated by a series of springs and dashpots representing a theoretical half-space surrounding the base of the structure. The structure investigated herein is presumed to be linear and viscously damped and is supported at the surface of a homogeneous, isotropic, elastic half-space and is excited at the base. However, to accurately represent the elastic half-space, the properties of springs and dashpots are required to be dependent on the frequency of excitation. Thus, the governing equations for the structure foundation system are expressed and solved in the Fourier-transformed frequency domain. The governing equations are developed considering the motions of each floor and the whole system. In this method, initially structural deformations are obtained in terms of foundation displacement using a linear combination of vibrational modes of the building on a rigid foundation, in combination with the dynamic SSI force displacement relationships. In this study, the Fourier transforms are computed very efficiently by the fast Fourier transform algorithm, where they are treated as discrete transforms (Humar and Via, 1993). The numerical approximations of the impedance functions used in the following study are taken from Veletsos and Wei (1971) for lateral and rocking vibration and from Veletsos and Nair (1974) for torsional vibration. To demonstrate the proposed method, a detailed parametric study is made of torsional coupling in a multistory building excited by real earthquake ground motion (Erzincan, 1992, E-W) for different soil stiffnesses by using an original program developed and applied by the authors. For practical purposes the definite part of the record time including the peak value of acceleration of this input motion is considered. The results represented in frequency response curves indicate that this efficient method produces a good approximation to the exact response obtained by the direct method illustrated in previous studies (Çelebi and Gündüz, 2000, 2001). The results presented in this paper show that the seismic responses of asymmetric buildings including soil structure interaction can be significantly different from those without substructure interaction.

Structure-Foundation Model and Analytical Procedures

Assumptions and equation of motion

The building foundation system considered, as shown in Figure 1, is represented by an N-story 3-dimensional superstructure, consisting of shear frames, resting on a rigid square foundation of mass m_o with a negligible thickness on the surface of a linear homogeneous elastic half-space. The mass of this building is considered to be concentrated at each floor level, and the floor systems are assumed to be rigid rectangular floor decks supported by relatively massless, axially inextensible columns. The lateral load resisting elements are assumed to be arranged

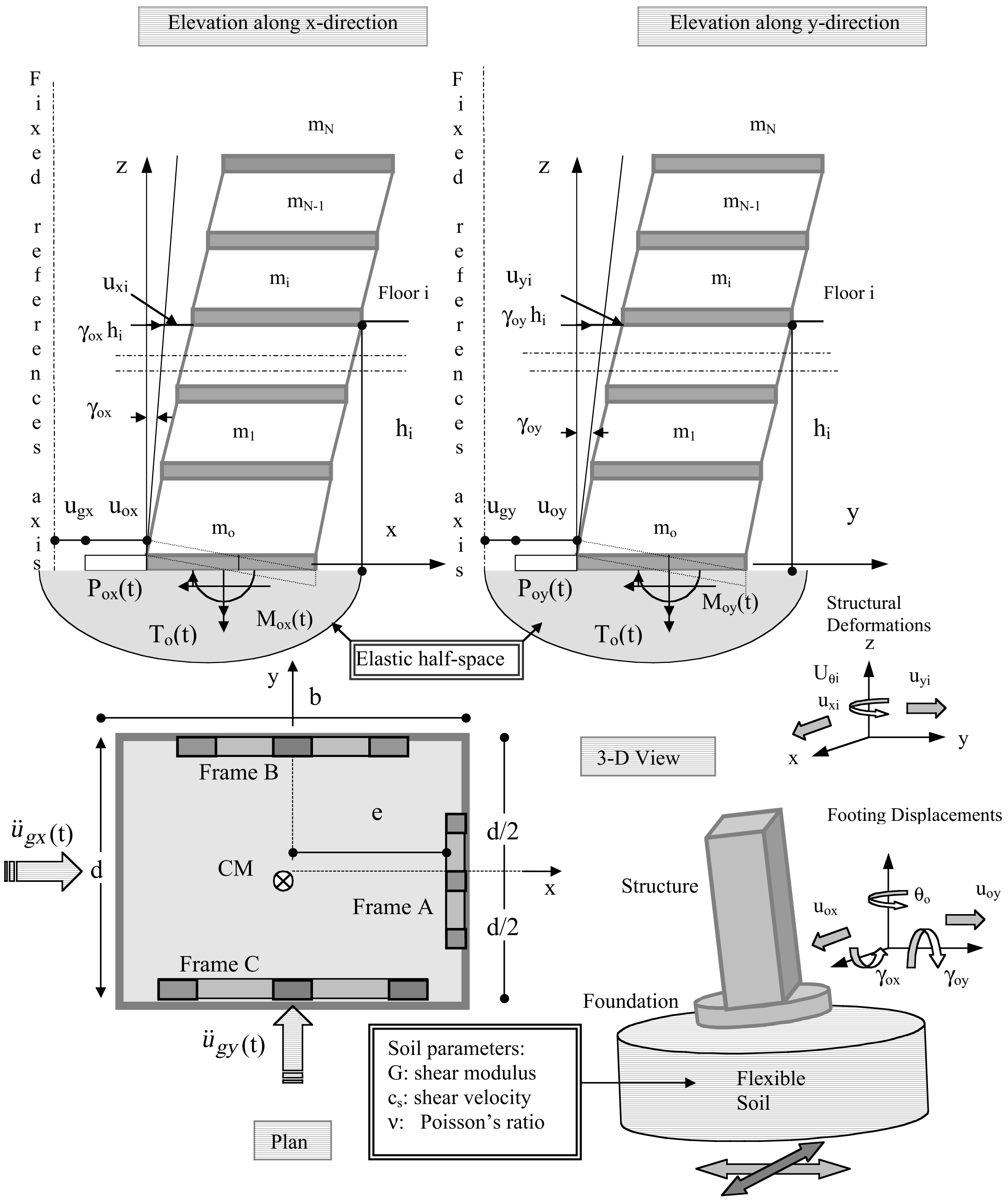

Figure 1. Idealized 3-dimensional asymmetric building foundation system on an elastic half-space.

so that the system has no axes of symmetry. Thus the structure is a 2-way torsionally coupled system with 3 degrees of freedom at each floor, namely horizontal displacements u_{xi}, u_{yi} and rotation about the vertical axis $u_{\theta i}$ for the ith floor. In addition, due to the deformability of the foundation, the system has 5 more displacement degrees of freedom, namely horizontal translations of the foundation u_{ox}, u_{oy}, rocking rotations of the foundation γ_{ox}, γ_{oy} and the twist of the foundation θ_o(Figure 1). The earthquake excitation is defined by $\ddot{u}_{gx}(t)$ and $\ddot{u}_{gy}(t)$, the x and y components of the ground acceleration measured at the surface of the homogeneous half-space in the far field, and $\ddot{u}_{g\theta}(t), \ddot{\gamma}_{gx}(t), \ddot{\gamma}_{gy}(t)$ are the rotational acceleration of the base of the building about the vertical and horizontal axes, respectively. The equations of motion for a SSI system with 3 N + 5 degrees of freedom can be written in the usual matrix form

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} + [M](\{\ddot{u}_{o1}(t)\}+ \{\ddot{\gamma}_{oh}(t)\} + \{\ddot{u}_{g1}(t)\} + \{\ddot{\gamma}_{gh}(t)\}) = \{0\} \tag{1a}$$

where

$$\{u(t)\} = \begin{Bmatrix} \{u_x(t)\} \\ \{u_y(t)\} \\ \{u_\theta(t)\} \end{Bmatrix} \{\ddot{u}_{o1}(t)\} = \begin{Bmatrix} \ddot{u}_{ox}(t)\{1\} \\ \ddot{u}_{oy}(t)\{1\} \\ \ddot{u}_{o\theta}(t)\{1\} \end{Bmatrix}$$

$$\{\ddot{\gamma}_{oh}(t)\} = \begin{Bmatrix} \ddot{\gamma}_{ox}(t)\{h\} \\ \ddot{\gamma}_{oy}(t)\{h\} \\ \{0\} \end{Bmatrix} \{\ddot{u}_{g1}(t)\} = \begin{Bmatrix} \ddot{u}_{gx}(t)\{1\} \\ \ddot{u}_{gy}(t)\{1\} \\ \ddot{u}_{g\theta}(t)\{1\} \end{Bmatrix}$$

$$\{\ddot{\gamma}_{gh}(t)\} = \begin{Bmatrix} \ddot{\gamma}_{gx}(t)\{h\} \\ \ddot{\gamma}_{gy}(t)\{h\} \\ \{0\} \end{Bmatrix} \tag{1b}$$

In the above expression $[M], [C]$, and $[K]$ represent structural property matrices. Viscous damping is assumed to be in such a form that the building on a rigid foundation admits decomposition into classical normal modes (Vaidya *et al.*, 1986). Furthermore, $\{u(t)\}$ and $\{h\}$ refer to column vectors of the structural displacements relative to the rigid foundation and the height of floors from the ground level, respectively. The acceleration vectors of the rigid foundation motion are $\{\ddot{u}_{o1}(t)\}$ and $\{\ddot{\gamma}_{oh}(t)\}$, respectively.

In terms of the linear transformation matrix, Eq. (1) can be simplified to

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\}+ [M][T]\{\ddot{u}_{ot}(t)\} = \{0\} \tag{2}$$

where $[T]$ represents the kinematic transfer matrix for transmitting total motion of the rigid foundation to the structure and the total displacement vector, $\{\ddot{u}_{ot}(t)\}$, and includes horizontal, rocking and torsional degrees of freedom of the rigid foundation in addition to the free field motion. The rotation and rocking acceleration components of the free field ground motion will be disregarded as indicated in Eq. (3). That is,

$$\{\ddot{u}_{ot}(t)\} = \begin{Bmatrix} \ddot{u}_{oxt}(t) \\ \ddot{u}_{oyt}(t) \\ \ddot{\theta}_o(t) \\ \ddot{\gamma}_{ox}(t) \\ \ddot{\gamma}_{oy}(t) \end{Bmatrix};$$

$$[T] = \begin{bmatrix} \{1\} & \{0\} & \{0\} & \{h\} & \{0\} \\ \{0\} & \{1\} & \{0\} & \{0\} & \{h\} \\ \{0\} & \{0\} & \{1\} & \{0\} & \{0\} \end{bmatrix}_{3N\times 5}; \tag{3}$$

$$\ddot{u}_{oxt}(t) = \ddot{u}_{ox}(t) + \ddot{u}_{gx}(t)$$
$$\ddot{u}_{oyt}(t) = \ddot{u}_{oy}(t) + \ddot{u}_{gy}(t)$$

In addition, 5 more equations are needed in order to completely solve the problem. These equations are developed by considering the equilibrium of the whole system. They can be further written in open form as

$$(m_o + \{1\}^T[m_x]\{1\})\ddot{u}_{ox}(t) + \{1\}^T[m_x]\{h\}\ddot{\gamma}_{ox}+ \{1\}^T[m_x]\{\ddot{u}_x\}+P_{ox}(t)=-(m_o+\{1\}^T[m_x]\{1\})\ddot{u}_{gx}(t) \tag{4a}$$

$$(m_o + \{1\}^T[m_y]\{1\})\ddot{u}_{oy}(t) + \{1\}^T[m_y]\{h\}\ddot{\gamma}_{oy}+ \{1\}^T[m_y]\{\ddot{u}_y\}+P_{oy}(t)=-(m_o+\{1\}^T[m_y]\{1\})\ddot{u}_{gy}(t) \tag{4b}$$

$$\{1\}^T[m_x]\{h\}\ddot{u}_{ox}(t) + (I_{tx} + \{h\}^T[m_x]\{h\})\ddot{\gamma}_{ox}+$$
$$\{h\}^T[m_x]\{\ddot{u}_x\} + M_{ox}(t) = -\{1\}^T[m_x]\{h\}\ddot{u}_{gx}(t) \tag{4c}$$

$$\{1\}^T[m_y]\{h\}\ddot{u}_{oy}(t) + (I_{ty} + \{h\}^T[m_y]\{h\})\ddot{\gamma}_{oy}+$$
$$\{h\}^T[m_y]\{\ddot{u}_y\} + M_{oy}(t) = -\{1\}^T[m_y]\{h\}\ddot{u}_{gy}(t) \tag{4d}$$

$$(m_o r_o^2 + \{1\}^T[m_\theta]\{1\})\ddot{\theta}_o(t) + \{1\}^T[m_\theta]\{\ddot{u}_\theta(t)\}+$$
$$T_o(t) = 0 \tag{4e}$$

In these equations, $\{1\}$ is the column vector where each element is unity, and m_o and r_o are the mass and the radius of gyration of the foundation, respectively. I_{tx}, I_{ty} are the total of the mass moment of inertia of the floors and the foundation mat with respect to the x and y axes. $\{h\}$ is the column vector of foundation to story heights, $[m_x]$, $[m_y]$ consist of floor masses of the structure in terms of the x and y axes, and $[m_\theta]$ is the mass polar moment of inertia about the z axis of the floor mass. P_{ox}(t), P_{oy}(t), M_{ox}(t), M_{oy}(t), T_o(t) are the horizontal SSI forces, rocking moments and torsional moment, respectively. It is noted that they are the dynamic loads imposed by the structure on the foundation. When the foundation medium is flexible these forces can be related to foundation displacements u_{ox}, u_{oy}, γ_{ox}, γ_{oy} and θ_o. Dynamic SSI force displacement equations may be specified in the frequency domain. Equation (4) can be expressed more concisely in the following form:

$$[M_1]\{\ddot{u}(t)\} + [M_2]\{\ddot{u}_{ot}(t)\} + \{P_o(t)\} = \{0\};$$
$$\{P_o(t)\}^T = \{P_{ox}(t) \quad P_{oy}(t) \quad M_{ox}(t) \quad M_{oy}(t) \quad T_o(t)\} \tag{5}$$

where $[M_1] = [T]^T[M]$ and $[M_2]$ includes components of superstructure, and these refer to the total foundation mass matrix. They are given by

$$[M_1] = \begin{bmatrix} \{1\}^T[m_x] & \{0\}^T & \{0\}^T \\ \{0\} & \{1\}^T[m_y] & \{0\}^T \\ \{0\}^T & \{0\}^T & \{1\}^T[m_\theta] \\ \{h\}^T[m_x] & \{0\}^T & \{0\}^T \\ \{0\}^T & \{h\}^T[m_y] & \{0\}^T \end{bmatrix}_{5\times 3N}$$

$$[M_2]=\begin{bmatrix} m_o + \{1\}^T[m_x]\{1\} & 0 & \{1\}^T[m_x]\{h\} & 0 & 0 \\ 0 & m_o + \{1\}^T[m_y]\{1\} & 0 & \{1\}^T[m_y]\{h\} & 0 \\ 0 & 0 & 0 & 0 & m_\theta + \{1\}^T[m_\theta]\{1\} \\ \{1\}^T[m_x]\{h\} & 0 & I_{tx} + \{h\}^T[m_x]\{h\} & 0 & 0 \\ 0 & \{1\}^T[m_y]\{h\} & 0 & I_{ty} + \{h\}^T[m_y]\{h\} & 0 \end{bmatrix}_{5\times 5} \tag{6}$$

Dynamic soil-structure interaction

The soil is assumed to be an elastic homogeneous isotropic half-space modeled by a massless rigid plate supported by equivalent translational, rotational and torsional springs and dashpots. Because the foundation stiffness and damping coefficients are dependent on excitation frequency, it is most convenient to consider the response of the building foundation system to harmonic ground motion. Due to these frequencies-based expressions for interaction shear force, moment, and torque, the interaction problem lends itself readily to formulation in the frequency domain. Thus, the Fourier transform is applied to the equations of motion (2) and (5), and the following, very simple, results are obtained:

$$(-\omega^2[M]+i\omega[C]+[K])\{\tilde{u}(\omega)\}-\omega^2[M][T]\{\tilde{u}_{ot}(\omega)\}=\{0\};$$
$$-\omega^2[M_1]\{\tilde{u}(\omega)\}-\omega^2[M_2]\{\tilde{u}_{ot}(\omega)\}+\left[\tilde{S}(\omega)\right]\{\tilde{u}_{ot}(\omega)\}=$$
$$\left[\tilde{S}(\omega)\right]\{\tilde{u}_g(\omega)\} \tag{7}$$

where $\{\tilde{u}_g(\omega)\}$ represents the Fourier transform of free field ground translation. The transformed equations can be rewritten in the matrix notation as follows:

$$\left[\tilde{K}(\omega)\right]\left\{\tilde{U}(\omega)\right\}=\left\{\tilde{P}(\omega)\right\} \tag{8}$$

The Fourier transformed dynamic stiffness matrix and the displacement vector consisting of structural and foundation displacement components take the following form:

$$\left[\tilde{K}(\omega)\right]=\begin{bmatrix}-\omega^2[M]+i\omega[C]+[K] & -\omega^2[M][T]\\ -\omega^2[M_1] & \left[\tilde{S}(\omega)\right]-\omega^2[M_2]\end{bmatrix}$$
$$\left\{\tilde{U}(\omega)\right\}=\left\{\begin{array}{c}\{\tilde{u}(\omega)\}\\ \{\tilde{u}_{ot}(\omega)\}\end{array}\right\} \tag{9}$$

where $\{\tilde{u}(\omega)\}$ is the Fourier transform of the $\{u(t)\}$. For harmonic excitation of frequency ω the dynamic stiffness matrix $\left[\tilde{S}(\omega)\right]$ of the foundation defined as the ratio of the amplitude of the applied load $\left\{\tilde{P}(\omega)\right\}$ to that resulting displacement $\{\tilde{u}_{ot}(\omega)\}$ is written as

$$\left[\tilde{S}(\omega)\right]=\begin{bmatrix}K_{hx}(\omega) & 0 & K_{hxrx}(\omega) & 0 & 0\\ 0 & K_{hy}(\omega) & 0 & K_{hyry}(\omega) & 0\\ K_{hxrx}(\omega) & 0 & K_{rx}(\omega) & 0 & 0\\ 0 & K_{hyry}(\omega) & 0 & K_{ry}(\omega) & 0\\ 0 & 0 & 0 & 0 & K_t(\omega)\end{bmatrix} \tag{10}$$

with Reissner's dimensionless frequency parameter $a_o=\omega r/c_s$ (shear-wave velocity c_s). The numerical approximations for the impedance functions used in this study are taken from Veletsos and Wei (1971) for lateral and rocking vibration and from Veletsos and Nair (1974) for torsional vibration. The foundation impedances can be obtained from the solution of a mixed boundary value problem in elastodynamics and are generally functions of soil properties, foundation size and exciting frequency. The values taken by the coupling terms are minor, above all for the usual values of Poisson's ratio of soil between 0.3 and 0.5. For this reason, the terms K_{hxrx} and K_{hyry} are disregarded for superficial foundations (Schmid *et al.*, 1988; Siefert, 1996). Assuming exterior terms of the diagonal of the dynamic stiffness matrix, which introduces neglible errors for most practical purposes, the impedance functions are described for the terms of the main diagonal of the matrix $\left[\tilde{S}(\omega)\right]$ as

$$K_j=K_{s\infty j}(k_j(a_o)+ia_oc_j(a_o))(1+2i\beta) \tag{11}$$

The internal damping of soil is also taken into consideration and is characterized by the damping ratio β. In this complex variable notation k_j represents the dimensionless spring coefficient and c_j represents the corresponding damping coefficient depending on a_o and the Poisson's ratio ν. The real parts of the impedance functions signify force components in phase with the displacements and can be termed dynamic stiffness for the foundation. On the other hand, the imaginary parts are force components in phase with the velocities and can be interpreted as energy dissipation by radiation of waves away from the foundation into the soil. Therefore, they may be termed foundation damping coefficients.

Static stiffness coefficients may be defined as $K_{S\infty j}$ for each degree of freedom – or mode – (for indices: hx and hy, horizontal displacements along the x and y axes, and for indices rx, ry and t, rotations along the x, y and z axes, respectively). It is noted that $K_{hx}=K_{hy}=8Gr/(2\text{-}\nu)$ is the static horizontal force necessary to produce a unit horizontal displacement of the disk with no restriction on the value of the resulting rotation, $K_{\gamma rx} = K_{\gamma ry} = 8Gr^3/3(1\text{-}\nu)$ is the static rocking moment necessary to rotate the disk through a unit angle with no restriction on the value of the resulting horizontal displacement, $K_t = 16c_s^2\rho r^3/3$ is the static torque necessary to twist the disk through a unit angle where ρ denotes the mass density of soil. The SSI relationships given by Eq. (8) were developed for a massless circular rigid disk. In this study, the research has been applied to a rectangular foundation in an approximate way by use of equivalent values for the radius of the rigid disk r_o such that the resulting static stiffness coefficients are the same as those corresponding to the rectangular foundation proposed by Thomson and Kobori (1963).

Modal Analysis

Based on the assumption that the fixed-base structure possesses classical normal modes, the structural response in the complex frequency domain can be expressed in terms of the mode shapes as

$$\{\tilde{u}(\omega)\} = \left\{ \begin{array}{c} \{\tilde{u}_x(\omega)\} \\ \{\tilde{u}_y(\omega)\} \\ \{\tilde{u}_\theta(\omega)\} \end{array} \right\} = [\phi]\{\tilde{q}(\omega)\} \tag{12}$$

where $\{\tilde{q}(\omega)\}$is the column vector of the Fourier transform of the modal displacements and $[\phi]$ is the matrix consisting of normal mode shapes of the fixed-base structure as

$$\{\tilde{q}(\omega)\} = \left\{ \begin{array}{c} \tilde{q}_1(\omega) \\ \tilde{q}_2(\omega) \\ \vdots \\ \tilde{q}_{3N}(\omega) \end{array} \right\}_{(3N\times 1)} ; \tag{13}$$

$$[\phi] = \begin{bmatrix} [\phi_{xx}] & [\phi_{xy}] & [\phi_{x\theta}] \\ [\phi_{yx}] & [\phi_{yy}] & [\phi_{y\theta}] \\ [\phi_{\theta x}] & [\phi_{\theta y}] & [\phi_{\theta\theta}] \end{bmatrix}_{(3N\times 3N)}$$

Substituting Eq. (12) into $\left[\tilde{K}(\omega)\right]\left\{\tilde{U}(\omega)\right\} = \left\{\tilde{P}(\omega)\right\}$ equations of motion of a building system with a rigid foundation in the frequency domain and applying the orthogonality conditions, Eq. (8), when the first row is premultiplied by $[\phi]^T$, introduces

$$\left[\tilde{K}_H(\omega)\right]\left\{\tilde{U}_H(\omega)\right\} = \left\{\tilde{P}_H(\omega)\right\} \tag{14a}$$

and also note that

$$\left[\tilde{K}_H(\omega)\right] = \begin{bmatrix} [H(\omega)] & -\omega^2[\Gamma] \\ -\omega^2[\Gamma]^T & \left[\tilde{S}(\omega)\right] - \omega^2[M_2] \end{bmatrix}$$

$$\left\{\tilde{U}_H(\omega)\right\} = \left\{ \begin{array}{c} \{\tilde{q}(\omega)\} \\ \{\tilde{u}_{ot}(\omega)\} \end{array} \right\} \tag{14b}$$

where $[\Gamma]$ represents the modal participation matrix defined as

$$[\Gamma] = [\phi]^T[M][T] \tag{15}$$

A diagonal matrix whose elements of modal structural transfer functions and typical element corresponding to the k^{th} mode are written as

$$[H(\omega)] = \begin{bmatrix} H_1(\omega) & 0 & 0 & 0 \\ 0 & H_2(\omega) & 0 & 0 \\ 0 & 0 & . & 0 \\ 0 & 0 & 0 & H_{3N}(\omega) \end{bmatrix}_{3N\times 3N} \tag{16}$$

$$H_k(\omega) = -\omega^2 + 2i\xi_k\omega_k\omega + \omega_k^2 \quad k = 1, 2, 3.............3N$$

The modal displacement of the structure can then be written in terms of foundation deformations from the first row of Eq. (14a) as given below; the frequency variation of the configuration vector can be subsequently computed, by summing up modal responses, as

$$\{\tilde{q}(\omega)\} = \omega^2[H(\omega)]^{-1}[\Gamma]\{\tilde{u}_{ot}(\omega)\} \tag{17}$$

Substituting Eq. (17) in the second row of Eq. (14b), the foundation displacement components can be obtained from the following equation:

$$\{\tilde{u}_{ot}(\omega)\} = \left[\tilde{D}(\omega)\right]^{-1}\left[\tilde{S}(\omega)\right]\{\tilde{u}_g(\omega)\} \quad \tilde{u}_{gx} = 0 \tag{18}$$

in which the total dynamic stiffness matrix, $\left[\tilde{D}(\omega)\right]$, is expressed as

$$\left[\tilde{D}(\omega)\right] = \left[\tilde{S}(\omega)\right] - \omega^2([M_2] + \omega^2[\Gamma]^T[H(\omega)]^{-1}[\Gamma]) \tag{19}$$

After Eq. (18) is solved for the foundation displacement vector, the modal structural response in the complex frequency domain can be calculated according to Eq. (17). Finally, the structural displacements of each floor in the frequency domain can be calculated by the mode superposition relations as

$$\{\tilde{u}(\omega)\} = \left\{ \begin{array}{c} \{\tilde{u}_x(\omega)\} \\ \{\tilde{u}_y(\omega)\} \\ \{\tilde{u}_\theta(\omega)\} \end{array} \right\} = [\phi]\{q(\omega)\} \tag{20}$$

The time-domain solution can be obtained through the inverse Fourier transform of Eq. (21) as follows:

$$\{u(t)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \{\tilde{u}(\omega)\}e^{i\omega t}d\omega \tag{21}$$

With the above computations, a simplified analysis method performed by discrete Fourier transform techniques using the fast Fourier transform algorithm based on modal decomposition with all vibration modes to solve the interaction problem may be summarized in the following analysis procedure:

Defining the structural and soil properties with an earthquake ground acceleration record,

Step 1. Modal analysis on the superstructure;

a) Solve the eigenvalue problem, determine the natural frequency and mode shapes.

$$[M], [K] \Rightarrow \mathrm{diag}\big[\omega_k^2\big], [\phi](k = 1....3N).$$

b) Evaluate the kinematic transfer matrix from Eq. (3)$\Rightarrow [T]$.

c) Set up total foundation mass matrices from Eq. (6) $\Rightarrow [M_1]$, $[M_2]$.

d) Determine the modal participation matrix from Eq. (15) $\Rightarrow [\Gamma]$.

Step 2. Frequency domain analysis at each frequency (ω);

a) Fourier transform of ground acceleration, $[\ddot{u}_g(t)] \Rightarrow [\ddot{u}_g(\omega)]$.

b) Determine the modal structural transfer functions from Eq. (16) $\Rightarrow [H(\omega)]$.

c) Compute the static soil spring constants $\Rightarrow K_{S\infty j}$ (for j = hx, hy, rx, ry and t).

d) Define the foundation impedance functions depending on soil properties (ρ, ν, β, and G) from Eq. (11) $\Rightarrow K_j = K_{s\infty j}(k_j(a_o) + ia_oc_j(a_o))(1 + 2i\beta)$.

e) Set up a total dynamic stiffness matrix from Eq. (19) $\Rightarrow \left[\tilde{D}(\omega)\right]$.

f) Calculate the translational, rotational and torsional displacements of the foundation from Eq. (18) $\Rightarrow \{\tilde{u}_{ot}(\omega)\}$.

g) Compute the modal displacements for each floor from Eq. (17) $\Rightarrow \{\tilde{q}(\omega)\}$.

h) Define the structural displacements from Eq. (20) $\Rightarrow \{\tilde{u}(\omega)\}$.

Step 3. Time domain analysis;

a) Determine the displacement response by using the inverse Fourier transform from Eq. (21) $\Rightarrow \{u(t)\}$.

b) Calculate the design story shear forces.

c) Design of members.

It is clear that the above design procedure is similar to that for the analysis of a fixed base structure with the addition of steps 2c-2f and the alteration of step 2g to include the deformability of the soil medium.

Numerical Analysis

The application of the proposed method of analysis including all modes of vibration in the calculations to evaluate the dynamic SSI effects on the 3-dimensional buildings with eccentricity in only one direction is considered. The configuration of the soil-structure system is shown in Figure 2. The main parameters of the dynamical structure foundation model and properties of the soil are summarized in this figure. The monosymmetric 2-story building resting on homogeneous elastic soil through a rigid square foundation consists of reinforced concrete frames joined at each floor level by a rigid diaphragm. The floors, including the foundation, were assumed to be identically rectangular, with the length of the building d = 12 m and the width of the building b = 12 m. In this example, the center of stiffness is assumed to lie at eccentricity e = 1.0 m from the center of mass. It has been assumed that the floor slab and the foundation mat have the same eccentricity along the x axis in a direction perpendicular to the input motion, and no eccentricity along the y axis. The mass $m_i = 50$ t and torsional radius of gyration $r_i = 5$ m at each floor level (i = 1,2) are the same. The rigid foundation mat is idealized as a circular plate of radius $r_o = 5$ m and its mass also taken to be $m_o = 50$ t. It should be noted that the radius of the base mass is taken as the radius of a circle having the same area as the plane of the floor.

It is considered that the structure has the same translational stiffness in the x and y directions, and that the SSI translational and rocking stiffnesses are respectively the same in both horizontal directions. The story stiffnesses of these frames are K_{yAi}, K_{xCi} and $K_{xBi} = 26500$ kN/m, and the story heights are $h_i = 3$ m, also the same for all floors (i = 1,2). The damping ratio of the superstructure in each mode of vibration is taken as 5%.

The viscoelastic foundation medium is assumed to have a density of 20 kN/m^3 and a Poisson's ratio of $\nu = 0.33$, and the material damping ratio is $\beta = 0.05$. To indicate the significance of SSI effects on structural response, the shear wave velocity (c_s) of elastic half-space material was selected in a range of $300 \leq c_s \leq 1500$ m/s. The upper limit of 1500 m/s for the shear wave velocity of the soil may be assumed to refer to a very stiff ground condition (fixed base). The lower limit of 300 m/s for the shear velocity is chosen for defining soft soil conditions.

For a parametric study, the key parameters controlling the dynamic structural response including the effects of foundation interaction are chosen as the ratio, m/m_o, of the superstructure mass to the mass of the foundation, the ratio, H/r, of the total height of the structure to the radius of the foundation base taken here as the same as the floor slabs, and the ratio, e/r, of the eccentricity to the radius of the foundation base. The practical range of variables considered in this study is taken as $0.5 \leq m/m_o \leq 3$ for the mass ratio, $0.05 \leq e/r \leq 0.25$ for the eccentricity ratio, and $0.5 \leq H/r \leq 2$ for the height ratio.

The responses of this building foundation system to the torsional effects were obtained when it was subjected to the Erzincan, 1992, earthquake (E-W component, M = 6.8) as shown in Figure 3 with peak acceleration of 0.5 g as the free field ground motion in a direction perpendicular to the direction of the eccentricity.

In Figure 4, the variation of lateral deflection, rocking and twisting (torsional) components of the foundation base for different shear wave velocities of soil are plotted as a function of time and compared to the corresponding values for a fixed base structure. The maximum values of the foundation displacements for Erzincan, 1992, excitations are $u_o = 0.028$ m, $\gamma_o = 8.85 \times 10^{-3}$rad and $\theta_o = 4.25 \times 10^{-3}$rad for a shear velocity of $c_s = 300$ m/s (soft soil condition) in the case of intermediate high buildings with moderate eccentricity where $m/m_o = 1$, H/r = 1.2, and e/r = 0.2. However, the corresponding values are $u_o = 0.0025$ m, $\gamma_o = 9 \times 10^{-4}$rad and $\theta_o = 4.2 \times 10^{-4}$rad for relatively rigid based buildings, in which case the structure is considered to be supported by the soil with a shear wave velocity of $c_s = 1500$ m/s, as shown in Figure 4. It may be noted that the deformability of the foundation soil significantly influences the foundation displacements. Therefore, the SSI effects on the structural response are shown to be more important when the soil becomes softer, in soil with a lower value of c_s.

Furthermore, in order to demonstrate the application of this modal superposition method to practical problems, a detailed parametric analysis of the same idealized 3-dimensional model has been solved with various controlling parameters (m/m_o, H/r, and e/r) to estimate the dynamic behavior of the soil structure system for 2 ground conditions, for stiffer soil conditions, $c_s = 1500$ m/s, and for softer soil conditions, $c_s = 500$ m/s, as shown in Figures 5-7, respectively.

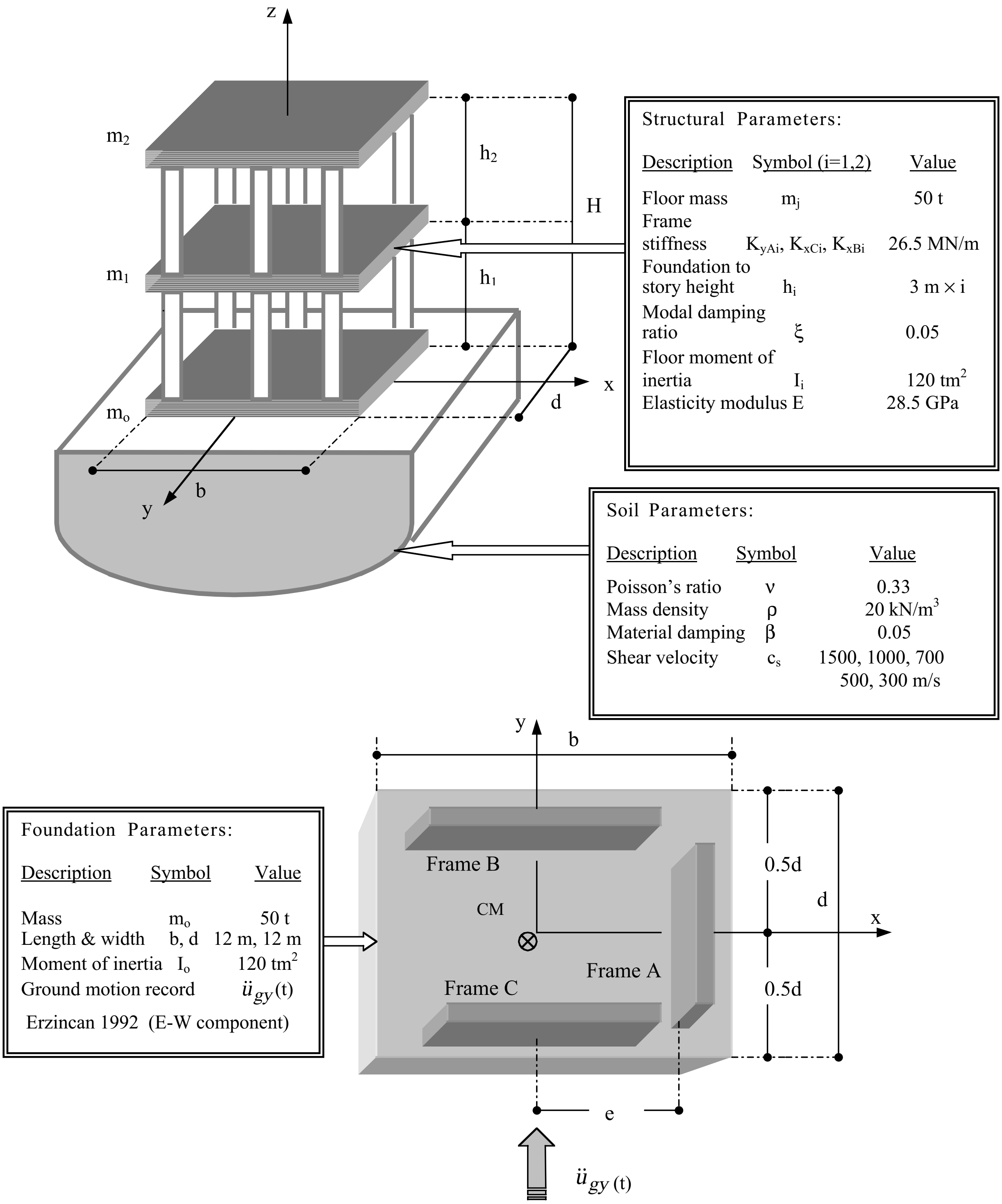

Figure 2. Configuration of 2-story building and model parameters for numerical examples.

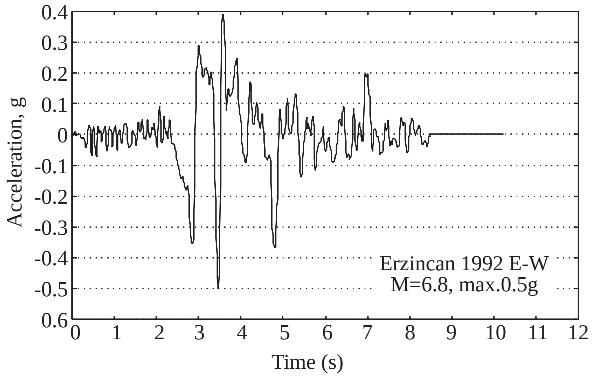

Figure 3. Time history acceleration of free-field input motion at the surface.

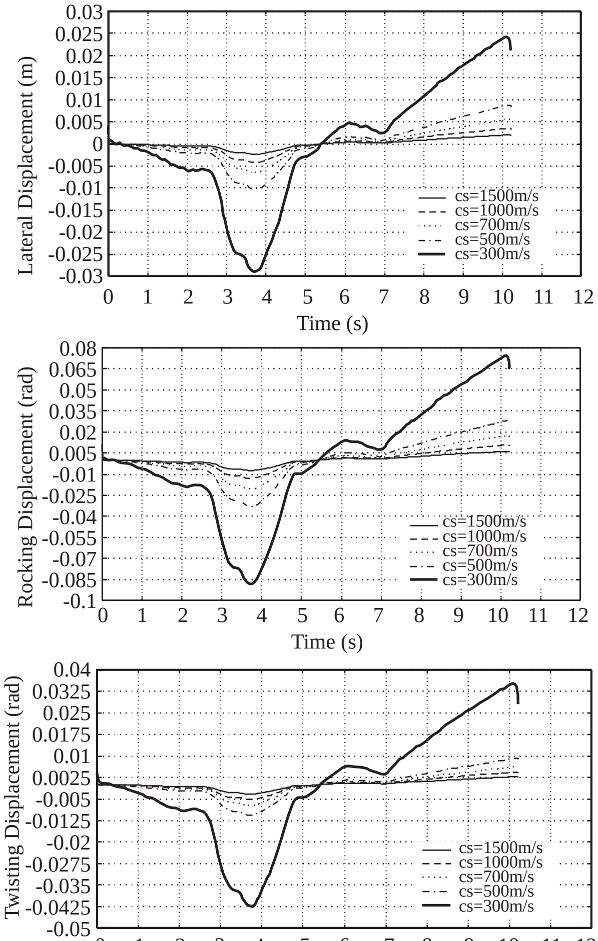

Figure 4. Variation of the displacement components of the foundation base depending shear velocity in the case of m/m$_o$ = 1, H/r = 1.2 and e/r = 0.2.

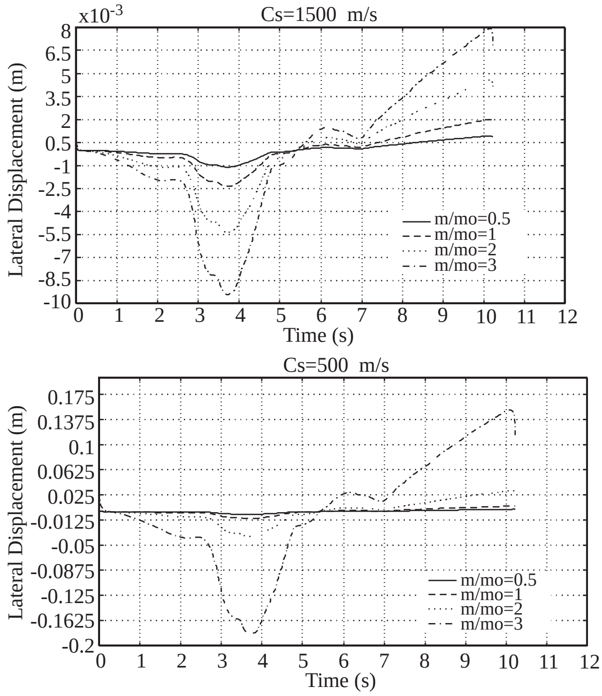

Figure 5. Variation of the lateral displacement components of foundation base depending mass ratio for shear wave velocity of 1500 m/s and 500 m/s in the case of H/r = 1.2 and e/r = 0.2.

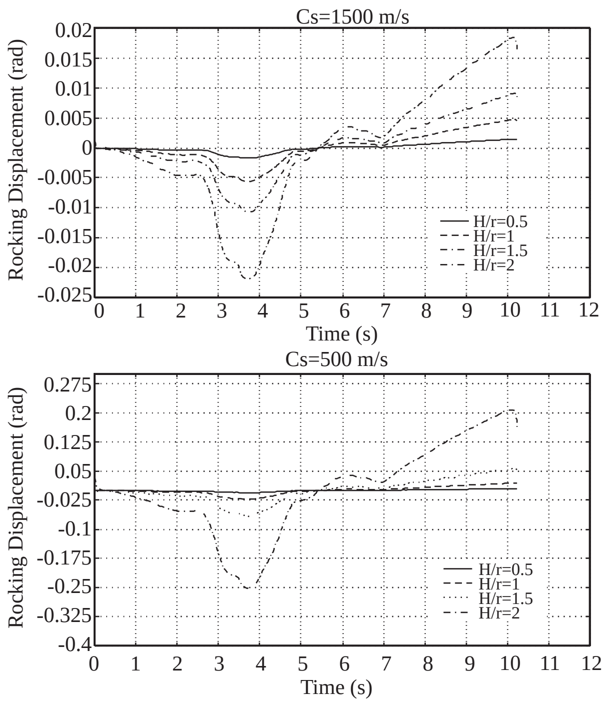

Figure 6. Variation of the rocking displacement components of the foundation base depending on height ratio for a shear wave velocity of 1500 m/s and 500 m/s in the case of m/m$_o$ = 1 and e/r = 0.2.

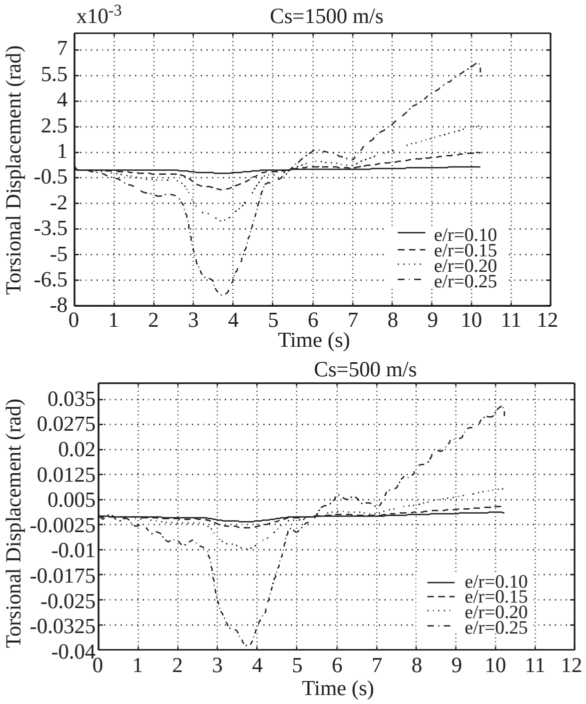

Figure 7. Variation of the torsional displacement components of foundation base depending on the eccentricity ratio for a shear wave velocity of 1500 m/s and 500 m/s in the case of m/m_o = 1 and H/r = 1.2.

Four different values of height to gyration radius ratio (h/r), eccentricity to gyration radius ratio (e/r), and mass of the structure to mass of the foundation ratio (m/m_o), depending on the shear wave velocity of the soil medium, are considered.

It should be noted that an apparent increase occurs in the foundation displacement components as the shear wave velocity of the soil medium decreases. From those time-domain responses, it is observed that in rigid and soft soil conditions, an obvious increase occurs in the horizontal displacement of the foundation as the mass ratio, m/m_o increases, as shown in Figure 5. For instance, the maximum value of the foundation lateral displacement is 2.25×10^{-3}m in the case of $m/m_o = 1$ for the rigid foundation medium, whereas the corresponding value is 9.35×10^{-3}m in the case of $m/m_o = 3$. In this situation, the foundation mass is assumed to be reasonably small compared to the mass of the superstructure.

Additionally, the lateral displacement of the foundation based on soil with $c_s = 500$ m/s increases to almost 200 times the lateral displacement of the foundation based on soil with a shear velocity of $c_s =$ 1500 m/s, for the ratio of $m/m_o = 3$. Furthermore, the rocking rotation of the foundation is affected by any change in the value of the height ratio, H/r, especially when the soil is softer, as shown in Figure 6. It modifies the rocking rotation of the foundation, decreases for short, squat structures and increases for tall, slender structures. The smaller the value of c_s and the greater the value of the height ratio, H/r, the more pronounced the interaction effects become on foundation rotation. For instance, for H/r = 2 and c_s = 1500 m/s, the peak of the rotation angle is approximately 0.022 rad, whereas for c_s = 500 m/s this value is 0.25 rad. For squat structures, these rocking displacements decrease by up to 1.5×10^{-3}rad and 0.022 rad for rigid and soft soil conditions, respectively.

From Figure 7 it can be seen that the peak values of torsional displacement of the foundation increased significantly in the case of intermediate high buildings with a moderate mass ratio for low soil foundation stiffness when compared to those obtained for rigidly based structures as the eccentricity ratio, e/r, increases. The maximum torsional rotations of the foundation obtained in this analysis are approximately 0.00725 rad and 0.0385 rad for systems having a shear wave velocity of soil with c_s = 1500 m/s and c_s = 500 m/s, respectively, when the eccentricity ratio is taken as e/r = 0.25. Furthermore, it should be noted that when the eccentricity ratio increased from 0.2 to 0.25, the torsional rotations also increased by 2.5 times and 4 times for buildings founded on stiff and soft soils, respectively.

Conclusions

In this study a simplified method of analysis is presented using modal decomposition considering all modes of vibration in the calculation to obtain the structural responses of torsionally asymmetric buildings, including soil-structure interaction effects in the frequency domain, by using the fast Fourier transform. Applying this algorithm, the advantage of classical normal mode approximation is used for almost all vibration modes, and the interaction problem is solved easily and effectively within the framework of the Fourier-transformed frequency domain analysis for a fixed base structure. The matrix formulation of this method produces accurate approximation with less computational effort, despite using frequency dependent impedance functions.

The results presented in this paper indicate that the earthquake response of a soil-torsionally coupled structure interaction can be signicantly different from that calculated with a fixed base model. This work has been based on the assumptions that the foundation is supported at the surface of a viscoelastic half-space and that the superstructure responds within the elastic range. From these time-domain responses, it has been shown that an obvious increase occurs in the horizontal and rocking displacements of the foundation as the predefined mass ratio and height ratio increase, respectively, for soft soil conditions compared to the associated fixed-base system.

This detailed parametric study shows that the effects of deformability of the foundation medium cause the dynamic behavior of the structure foundation system to differ, particularly when the shear wave velocity representing the soil stiffness is lower than 1000 m/s.

For a medium highrise building with moderate or large eccentricity, increased torsional loadings must be taken into account for the combined lateral-torsional response for such structures supported on a moderately and very flexible foundation medium.

Nomenclature

Roman Symbols

a_o	Reissner parameter
c_j	foundation damping coefficients for each degree of freedom - or the mode j
c_s	velocity of shear wave in the soil
$[C]$	viscous damping matrix of structure
$\left[\tilde{D}(\omega)\right]$	total dynamic stiffness matrix
e	structural (static) eccentricity
G	shear modulus of soil
$\{h\}$	column vector of heights
$[H(\omega)]$	modal structural transfer functions
I_{tx}, I_{ty}	the total of the mass moment of inertia of the floors and the foundation mat with respect to the x and y axes
k_j	dimensionless spring coefficients for mode j
K_{hx},K_{hy}	static translational stiffness of foundation in the x and y directions, respectively
$K_{\gamma rx}, K_{\gamma ry}$	static rocking moment of foundation about the x and y axes, respectively
K_t	static torque of foundation
K_j	foundation impedance functions for the mode j
$K_{S\infty j}$	static stiffness for the half-space for the mode j
$[K]$	stiffness matrix of structure
m_i, m_o	floor and foundation mass
$M_{ox}(t)$, $M_{oy}(t)$	overturning moments at foundation-soil interface in the x and y directions
$[m_x], [m_y]$	mass matrices consisting of floor masses with respect to the x and y axes
$[m_\theta]$	mass polar moment of inertia, about the z axis of the floor mass
$[M]$	mass matrix of structure
$P_{ox}(t)$, $P_{oy}(t)$	horizontal soil-structure interaction forces in the x and y directions, respectively
$\left\{\tilde{P}(\omega)\right\}$	frequency dependent external force vector
$\{\tilde{q}(\omega)\}$	column vector of Fourier transform of the modal displacements
r, r_o	radius of floor and foundation (base) disk, respectively
$\left[\tilde{S}(\omega)\right]$	dynamic-stiffness matrix of the foundation defined as the ratio of the amplitude of the applied load $\left\{\tilde{P}(\omega)\right\}$
$T_o(t)$	torsional soil-structure interaction moment
$[T]$	kinematic transfer matrix
u_{xi}, u_{yi}	horizontal displacements of ith floor in the x and y directions, respectively
$u_{\theta i}$	rotation about vertical axis
u_{ox}, u_{oy}	horizontal translations of the foundation with respect to the x and y axes
$\ddot{u}_{gx}(t), \ddot{u}_{gy}(t)$	free field ground acceleration measured at the surface
$\{u(t)\}$	column vectors of the structural displacements relative to the rigid foundation
$\{\tilde{u}(\omega)\}$	Fourier transform of the $\{u(t)\}$
$\{\ddot{u}_{g\theta}(t)\}$	the rotational acceleration of the base of the building about the vertical axis
$\{\ddot{u}_{o1}(t)\}$	the acceleration vector of the rigid foundation motion
$\{\tilde{u}_{ot}(\omega)\}$	Fourier transform of the total displacement vector of the rigid foundation in addition to the free field motion

Greek Symbols

β	internal damping ratio of soil
$[\phi]$	normal mode matrix

γ_{ox}, γ_{oy}	rocking rotations of the foundation in the directions of the x and y axes, respectively	ν	Poisson's ratio of soil
$\ddot{\gamma}_{gx}(t), \ddot{\gamma}_{gy}(t)$	the rotational acceleration of the base of the building along horizontal axes	ρ	soil mass density
$[\Gamma]$	modal participation matrix	θ_o	twist of the foundation
		ω	circular frequency of excitation
		ω_k	natural frequency of kth mode

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