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A Study of Pressure Distribution of A Slider Bearing Lubricated with Powell-Eyring Fluid

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Abstract

A slider bearing consisting of connected surfaces with Powell-Eyring fluid as a lubricant is analyzed in the present study. A Powell-Eyring model has been used as a non-Newtonian lubricant in a slider bearing. The analysis is based on the perturbation technique. Under the assumptions of the order of magnitudes of the variables, it is seen that only viscous and non-Newtonian terms have effects, whereas inertia terms are negligible. Choosing non-Newtonian effects smaller than viscous effects, a perturbation solution is constructed. The pressure distribution in the bearing is calculated approximately.

Key words: Slider bearing, Non-newtonian fluids, Perturbation analysis, Powell-eyring model, Lubrication.

Introduction

The lubrication of bearings is an important technological problem. The pressure distribution in the bearing should be known for proper functioning. Much work has been done on Newtonian-type lubrication. However, additives are frequently used in lubricating fluids, which makes the flow non-Newtonian.

Some relevant studies on non-Newtonian lubrication in bearings have been published. Ng and Saibel (1962) used a special third-grade fluid (second-grade terms neglected) and studied the flow occurring in a slider bearing. Harnoy and Hanin (1974) and Harnoy and Philippoff (1976) studied the flow of a second-grade fluid in a journal bearing. Bourgin and Gay (1983) used a model similar to that of Ng and Saibel (1962) to investigate the behavior of flow in a journal bearing. Buckholz (1985) used a power-law model as a non-Newtonian lubricant in a slider bearing. More recently, Kacou *et al.* (1987) studied the flow of a third-grade fluid in a journal bearing and constructed a perturbative solution. The work is extended by the same authors (Kacou *et al.* [1988]) by including thermal effects. Yürüsoy and Pakdemirli (1999) studied the flow of a special third-grade fluid in a slider bearing. Yürüsoy (2002) has investigated second- and third-grade fluids in a slider bearing and used a perturbative solution. Bujurke and Jagadeeswar (1992) used second-grade fluid as a non-Newtonian lubricant in a taper-flat slider bearing and constructed a von-Karman momentum integral solution. Na (1994) investigated the boundary layer flow of the Reiner-Philippoff model. Hansen and Na (1968) considered the similarity solution of the laminar boundary layer problem of the Powell-Eyring model.

In this study, a Powell-Eyring model was used as a non-Newtonian lubricant in a slider bearing. The Powell-Eyring model, although mathematically more complex, deserves our attention for at least two reasons. Firstly, it can be deduced from a kinetic theory of gases rather than the empirical relation as in the power-law model. Secondly, it correctly reduces to Newtonian behavior for low and high shear rates for otherwise pseudoplastic systems, whereas the power-law model indicates an infinite effective viscosity for low shear rate, thus limiting its range of applicability. Mathematically, the Powell-Eyring model can be written as (see reference Hansen and Na [1968])

$$\tau_{yx} = \mu \frac{\partial u^*}{\partial y^*} + \frac{1}{B} \sinh^{-1} \frac{1}{C} \frac{\partial u^*}{\partial y^*} \tag{1}$$

where τ_{yx} is shear stress, μ is viscosity and B and C are constants of the Powell-Eyring model.

First, the equations of motion for a Powell-Eyring fluid in a slider bearing will be derived. Under the thin film assumption, viscous, non-Newtonian effects remain significant whereas the inertial term can be neglected in a slider bearing flow. Then assuming that non-Newtonian effects, are small compared to the viscous effects a perturbation type of solution is constructed. The first term in the solution is due to Newtonian behavior and non-Newtonian terms are added to the Newtonian solution as corrections. The pressure distributions are calculated approximately and the effect of non-Newtonian behavior is shown in figures.

Equation of Motion

The slider bearing is shown in Figure 1. The continuity and linear momentum equations are

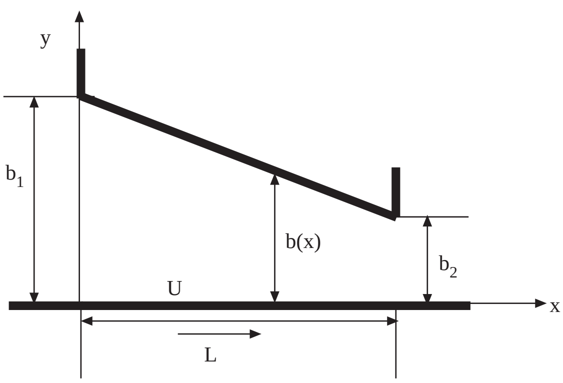

Figure 1. Slider bearing.

$$div\ \mathbf{v} = 0 \tag{2}$$

$$div \tau = \rho \frac{dv}{dt} \tag{3}$$

Let us introduce the following non-dimensional parameters:

$$x = \frac{x^*}{L},\ y = \frac{y^*}{b_1},\ u = \frac{u^*}{U},\ v = \frac{Lv^*}{b_1 U},\ b = \frac{b^*}{b_1},$$
$$p = \frac{p^*}{\rho U^2} \frac{b_1}{L} \tag{4}$$

Substituting Eqs. (1) and (4) into (2) and (3), gives

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}$$

$$\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{1}{\delta}\frac{\partial p}{\partial x} + \frac{1}{Re}\frac{1}{\delta^2}\frac{\partial^2 u}{\partial y^2} + \widehat{\alpha}\frac{1}{\delta^2}\frac{\frac{\partial^2 u}{\partial y^2}}{\sqrt{\beta\left(\partial u/\partial y\right)^2 + 1}} \tag{6}$$

$$\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{1}{\delta^3}\frac{\partial p}{\partial y} + \widehat{\alpha}\frac{1}{\delta^2}\frac{\frac{\partial^2 u}{\partial y \partial x}}{\sqrt{\beta\left(\partial u/\partial y\right)^2 + 1}} \tag{7}$$

In the above equations, the non-dimensional parameters are

$$Re = \frac{\rho U L}{\mu},\quad \widehat{\alpha} = \frac{1}{LBCU\rho},\quad \beta = \frac{U^2}{b_1^2 C^2},\quad \frac{1}{\delta} = \frac{L}{b_1} \tag{8}$$

In Eqs. (5)-(7), only the largest terms in each group are retrieved. We may now assume that 1/Re is of order δ, $\widehat{\alpha}$ of order δ ($\widehat{\alpha}=\delta\widehat{\gamma}$). Under these assumptions, the largest terms in Eqs. (5)-(7) are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{9}$$

$$\frac{dp}{dx} = \frac{\partial^2 u}{\partial y^2} + \widehat{\gamma}\frac{\frac{\partial^2 u}{\partial y^2}}{\sqrt{\beta\left(\partial u/\partial y\right)^2 + 1}} \tag{10}$$

$$\frac{\partial p}{\partial y} = 0 \tag{11}$$

The boundary conditions for the problem are

$$u(0) = 1, u(b) = 0, v(0) = 0, v(b) = 0 \tag{12}$$

Velocity profile

In this section, velocity profile will be calculated approximately. Assuming that the non-Newtonian term is small compared to the viscous term, one may write

$$\widehat{\gamma} = \varepsilon\gamma \tag{13}$$

where ε is a small parameter. When the non-Newtonian term of equation (10) is expanded into power series, we have

$$\frac{dp}{dx} = \frac{\partial^2 u}{\partial y^2} + \widehat{\gamma}\frac{\partial^2 u}{\partial y^2}\left(1 - \frac{1}{2}\beta\left(\frac{\partial u}{\partial y}\right)^2 + O(\beta^2)\right) \tag{14}$$

where $\widehat{\gamma}$ and β are dimensionless material constants in the Powell-Eyring model. The solution procedure attempted here is that of the perturbation technique of Nayfeh (1981). The approximate velocity profile in the x and y directionb and the approximate pressure profile can then be written as

$$u = u_0 + \varepsilon u_1 v = v_0 + \varepsilon v_1 \quad p = p_0 + \varepsilon p_1 \tag{15}$$

Substituting Eqs. (15) and (13) into Eqs.s (14), (9) and (12) one has

Order 1:

$$\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} = 0 \tag{16a}$$

$$\frac{\partial^2 u_0}{\partial y^2} = \frac{dp_0}{dx} \tag{16b}$$

$$u_0(0) = 1, u_0(b) = 0, v_0(0) = 0, v_0(b) = 0 \tag{16c}$$

Order ε:

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \tag{17a}$$

$$\frac{\partial^2 u_1}{\partial y^2} = \frac{dp_1}{dx} - \gamma\frac{\partial^2 u_0}{\partial y^2}\left(1 - \beta\left(\frac{\partial u_0}{\partial y}\right)^2\right) \tag{17b}$$

$$u_1(0) = 0, u_1(b) = 0, v_1(0) = 0, v_1(b) = 0 \tag{17c}$$

Equations (16a-c) represent the Newtonian problem with the well-known solution

$$u_0 = \frac{1}{2}\frac{dp_0}{dx}\left(y^2 - yb\right) + \left(1 - \frac{y}{b}\right) \tag{18}$$

$$v_0 = -\frac{1}{2}\frac{d}{dx}\left(\frac{dp_0}{dx}\left(\frac{y^3}{3} - b\frac{y^2}{2}\right)\right) - \frac{y^2}{2b^2}\frac{db}{dx} \tag{19}$$

$$\frac{d}{dx}\left(\frac{dp_0}{dx}b^3\right) = 6\frac{db}{dx}, \qquad p_0(0) = p_0(1) = 0 \tag{20}$$

Equation (20) determines the Newtonian pressure. Substituting equation (18) into equation (17b) and using the boundary conditions, one gets the correction term to the velocity profile

$$\begin{aligned} u_1 = {} & \tfrac{dp_1}{dx}\left(\tfrac{y^2}{2} - \tfrac{by}{2}\right) + \gamma\beta\left(\left(\tfrac{y^2b^2}{16} + \tfrac{y^4}{24} - \tfrac{y^3b}{12} - \tfrac{yb^3}{3}\right)\left(\tfrac{dp_0}{dx}\right)^3 + \left(\tfrac{y^2}{4} - \tfrac{y^3}{6b} - \tfrac{yb}{12}\right)\left(\tfrac{dp_0}{dx}\right)^2\right. \\ & \left. + \left(\tfrac{y^2}{4b^2} - \tfrac{y}{4b}\right)\left(\tfrac{dp_0}{dx}\right) + \tfrac{1}{\beta}\left(\tfrac{by}{2} - \tfrac{y^2}{2}\right)\left(\tfrac{dp_0}{dx}\right)\right) \end{aligned} \tag{21}$$

Hence, the solution can be written as

$$\begin{aligned} u = {} & \tfrac{dp}{dx}\left(\tfrac{y^2}{2} - \tfrac{by}{2}\right) + \left(1 - \tfrac{y}{b}\right) + \widehat{\gamma}\beta\left(\left(\tfrac{y^2b^2}{16} + \tfrac{y^4}{24} - \tfrac{y^3b}{12} - \tfrac{yb^3}{3}\right)\left(\tfrac{dp_0}{dx}\right)^3 + \left(\tfrac{y^2}{4} - \tfrac{y^3}{6b} - \tfrac{yb}{12}\right)\left(\tfrac{dp_0}{dx}\right)^2\right. \\ & \left. + \left(\tfrac{y^2}{4b^2} - \tfrac{y}{4b}\right)\left(\tfrac{dp_0}{dx}\right) + \tfrac{1}{\beta}\left(\tfrac{by}{2} - \tfrac{y^2}{2}\right)\left(\tfrac{dp_0}{dx}\right)\right) \end{aligned} \tag{22}$$

Using Eqs. (17a), (17c) and (15), we have

$$v = -\frac{d}{dx}\left(\frac{dp}{dx}\left(\frac{y^3}{6} - \frac{by^2}{4}\right)\right) - \frac{y^2}{2b^2}\frac{db}{dx} - \widehat{\gamma}\beta\frac{d}{dx}\left(\left(\frac{y^3b^2}{48} + \frac{y^5}{120} - \frac{y^4b}{48} - \frac{y^2b^3}{6}\right)\left(\frac{dp_0}{dx}\right)^3 + \left(\frac{y^3}{12} - \frac{y^4}{24b} - \frac{y^2b}{24}\right)\left(\frac{dp_0}{dx}\right)^2 + \left(\frac{y^3}{12b^2} - \frac{y^2}{8b}\right)\left(\frac{dp_0}{dx}\right) + \frac{1}{\beta}\left(\frac{by^2}{4} - \frac{y^3}{6}\right)\left(\frac{dp_0}{dx}\right)\right) \tag{23}$$

$$\frac{dp_1}{dx} = -\frac{19\beta\gamma}{10}b^2\left(\frac{dp_0}{dx}\right)^3 - \frac{\beta\gamma}{2b^2}\left(\frac{dp_0}{dx}\right) + \gamma\left(\frac{dp_0}{dx}\right) + C_1$$
$$p_1(0) = p_1(1) = 0 \tag{24}$$

where C_1 is a constant. Equation (23) is a non-Newtonian pressure equation. The pressure distribution $p_0(x)$ and $p_1(x)$ remain unknown in Eqs. (20) and (24) respectively. The goal would then be to determine the pressure distribution approximately.

Pressure Distribution

The solution to Eq. (20), which is the Newtonian solution, subject to the given boundary conditions is

$$p_0 = \frac{6x(b-r)}{b^2(1+r)} \tag{25}$$

where b and r are defined to be

$$b = (1-(1-r)x), r = b_2/b_1 \tag{26}$$

Inserting this Newtonian pressure distribution into equation (23) and applying the associated boundary conditions one finally obtains

$$p_1 = \beta\gamma\left(-\frac{2736r^3}{5\,(r-1)\,(1+r)^3b^6} + \frac{24624r^2}{25(1+r)^2(r-1)b^5} - \frac{6171r}{10(r-1)(1+r)b^4} + \frac{689}{5(r-1)b^3} + \frac{(r-1)^2(-2077+4054r-2077r^2)}{50r^3(r+1)^2}x - \frac{6890-10185r+8208r^2-2077r^3}{50(r-1)(r+1)^3}\right) + \gamma\frac{6x(b-r)}{b^2(r+1)} \tag{27}$$

The final pressure distribution would then be

$$p = \frac{6x(b-r)}{b^2(1+r)} + \beta\widehat{\gamma}\left(-\frac{2736r^3}{5(r-1)(1+r)^3b^6} + \frac{24624r^2}{25(1+r)^2(r-1)b^5} - \frac{6171r}{10(r-1)(1+r)b^4} + \frac{689}{5(r-1)b^3} + \frac{(r-1)^2(-2077+4054r-2077r^2)}{50r^3(r+1)^2}x + \frac{1}{\beta}\frac{6x(b-r)}{b^2(r+1)} - \frac{6890-10185r+8208r^2-2077r^3}{50(r-1)(r+1)^3}\right) \tag{28}$$

In the next section, the numerical plots of pressure distribution will be given.

Results and Discussion

The analytical study of a slider bearing with Powell-Eyring fluid as lubricant is considered. The pressure distributions and velocity profiles in x and y directions in the slider bearing are calculated approximately using the perturbation method. Bearing characters can be analyzed for any value of Powell-Eyring constants by solving a set of algebraic equations. The pressure distribution in the bearing is determined for various values of the parameters $\widehat{\gamma}$ and β. Figure 2 indicates the variation of the pressure with respect to x when $\beta = 0$ and $\widehat{\gamma}$ is varied. It is seen that the pressure increases with increasing $\widehat{\gamma}$. Figure 3 illustrates the manner in which pressure varies with $\widehat{\gamma}$, when β is held fixed at some nonzero value. As before, increasing $\widehat{\gamma}$ increases the pressure, which means higher loading capacity for the bearing. In Figure 4 for different β, $\widehat{\gamma}$ is held fixed. It is seen that when $\widehat{\gamma} > 0$ the pressure decreases with increasing β, which means lower loading capacity for the bearing. Lubricants possessing higher $\widehat{\gamma}$ values of the Powell-Eyring model bear higher load capacities. In Figure 5 for $\beta = \widehat{\gamma} = 0.01$ the dimensionless length versus dimensionless pressure is plotted for different clearance ratios. Similar to Newtonian behavior, in the non-Newtonian case pressure builds up in the bearing for lower clearance ratios. The maximum load capacity of a bearing depends on both parameter $\widehat{\gamma}$ of a lubricant and clearance ratios. The present analysis with slider bearings suggests that the load capacity of a bearing lubricated with Powell-Eyring fluid can be obtained after giving an appropriate design to the bearings.

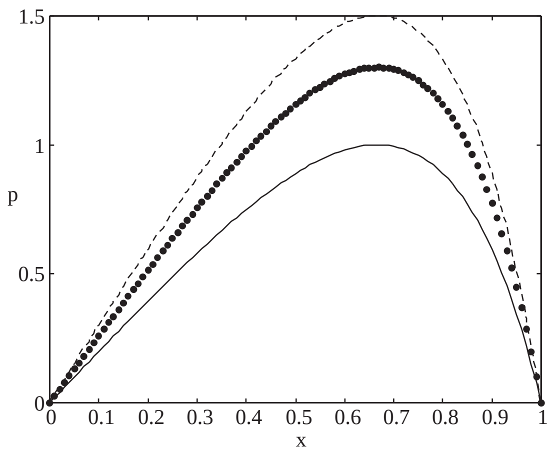

Figure 2. Pressure distribution in the bearing corresponding to various non-Newtonian effects for r = 0.5 (— $\alpha = \beta = 0$ (Newtonian); ..... $\alpha = 0.3$, $\beta = 0$; - - - $\alpha = 0.5$, $\beta = 0$).

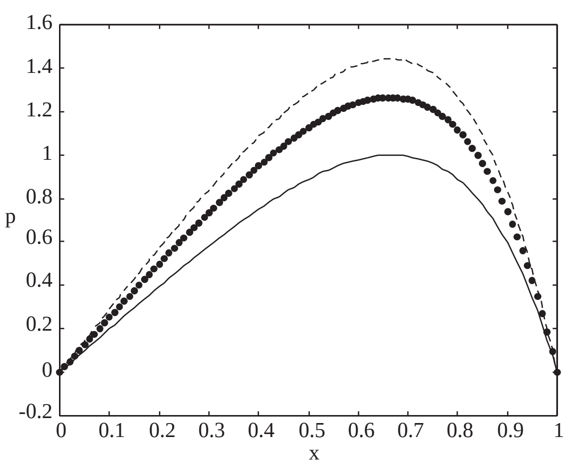

Figure 3. Pressure distribution in the bearing corresponding to various non-Newtonian effects for r = 0.5 (— $\alpha = \beta = 0$ (Newtonian); ..... $\alpha = 0.3$, $\beta = 0.01$; - - - $\alpha = 0.5$, $\beta = 0.01$).

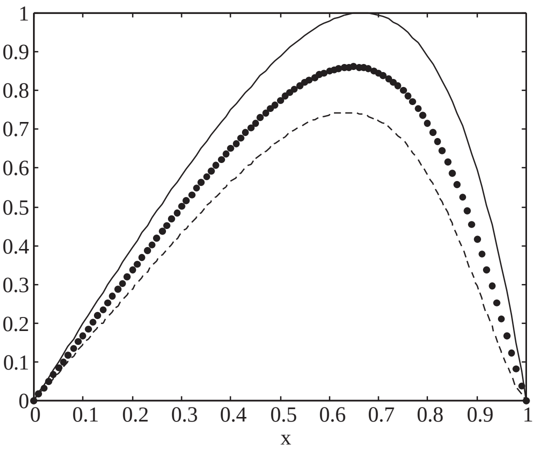

Figure 4. Pressure distribution in the bearing corresponding to various non-Newtonian effects for r = 0.5 (— $\alpha = \beta = 0$ (Newtonian); ..... $\alpha = 0.1$, $\beta = 0.2$; - - - $\alpha = 0.1$, $\beta = 0.3$).

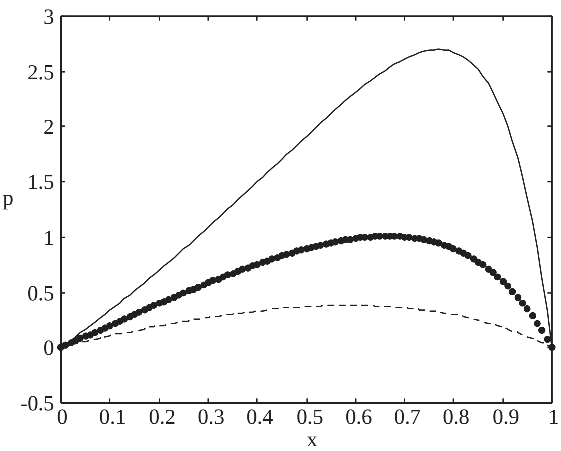

Figure 5. Pressure distribution in the bearing corresponding to different clearance ratios for $\alpha = \beta = 0.01$ (- - - r = 0.3) (.....r = 0.5)(- - - r = 0.7).

The paper deals only with the theoretical part of lubrication and it may inspire mechanical engineers to search for the validity of our predicted theoretical results and investigate the design of bearings to extract greater economical benefits from lubricants.

Nomenclature

b	bearing clearance
b^*	dimensional bearing clearance
b_1	clearance at left
b_2	clearance at right
B,C	constants in the Powell-Eyring model

$\frac{dp}{dx}$	dimensionless pressure gradient
L	bearing length
p	non-dimensional pressure
p*	dimensional pressure
p_0	first term in perturbation expansion of pressure
p_1	second term in perturbation expansion of pressure
r	clearance ratios of slider bearing ($\frac{b_2}{b_1}$)
Re	Reynolds number
U	velocity of the moving surface
v	velocity vector
u,v	velocity component in x and y directions
u^*,v^*	dimensional velocity component in x and y directions
u_0	first term in perturbation expansion of velocity component x direction
u_1	second term in perturbation expansion of velocity component x direction
v_0	first term in perturbation expansion of velocity component y direction
v_1	second term in perturbation expansion of velocity component y direction
x	coordinate along the bearing length
x*	dimensional coordinate along the bearing length
y	coordinate along thickness
y*	dimensional coordinate along thickness
$\widehat{\alpha}, \widehat{\gamma}, \gamma, \beta$	dimensionless material constants
ε	perturbation parameter
μ	viscosity
ρ	density
δ	a small parameter($\frac{b_1}{L}$)
τ_{yx}	shearing stress

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