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ANALYTICAL SOLUTIONS FOR WELL DRAWDOWN WITH WELL LOSSES
1. MULTIPLE WELL SYSTEM NEAR A BOUNDARY

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This paper deals with two cases. The first case concerns a solution for the drawdown in a given well which makes it possible to monitor and/or model the drawdown in other wells in the system and determine whether a critical well depth and, consequently, a critical groundwater flow velocity at the well has not been exceeded. Excessive velocity at the well results in the rinsing of fine soil particles, the impairment of filtration stability and subsequently in the clogging of the well filter pack. Significant additional resistance to flow into the well, often referred to as "skin effect", will follow this clogging and result well losses. In a final phase, high flow velocities can result in complete clogging of the filter pack and the breakdown of the well. The second case concerns the drawdown in a system of wells including boundary conditions and changes in water levels due to different pump start times and discharge rates. The solution is based on the assumptions of additional resistance in the well, any number of discharge and injection wells, and a constrained water level.

INTRODUCTION

In a saturated system we assume that independence of the coefficients of transmissivity and storage capacity over time is valid. If the relative changes of the values of the transmissivity are not large (small drawdown), it is possible to apply these relations to a system with a free water surface. If these changes are large, it is possible to use a conversion to the height of a free water surface.

The solution of a well system makes use of the principle of superposition, which means that if there are numerous discharge or inflow points in a saturated system, the piezometric ground water level equals the algebraic sum of the piezometric levels due to individual inflows or discharges measured from the initial state, or the ground water level before the start of operation of the inflows and discharges.

THEORY

For the case of four wells (Figure 1) the drawdown in point B will be determined by the relation

$$s_B = s_{1,B} + s_{2,B} + s_{3,B} + s_{4,B} \tag{1}$$

where:

$s_{1,B}$, $s_{2,B}$, $s_{3,B}$, $s_{4,B}$ are drawdowns produced by wells 1, 2, 3, 4

s_B is the total drawdown in point B.

Solution of a well system with boundary conditions

Let us consider non-stationary flow regime. Figure 2 shows two "real" wells V1 and V2 in an area bordered by an impermeable and a permeable boundary meeting at an angle of 90°. The influence of the boundaries is replaced with image infiltration and discharge wells VIC and VIN (after Walton 1970).

The overall ground water level drawdown in a real well is given by

$$sv\_celk(ii) = s_V\,(ii) - \sum_{\substack{jj=1\\kk=1}}^{\substack{p\_v\\pim\_n(k)}} sim\_nv(ii, jj, kk) + \sum_{\substack{jj=1\\kk=1}}^{\substack{p\_v\\pim\_c(k)}} sim\_cv(ii, jj, kk) + \sum_{ll=1}^{p\_v} s\,(ll, ii) \tag{2}$$

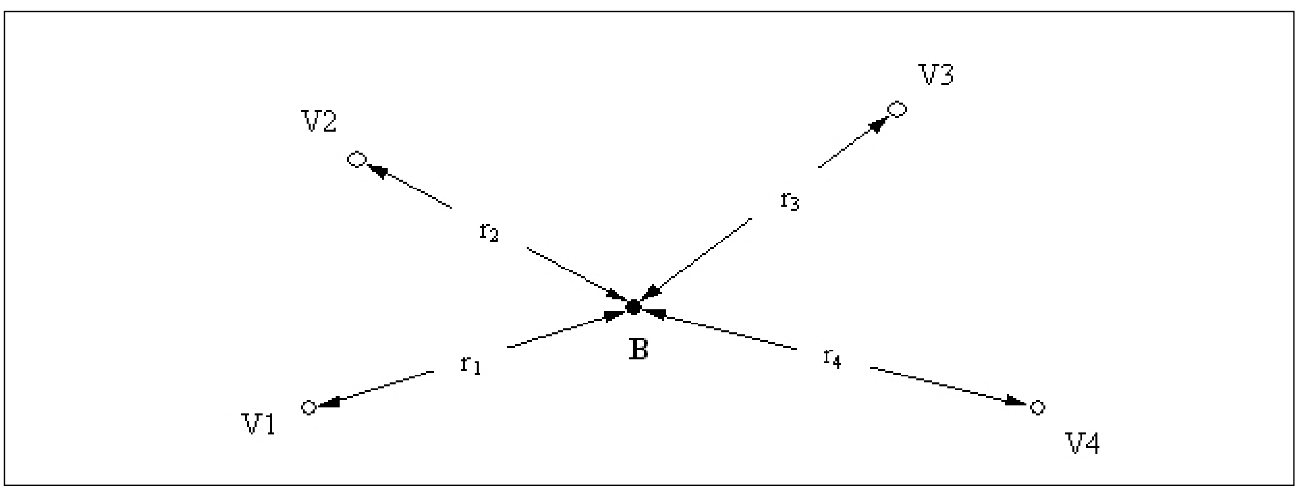

Figure 1. Superposition of four wells.

where:

sv_celk(ii) = the total drawdown in the ii-th real well

sv (ii)= drawdown in the ii-th real well, if the influence of other real and imaginary well is not taken into account (drawdown in one real discharge well)

sim_nv (ii,jj,kk) = part of the drawdown in the ii-th real well due to the effect of the jj-th imaginary injection well belonging to the kk-th real well

sim_cv (ii,jj,kk) = part of the drawdown in the ii-th real well due to the effect of the jj-thimaginary discharge well belonging to the kk-th real well

pim_n(k) = total number of imaginary injection wells considered in the kk-th real well

pim_c(k) = total number of imaginary discharge wells considered in the kk-th real well

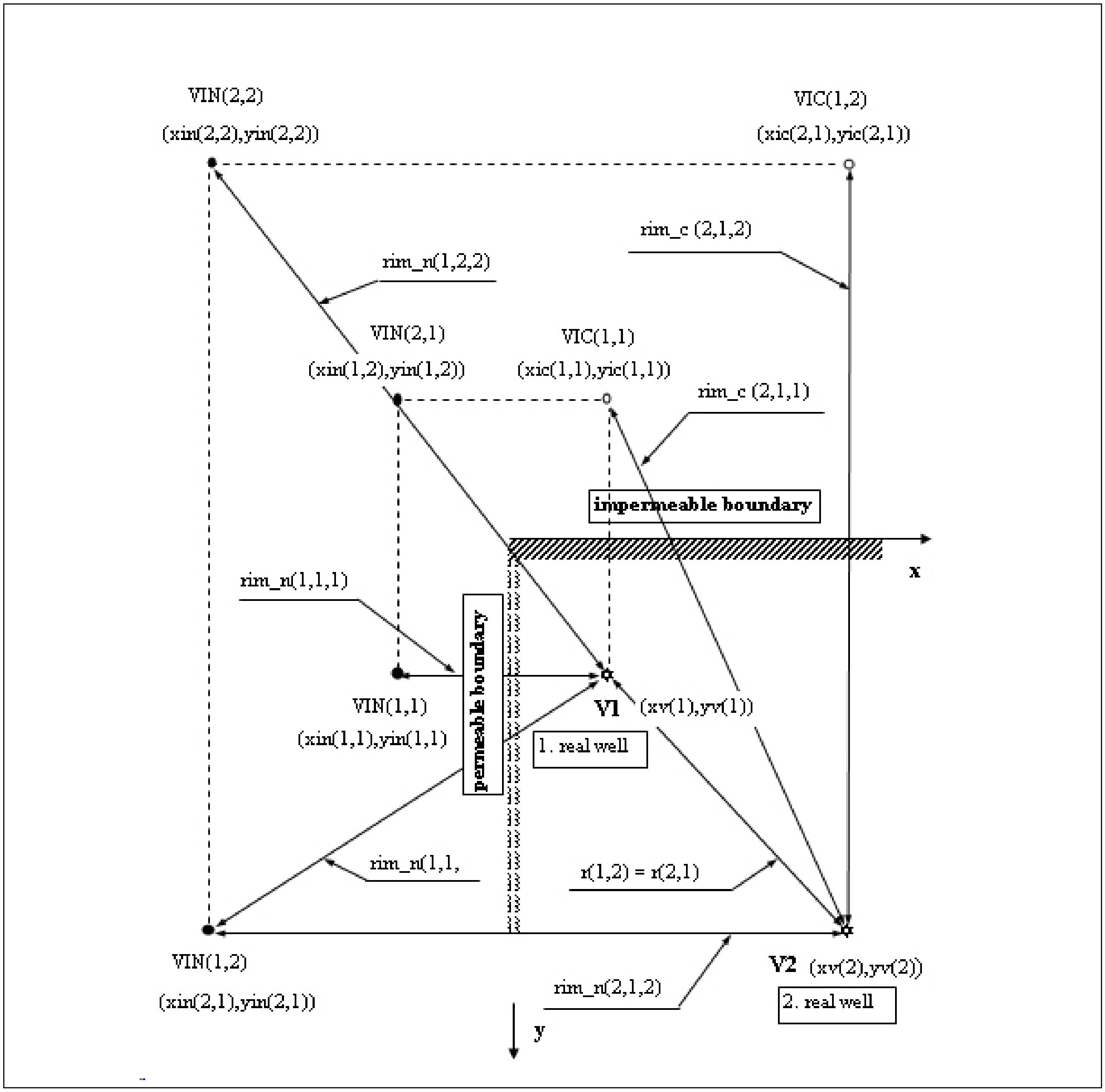

Figure 2. Image-well system bordered by impermeable and a permeable boundaries.

p_v = total number of real wells in the area concerned

s (ll,ii) = part of the drawdown in the ll-th real well due to the effect of the ll-th real well

The terms in Figure 2 are defined as:

xin(kk,jj), *yin(kk,jj)* = *x*, *y* coordinates of the imaginary infiltration well belonging to the kk-th real well

xic(kk,jj), *yic(kk,jj)*= *x*, *y* coordinates of the jj-th imaginary pumped well belonging to the kk-th real well

xv(ii), *yv(ii)* = *x*, *y* coordinates of the ii-th real well

The individual terms of the right-hand side of Equation (2) can be expressed as follows:

1. In real discharge wells we consider the influence of both the additional resistance in the well and its close environs and the actual well volume characterized by well storage capacity. Departing from Pech (1997):

1.a. The first term is

$$s_V\left(ii\right)=\frac{Q\left(ii\right)}{2\pi T}\sum_{j=1}^{k}con\left(j,k\right)\sum_{i=0}^{m}\binom{m}{i}\left(-1\right)^{i}\cdot$$

$$\frac{K_0\left(c^{1/2}\right)-W\left(ii\right)c^{1/2}\;K_1\left(c^{1/2}\right)}{c^{3/2}\left[c^{1/2}\;K_1\left(c^{1/2}\right)+C_D\left(ii\right)c^{1/2}\left(K_0\left(c^{1/2}\right)+W\left(ii\right)c^{1/2}\;K_1\left(c^{1/2}\right)\right)\right]} \qquad (3)$$

where:

$K_0(x)$ = Bessel function of the second kind of order zero

$K_1(x)$ = Bessel function of the second kind of order one

Q (ii) = discharged (pumped) water quantity in the ii-th real well

T = transmissivity

S = storativity.

$$c = (m + i)\,(\ln 2)/\,t_{D\,t} \qquad (4)$$

$$m = k + 1 - j \qquad (5)$$

$$con\left(j,k\right)=\frac{\left(-1\right)^{j-1}}{k}\binom{k}{j}\,j\,m^{k-1}\,\frac{\ln 2}{t_D}\,\frac{\left(2m\right)!}{m!\left(m-1\right)!} \qquad (6)$$

dimensionless time is expressed as:

$$t_{Dt}=\frac{T\left(t_S-t_P\left(ii\right)-t_0\right)}{r_V\left(ii\right)^2\;S} \qquad (7)$$

$r_v\,(ii)$ = diameter of the ii-th real well

t_S = time at which the drawdown magnitude in the ii-th real well is determined

t_P = time of pumping start in the ii-th real well

t_0 = reference time for all wells (e.g. time at which pumping in the first real well in the area concerned has started)

W(ii) = additional resistance coefficient n the ii-th real well

$C_D(ii)$ = dimensionless well-bore storage capacity coefficient of the ii-th real well

1.b. The second term

$$sim\_nv(ii, jj, kk) = \frac{Q(kk)}{2\,\pi\,T} \sum_{j=1}^{k} con(j, k) \sum_{i=0}^{m} \binom{m}{i} (-1)^i \left(\frac{K_0\left(\frac{rim\_n(ii, jj, kk)}{r_V(kk)} . c^{1/2}\right)}{c^2\;K_1\left(c^{1/2}\right)} \right) \tag{8}$$

where:

Q (kk) = discharged (pumped) water quantity in the kk-th real well to which the jj-th imaginary injection well belongs

$t_P(kk)$ = time of pumping start in the kk-th real well to which the jj-th imaginary injection well belongs

rim_n(ii,jj,kk) = distance of the ii-th real well and the jj-th imaginary infiltration well belonging to the kk-th real well, determined by the relation

$$rim\_n(ii, jj, kk) = \sqrt{\left(xv(ii) - xin(kk, jj)\right)^2 + \left(yv(ii) - yin(kk, jj)\right)^2} \tag{9}$$

1.c. The third term:

$$sim\_cv(ii, jj, kk) = \frac{Q(kk)}{2\,\pi\,T} \sum_{j=1}^{k} con(j, k) \sum_{i=0}^{m} \binom{m}{i} (-1)^i \left(\frac{K_0\left(\frac{rim\_n(ii, jj, kk)}{r_V(kk)} . c^{1/2}\right)}{c^2\;K_1\left(c^{1/2}\right)} \right) \tag{10}$$

where:

rim_c(ii,jj,kk) = distance of the ii-th real well and the jj-th imaginary pumped well, belonging to the kk-th real well, determined by the relation

$$rim\_c(ii, jj, kk) = \sqrt{\left(xv(ii) - xic(kk, jj)\right)^2 + \left(yv(ii) - yic(kk, jj)\right)^2} \tag{11}$$

1.d. The fourth term:

$$s\left(ll,ii\right)=\frac{Q\left(ll\right)}{2\pi\ T}\sum_{j=1}^{k}con\left(j\,,k\right)\sum_{i=0}^{m}\binom{m}{i}\left(-1\right)^{i}\ \frac{K_0\left(\dfrac{r\left(ll\,,ii\right)}{r_V\left(ll\right)}\ c^{1/2}\right)}{c^2\ K_1\left(c^{1/2}\right)} \tag{12}$$

where:

s(ll,ii) = drawdown in the ii-th real well due to the effect of the ll-th real well

Q(ll) = pumped water quantity in the ll-th real well

rv (ll) = radius of the ll-th real well

r(ll,ii) = distance of the ii-th and the ll-th real wells determined from the relation

$$r\left(ll\,,ii\right)=\sqrt{\left(xv\left(ll\right)-xv\left(ii\right)\right)^2+\left(\ yv\left(ll\right)-yv\left(ii\right)\right)^2} \tag{13}$$

xv(ll), x(ii), yv(ll), y(ii) = *x*, *y* coordinates of the ll-th and ii-th real wells

2. In the cases when it is possible to neglect the influence of the volume of discharge wells on the course of the inflow test (i.e. if the well storage capacity equals zero) and only the influence of additional resistance in the well and its close environment is considered, we start with the Theis equation . The denomination of individual quantities is the same as in the preceding equations. We shall express the individual terms of Equation (2) as follows:

2.a. The first term:

$$s_V\ \left(ii\right)=\frac{Q\left(ii\right)}{4\pi\ T}\left(\ W\left(\frac{r_V\left(ii\right)^2\ S}{4\ T\left(t_S-t_P\left(ii\right)-t_0\right)}\right)+2\ W_{ii}\right) \tag{14}$$

where:

W_{ii} = additional resistance coefficient in the ii-th real well

2.b. The second term:

$$sim\_nv\left(ii\,,\ jj\,,kk\right)=\frac{Q\left(kk\right)}{4\pi\ T}\left(\ W\left(\frac{rim\_n\left(ii\,,\ jj\,,kk\right)^2\ S}{4\ T\left(t_S-t_P\left(kk\right)-t_0\right)}\right)\right) \tag{15}$$

2.c. The third term:

$$sim\_nc\left(ii\,,\ jj\,,kk\right)=\frac{Q\left(kk\right)}{4\pi\ T}\left(\ W\left(\frac{rim\_c\left(ii\,,\ jj\,,kk\right)^2\ S}{4\ T\left(t_S-t_P\left(kk\right)-t_0\right)}\right)\right) \tag{16}$$

2.d. The last term of the right-hand side of Equation 2

$$s\left(ll,ii\ \right)=\frac{Q\left(ll\right)}{4\pi\ T}\left(\ W\left(\frac{r\left(ll,ii,\right)^{2}\ S}{4\ T\left(t_{S}-t_{P}\left(ll\right)-t_{0}\right)}\right)\right) \tag{17}$$

3. For dimensionless time $t_D > 25$ it is possible to use the Jacob semilogarithmic approximation which has the following forms for the individual terms of the right-hand side of Equation 2:

3.a. For the first term

$$s_{V}\ \left(ii\right)=\frac{Q\left(ii\right)}{4\pi\ T}\left(\ ln\frac{2{,}246\ T\left(t_{S}-t_{P}\left(ii\right)-t_{0}\right)}{r_{V}\left(ii\right)^{2}\ S}+2\ W_{ii}\right) \tag{18}$$

3.b. For the second term

$$sim\_nv\left(ii,\ jj,\ kk\right)=\frac{Q\left(kk\right)}{4\pi\ T}\left(\ ln\frac{2{,}246\ T\left(t_{S}-t_{P}\left(ii\right)-t_{0}\right)}{rim\_nv\left(ii,\ jj,\ kk\right)^{2}\ S}\right) \tag{19}$$

3.c. For the third term

$$sim\_cv\left(ii,\ jj,\ kk\right)=\frac{Q\left(kk\right)}{4\pi\ T}\left(\ ln\frac{2{,}246\ T\left(t_{S}-t_{P}\left(ii\right)-t_{0}\right)}{rim\_nc\left(ii,\ jj,\ kk\right)^{2}\ S}\right) \tag{20}$$

3.d. For the last term on the right-hand side of Equation 2 it holds that

$$s\left(ll,ii\right)=\frac{Q\left(ll\right)}{4\pi\ T}\left(\ ln\frac{2{,}246\ T\left(t_{S}-t_{P}\left(ll\right)-t_{0}\right)}{r\left(ll,ii\right)^{2}\ S}\right) \tag{21}$$

Solution for the piezometric water level at any point in the area

In an area bordered on two sides by an impermeable and a feeding boundaries closing an angle of 90° let us consider one real well. To include the influence of boundaries into the solution we shall supplement the real well with two imaginary injection wells and one imaginary discharge well. The situation is shown schematically in Figure 3. The origin of the coordinate system is situated in the point of intersection of the impermeable and the feeding boundaries.

The terms in Figure 3 are defined as:

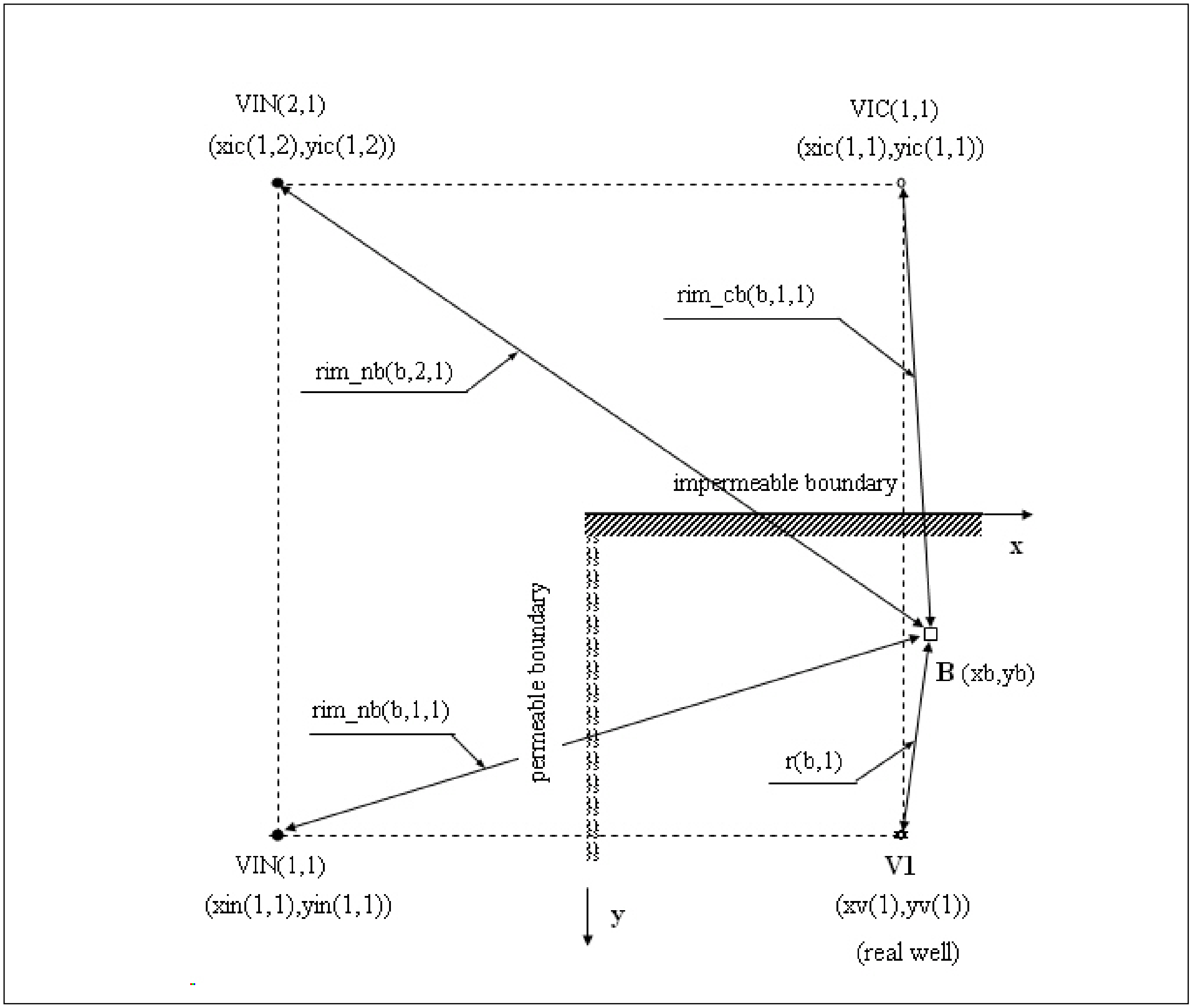

Figure 3. Schematic drawing of a real well and the impermeable and the permeable boundaries.

rim_cb (b,jj,kk) = distance of point B from the jj-th imaginary pumped well belonging to the kk-th real well

rim-nb(b,jj,kk) = distance of point B from the jj-th imaginary well belonging to the kk-th real well

We select an arbitrary point B with coordinates x_b and y_b situated in the "real" part of the area (i.e. the area with the real discharge well).

If we start with Equations (1) and (2), the overall drawdown in point B with coordinates x_b and y_b is

$$s(xb, yb) = \sum_{ii=1}^{p\_v} s(b, ii) - \sum_{\substack{jj=1 \\ kk=1}}^{\substack{p\_v \\ pim\_n(k)}} sim\_nb(b, jj, ii) + \sum_{\substack{jj=1 \\ kk=1}}^{\substack{p\_v \\ pim\_c(k)}} sim\_cb(b, jj, ii) \quad (22)$$

where:

s(b,ii) = part of the drawdown in point B due to the effect of the ii-th real well.

The meaning of individual quantities is the same as in Equation 2.

sim_nb(b.jj,ii) = part of the drawdown in point B due to the effect of the jj-th imaginary

sim_cb(b,jj,ii) = part of the drawdown in point B due to the effect of jj-th imaginary pumped well

We can express the terms of the right-hand side of Equation (2) analogously to the preceding part. Only the non-stationary flow regime is considered for which we shall use for individual terms:

$$s(b,ii) = \frac{Q(ii)}{2\,\pi\,T} \sum_{j=1}^{k} con(j,k) \sum_{i=0}^{m} \binom{m}{i} (-1)^i \frac{K_0\left(\dfrac{r(b,ii)}{r_V(ii)}\, c^{1/2}\right)}{c^2\, K_1\left(c^{1/2}\right)} \tag{23}$$

where:

The quantities with index ii refer to the ii-th real well.

r(b,ii) = distance of point B from the ii-th real well which we shall determine as follows:

$$r(b,ii) = \sqrt{\left(xb - xv(ii)\right)^2 + \left(yb - yv(ii)\right)^2} \tag{24}$$

xb, yb = x, y coordinates of point B

$$sim\_nb(b,jj,kk) = \frac{Q(kk)}{2\,\pi\,T} \sum_{j=1}^{k} con(j,k) \sum_{i=0}^{m} \binom{m}{i} (-1)^i \left(\frac{K_0\left(\dfrac{rim\_n(b,jj,kk)}{r_V(kk)}\, c^{1/2}\right)}{c^2\, K_1\left(c^{1/2}\right)} \right) \tag{25}$$

where:

rim-nb(b,jj,kk) = distance of point B from the jj-th imaginary well belonging to the kk-th real well.

$$rim\_nb(b,jj,kk) = \sqrt{\left(xb - xin(jj,kk)\right)^2 + \left(yb - yin(jj,kk)\right)^2} \tag{26}$$

and

$$sim\_cb(b,jj,kk) = \frac{Q(kk)}{2\,\pi\,T} \sum_{j=1}^{k} con(j,k) \sum_{i=0}^{m} \binom{m}{i} (-1)^i \left(\frac{K_0\left(\dfrac{rim\_c(b,jj,kk)}{r_V(kk)}\, c^{1/2}\right)}{c^2\, K_1\left(c^{1/2}\right)} \right) \tag{27}$$

rim_cb (b,jj,kk) – distance of point B from the jj-th imaginary pumped well belonging to the kk-th real well

$$rim\_cb\left(b,jj,kk\right)=\sqrt{\left(xb-xic\left(jj,kk\right)\right)^{2}+\left(yb-yic\left(jj,kk\right)\right)^{2}} \quad (28)$$

xic (jj,kk), yic (jj,kk)= x,y coordinates of the jj-th imaginary infiltration well belonging to the kk-th real well

CONCLUSION

Relations are derived for the determination of drawdown in wells located in proximity to permeable or impermeable boundaries and they are applied to the solution of a system of wells taking into account these boundary conditions. These relations were used as the basis for the SIM 1 computer program by means of which it is possible to model the discharge of a system of wells influenced by boundary conditions, to monitor the history of water level drawdown in individual wells of the system during various pumping starts in individual wells and monitor whether the critical drawdown in the wells has not been exceeded. The model can regulate the maximum permissible discharge from the wells (i.e. the drawdown in the course of which the critical velocities on the well skin are exceeded). As shown by Todd (1980), if the critical values of water level drawdown in the well are not exceeded, the process of well aging may be retarded or completely stopped in many cases, thus substantially prolonging the well service life.

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