Turkish J. Eng. Env. Sci.
27 (2003) , 265 – 273.

Reliability–based Assessment of Erodible Channel Capacity

A. Melih YANMAZ
Middle East Technical University, Civil Engineering Department,
06531 Ankara-TURKEY
email: myanz@metu.edu.tr

Received 28.01.2003

Abstract

The surface drainage of an irrigation area is concerned with the collection and removal of storm runoff and excess water from irrigation applications. The engineering problem is to determine the shape, cross-sectional area and slope of a stable channel. A reliability-based design, which addresses and treats possible uncertainties associated with the design parameters, is of importance as it leads to realistic decision-making. In this paper, a reliability-based assessment of erodible channel capacity based on the regime concept is carried out in an example using the Advanced First Order Second Moment formulation and the Monte Carlo simulation technique. A sensitivity analysis is also carried out to observe the effects of coefficient of variation and probability distributions. The results obtained from both approaches are found to be in relatively good agreement.

Key words: Reliability, Erodible channel, Advanced first order second moment formulation, Monte carlo simulation.

Introduction

An erodible channel may experience various modes through the stabilization process. Depending on the sediment transport rate, a channel may be subject to scouring, accretion or both along its reaches. A stabilized channel, which attains its equilibrium in the the long term, may be considered as a channel having fixed boundaries, and hence its hydraulic behavior can be assessed using simpler relations than those required for movable boundaries. Long-term channel equilibrium can be assessed with reference to the concept of degree of freedom. A prismatic channel has one degree of freedom, i.e. the channel attains its mean equilibrium depth over a considerable period of time for the given steady flow rate, Q. Two degrees of freedom occur in a channel having fixed banks and that carries sediment load. The depth, y, and the bed slope, S_0, are adjusted for the given discharge. In the case of an erodible channel having loose boundaries, there exist three degrees of freedom. For the given steady discharge, an erodible channel adjusts its depth, y, surface width, W, and the bed slope, S_0. Lacey (1930) found that natural channels have a tendency to experience a stabilized section having a semi-elliptical shape. The coarser the sediment, the flatter the semi-ellipse and the greater the width at the water surface. Finer material would lead to the development of a semi-circular shape.

Stable channel design, which can be carried out for surface drainage systems as well as navigation channels and stream stabilization purposes, is one of the most intriguing applications in loose-boundary hydraulics. Although much work has been put in to the development of design guidelines, the proposed methods reported in the literature are subject to uncertainties because of simplifications made in their developments. Among several design methods the method of maximum permissible velocity, the tractive force approach and the regime concept are worthy of note. The maximum permissible velocity is defined as the greatest mean velocity that will not cause erosion of the channel boundaries. Erodible channel design based on this concept is subject to high uncertainties, as the accuracy of the maximum permissible velocities proposed by Scobey and Fortier (See Chow, 1959) for old straight channels of small slope having a 1 m flow depth are relatively low. The validity of the tractive force approach is based on the accuracies of the shear stress distributions and permissible unit tractive forces. The diagrams giving the permissible unit tractive forces proposed by the U.S. Bureau of Reclamation do not consider particle gradation and soil properties, such as degree of plasticity and chemical action among soil particles forming the channel boundaries. The regime approach, although it does not incorporate physical explanations for the phenomenon, is an empirical approach that has been developed by observing the characteristics of stable channels. Initial investigations have been carried out in India, Pakistan and Egypt. Extended investigations on this approach have been performed in the USA and Canada. The method was initiated by Kennedy (1895). Based on extensive field investigations, additional contributions to the regime concept were made by other researchers. Some of the works, which are of practical importance, are attributed to Lindley (1919), Blench (1957), Lacey (1966) and Simons and Albertson (1963). The regime equations have been verified using additional field data beyond those used in the development of a particular equation. This approach sizes an erodible channel such that its sections and slope are in equilibrium with discharges. Therefore, increments or decrements in Q modify W and y. That is why, after the annual periods, sections and slopes remain practically constant. With these characteristics, a regime equation, which is developed using extensive field data, is assumed to be adequate for the preliminary design. However, care must be taken in applying a particular regime equation to a specific site. The designer must verify that the local conditions are similar to those used in the derivation of that particular equation.

An erodible drainage channel such as a collector may attain a single degree of freedom because of the following reasoning. In practice, the size of the maximum permissible land allocated for agricultural activities is defined according to local conditions. Therefore, an almost constant channel surface width can be provided at the lowest ridges in the project area for the removal of drainage water. Furthermore, the channel bed slope is governed by the topographical characteristics of the area where irrigation practices are carried out. Hence, for the given discharge, specified surface width and bed slope of the channel, the mean equilibrium depth can be estimated using a suitable regime equation. To this end, the equation proposed by Maza Alvarez and Echavarrio Alforo (1973) is used. This equation, which is valid for a straight uncontracted reach having bed material with $D_{75} < 6$ mm, where D_{75} is the characteristic particle size for which 75% of the material is finer, is as follows:

$$y = 0.365 \left(\frac{Q^{0.784}}{W^{0.784} D_{50}^{0.157}} \right) \tag{1}$$

in which y is the mean equilibrium depth in m, Q is the discharge in m^3/s, W is the surface width in m, and D_{50} is the median sediment size in m. This method is based on data from other researchers together with field data from South American rivers.

Hydrosystems Reliability

Reliability analyses have been increasingly used in water resources engineering for decision-making in the design and operation of hydrosystems subject to uncertainty. To carry out reliability-based analyses, possible uncertainties associated with the variables need to be identified, quantified, and treated. The total uncertainty, i.e. the model and parameter uncertainty associated with a hydraulic phenomenon, cannot be quantified precisely for most of the applications because of the lack of relevant information. To offset this limitation, sets of elaborate measurements as well as judgment and experience are needed to collect relevant information for uncertainty analysis. In hydrologic problems, the statistical information available is often limited to the first two moments of the variables only. Values for the higher moments are normally either unreliable or unavailable. Therefore, second order approximation is not only difficult to evaluate but also impractical. That is why first order uncertainty analysis is usually carried out for the reliability evaluations of water resources systems (Yen *et al.,* 1986).

Uncertainties can be expressed in terms of coefficients of variations. However, the validity of the results of an uncertainty analysis is dependent on the correct choice of the coefficient of variation and the probability distribution of the variables involved in a physical phenomenon. The coefficients of variations in hydraulic parameters, such as flow depth and velocity, reflect the possible errors in the measurement of these variables that may be small in elaborate laboratory conditions under the control of an experienced hydraulician. However, these values may attain somewhat larger values in prototype conditions, depending on the location of measurement, sediment transport regime of the river, precision of the instrument and human-induced errors. The coefficients of variations of geometric variables, such as channel surface width, may arise due to measurement error, which is normally very small. Another difficulty in uncertainty analysis is the identification of the dependence of parameters involved in a phenomenon, which is based on the interpretation of the cross correlations between these parameters. Due to the complexity and random nature of the hydraulic phenomena, such information is normally not available.

Conventional hydraulic design is deterministic, i.e. it does not account for possible variations of the parameters involved in the phenomenon concerned. With the application of reliability theory to hydraulic engineering practices, probabilistic design approaches have been proposed that enabled the assessment of various reliability levels under different combinations of design parameters. In these applications, the probability distributions of parameters having specified coefficients of variations were considered. In the composite risk analysis of a hydrosystem using resistance-loading interference, the risks resulting from various sources of uncertainty can be incorporated to produce an overall risk assessment for the design of the system (Chow *et al.*, 1988). The loading, X, on a system is the measure of the impact of external events. The resistance, Y, is the measure of the ability of the system to withstand the loading. Therefore, the reliability, α, of a system can be expressed as the probability that the resistance of the system equals or exceeds the loading, $\alpha = P(X \leq Y)$ or $\alpha = P(SM \geq 0)$, where P is the probability and SM is the safety margin, Y-X. If the resistance and loading are dependent variables, the system reliability becomes (Mays and Tung, 1992):

$$\alpha = P(SM \geq 0) = \int_0^\infty \int_0^Y f_{X,Y}(X,Y)dXdY \qquad (2)$$

where $f_{X,Y}$(X,Y) is the joint probability density function of the resistance and loading. Composite reliability models for bridge pier scouring developed by Yanmaz and Çiçekdağ (2001) and Yanmaz and Üstün (2001) can be given as examples of the application of Equation (2). In a static reliability model, the system performance should be checked under a single worst loading condition. The level of reliability may be assessed by a safety factor, SF = Y/X. When the loading and resistance are independent, Equation (2) is changed to

$$\alpha = P(SM \geq 0) = \int_0^\infty f_Y(Y)\left[\int_0^Y f_X(X)dX\right]dY \qquad (3)$$

Dynamic reliability considers repeated loading during a specified service life of a hydrosystem. Recent examples of this application include Yanmaz (2000) and Yanmaz (2002). The determination of the exact system reliability using Equations (2) or (3) requires a knowledge of the probability density functions (PDFs) of the random resistance and loading. For most hydraulic applications, this information is difficult to obtain because of a lack of data or the expression is highly complex such that analytical solution is not possible. That is why some approximate methods have been developed to estimate the reliability in order to offset the difficulties in obtaining joint or individual PDFs for resistance and loading. In this study, the Advanced First Order Second Moment (AFOSM) formulation and the Monte Carlo simulation are applied in the reliability evaluation of erodible channel capacity. Basic information on these approaches is presented below.

The AFOSM formulation (Hasofer and Lind, 1974) is based on the definition of a proper failure function where SM = 0 composed of n random normalized variables, $g(z) = (z_1,\ z_2, \ldots, z_n)$. In this function $z_i = (x_i - \mu_i)/\sigma_i$, where x_i is a basic random variable composing resistance and loading terms and μ_i and σ_i are the mean and standard deviation of that variable, respectively. The reliability index, β_r, is defined as the shortest distance from the origin to the failure surface in the normalized coordinate system. The reliability is approximated by $\Phi(\beta_r)$, where $\Phi(\ )$ is the standard normal distribution function. When the failure surface is a linear function of the basic variables, the reliability index as defined by Cornell (1969) is easily proven to be $\beta_r = \mu_{SM}/\sigma_{SM}$, which coincides with the β_r value defined by Hasofer and Lind (1974). For a nonlinear failure surface, the aforementioned shortest distance, β_r, is not unique, as in the case of a linear failure surface, and it can be determined using an iterative solution (Ranganathan, 1990). The point on the failure surface with minimum distance to the origin, Z^*, is known as the most probable failure point or the design point satisfying $g(Z^*) = 0$, which can be found from

$$\beta = -\frac{\sum_{i=1}^{n} z_i^* \left(\frac{\partial g}{\partial z_i}\right)_*}{\left[\sum_{i=1}^{n} \left(\frac{\partial g}{\partial z_i}\right)_*^2\right]^{1/2}} \quad (4)$$

where the term $(\partial g/\partial z_i)_*$ indicates the derivative to be evaluated at the design point, $z_i^* = \alpha_i^* \beta$, in which α_i^* are the direction cosines, which are expressed as

$$\alpha_i^* = -\frac{\left(\frac{\partial g}{\partial z_i}\right)_*}{\left[\sum_{i=1}^{n} \left(\frac{\partial g}{\partial z_i}\right)_*^2\right]^{1/2}} \quad (5)$$

The procedure is initiated by defining the normalized limit state equation, g(z)=0. Appropriate initial values are selected for β and α_i, such that $\sum \alpha_i^2 = 1.0$. While choosing values for α_i, positive values are selected for load variables, whereas negative values are assigned to the resistance variables. The iterative procedure continues until almost the same β values are obtained in two successive iterations, as illustrated in the application (See Table 2). If the probability distributions of the random variables, x_i, are not normal, the reliability can be evaluated using equivalent normal distributions (Paloheimo and Hannus, 1974; Rackwitz and Fiessler, 1978). For an individual variate, the equivalent normal distribution for a nonnormal variate may be obtained such that the cumulative probability, $\Phi(\ )$, as well as the probability density ordinate, $\phi(\ )$, of the equivalent normal distribution is equal to those of the corresponding nonnormal distribution at the design point. The mean, $\mu_{X_i}^N$, and standard deviation, $\sigma_{X_i}^N$, of the equivalent normal distribution for x_i are obtained from (Ang and Tang, 1984)

$$\mu_{X_i}^N = x_i^* - \sigma_{X_i}^N \Phi^{-1}[F_{X_i}(x_i^*)] \quad (6)$$

$$\sigma_{X_i}^N = \frac{\phi\left(\Phi^{-1}[F_{X_i}(x_i^*)]\right)}{f_{X_i}(x_i^*)} \quad (7)$$

where F_x and f_x are the cumulative density and probability density of x, respectively. The failure point is determined from

$$x_i^* = -\alpha_i \beta \sigma_{X_i}^N + \mu_{X_i}^N \quad (8)$$

In the second order reliability method, the failure surface at the design point is approximated by a parabolic surface with the axis of the parabola along the direction of the design point. Hong (1999) proposes some correction factors for improving the second order reliability estimates. The reliability expression based on the definition of $\alpha = P(SM \geq 0)$ is nonlinear for most of the hydraulic applications. For a general nonlinear performance function, the correct reliability may be evaluated through large-sample Monte Carlo simulations (Ang and Tang, 1984). In the Monte Carlo analysis, random numbers between 0 and 1 are generated for the variables having uniform distribution. These random numbers are then transformed to the desired distribution through an inverse transform method using a computer algorithm. The use of the reliability methods will be illustrated in the following application.

Application

A main collector is to be designed for a storm runoff of 87 m^3/s. The allowable surface width of the straight channel is 30 m. The uniform foundation material forming the channel boundaries has $D_{50} = 4$ mm. The mean equilibrium depth is obtained from Equation (1) as 2 m. Using Equation (1), the channel capacity, Q_c, can be expressed as

$$Q_c = 3.62 W y^{1.276} D_{50}^{0.2} \quad (9)$$

The safety margin is $SM = Q_c - Q$, or

$$SM = 3.62 W y^{1.276} D_{50}^{0.2} - Q \quad (10)$$

where Q is the discharge passing through the channel, which can be regarded as the system loading. The safety margin can be considered to be a measure of the level of channel stability. The reliability will be estimated for various loading levels ranging from 60 to 85 m^3/s. The performance equation in the normalized system is

$$g(z) = 3.62\,(z_y\sigma_y + \mu_y)^{1.276}\,(z_W\sigma_W + \mu_W)$$
$$(z_{D_{50}}\sigma_{D_{50}} + \mu_{D_{50}})^{0.2} - (z_Q\sigma_Q + \mu_Q) \quad (11)$$

in which z_y, z_W, z_{D50}, and z_Q are the reduced variates of y, W, D_{50}, and Q, respectively.

The β and α coefficients are determined from Equations (4) and (5) in an iterative order. Appropriate coefficient of variation and probability distributions should be selected. To this end previous studies can be reviewed. By examining the available data on the variation of some hydraulic variables, which were reported by previous researchers, Johnson (1996) presented limited information on the coefficient of variation and associated probability distribution of some hydraulic variables. Yanmaz (2000) carried out an uncertainty analysis for a diversion canal. In another study conducted by Yanmaz and Çiçekdağ (2001), a comprehensive uncertainty analysis was carried out for bridge pier scouring. With reference to these studies the statistical information presented in Table 1 is selected. The iterative solution is presented in Table 2 for Q = 80 m^3/s. To compare the results of reliability obtained from the AFOSM formulation, the Monte Carlo simulation was also carried out.

Table 1. Statistical information for the application.

Variable	Unit	μ	Ω	PDF
y	m	2	0.05	Normal
W	m	30	0.01	Normal
D_{50}	mm	4	0.01	Normal
Q	m^3/s	60-85	0.1	Lognormal

In the Monte Carlo analysis, the number of simulation cycles, i.e. the number of trials to generate random numbers, influences the level of reliability. The number of cycles required in a Monte Carlo simulation to determine the exact reliability must be large in order to obtain a significant sampling of simulation events such that the sample can be considered as the population (Yanmaz, 2003). The accuracy of the mean reliability under a particular simulation cycle may be estimated by the coefficient of variation of reliability, Ω_r, which decreases with increasing sample size (Melchers, 2002). Therefore, simulations should be carried out several times for large cycles such that the corresponding value of Ω_r is relatively small. According to Johnson (1999), it is desirable to have $\Omega_r < 0.1$. The variation of Ω_r against the number of simulation cycles is shown in Figure 1, from which one can observe that as the number of simulation cycles increases, Ω_r approaches a constant value of approximately 0.012. Therefore, it can be accepted that a further increase in the number of simulation cycles would not lead to improved accuracy in the computations. To this end an 8000-cycle is taken in the analysis.

The results of both approaches are presented in Figure 2 in a format showing the reliability as a function of the safety factor, SF = Q_c/Q. This figure indicates that reliability increases as the safety factor rises. The reliability results obtained from both approaches using the statistical information given in Table 1 were found to be in very good agreement.

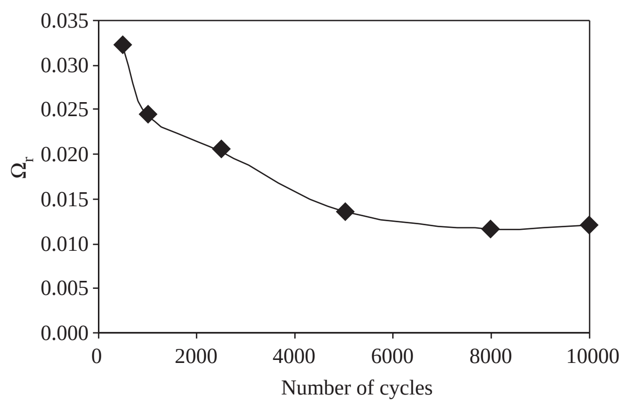

Figure 1. Variation of Ω_r against number of cycles.

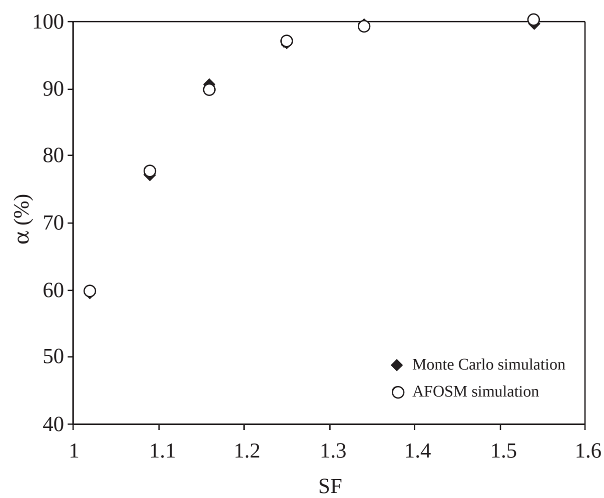

Figure 2. Variation of reliability against safety factor.

Sensitivity analyses were also carried out to observe the effect of the coefficient of variation and probability distribution. Since the coefficient of variation of the loading parameter, Ω_Q, is determined from a suitable frequency analysis that is compatible with the local hydrologic regime of the basin concerned, it is assumed that Ω_Q is considerably precise and reliable provided that the size of the data is enough and the data are of good quality. Therefore, only the resistance parameters were considered in the sensitivity analyses. The coefficient of variation of a particular resistance variable is increased by 100% while keeping the other coefficients of variations the same. The corresponding reliability is then computed. The results are summarized in Figure 3 in a format giving the absolute value of the percentage change of reliability relative to the case having the statistical information given in Table 1 with respect to the safety factor. Three different combinations of the coefficient of variation, i.e. case A is $\Omega_y = 0.10$, $\Omega_W = 0.01$, $\Omega_{D50} = 0.01$, case B is $\Omega_y = 0.05$, $\Omega_W = 0.02$, $\Omega_{D50} = 0.01$, and case C is $\Omega_y = 0.05$, $\Omega_W = 0.01$, $\Omega_{D50} = 0.02$ were considered. In this analysis, it is observed that 100% increases of Ω_W and Ω_{D50} have resulted in very small changes in the relative reliability. However, up to 10% of change in the relative reliability was observed for Ω_y, which may be due to the fact that the power of y is greater than the powers of W and D_{50}, and hence it dominates the accuracy of the reliability. For this case, the relative reliability was also found to decrease with an increasing safety factor.

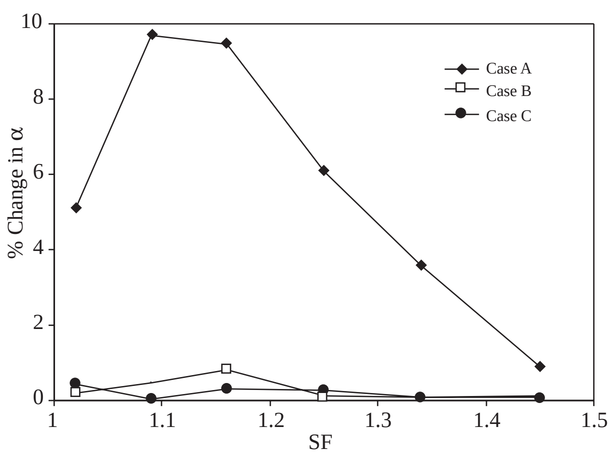

Figure 3. Sensitivity analysis for the effect of Ω.

Additional analysis has also been carried out to examine the effect of the choice of PDF different from those indicated in Table 1 using the same Ω values as given in Table 1. To this end triangular and uniform probability distributions have been assigned to the variables (See Figure 4). Inspection of Figure 4 reveals that the results obtained using the triangular distribution are in relatively good agreement with the normal distribution. Although uniform distribution has resulted in changes in the relative reliability up to 18% , these changes were observed to decrease for the increasing safety factor as shown in Figure 4. As a concluding remark it may be stated that the choice of Ω and PDF may gain importance for nonlinear equations having high powers. Therefore, to carry out a realistic reliability analysis, sufficient high precision data should be available.

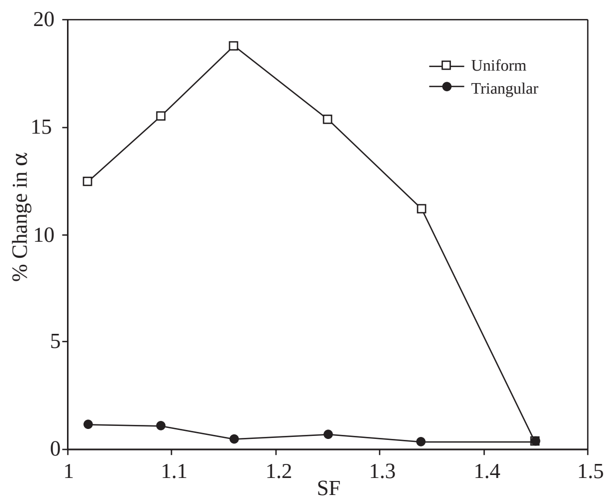

Figure 4. Sensitivity analysis for the effect of PDF.

Conclusions

A reliability-based assessment of erodible channel capacity based on the equation proposed by Maza Alvarez and Echavarrio Alforo, which can be used for a single degree of freedom, has been carried out. Possible sources of uncertainties involved in erodible channel design based on this equation have been interpreted. An example is given to illustrate the use of the AFOSM formulation and the Monte Carlo simulation technique. The results obtained using the statistical information presented in Table 1 were found to be in relatively good agreement for both approaches. Sensitivity analyses were carried out to observe the effects of coefficient of variation and probability distribution. The results of the analysis showed that a 100% increase of the coefficient of variation of flow depth resulted in changes in the reliability values up to 10% because of the greater power of the flow depth compared to the other variables.

Table 2. Iterations for AFOSM for Q = 80 m^3/s.

Variable	Initial value	Iteration											
		1	2	3	4	5	6	7	8	9	10	11	12
β	1.0	4.821	-0.132	1.134	0.594	0.834	0.730	0.775	0.755	0.764	0.760	0.762	0.761
α_1	-0.5	-0.544	-0.445	-0.575	-0.536	-0.549	-0.542	-0.545	-0.544	-0.544	-0.544	-0.544	-0.544
α_2	-0.5	-0.077	-0.070	-0.088	-0.083	-0.084	-0.083	-0.084	-0.083	-0.084	-0.084	-0.084	-0.084
α_3	-0.7	-0.016	-0.014	-0.018	-0.017	-0.017	-0.017	-0.017	-0.017	-0.017	-0.017	-0.017	-0.017
α_4	0.1	0.836	0.893	0.814	0.840	0.832	0.836	0.834	0.835	0.835	0.835	0.835	0.835
x_4^*	-	111.84	78.29	86.81	83.93	85.42	84.80	85.08	84.96	85.01	84.99	85.00	85.00
μ_4'	80	73.8	79.6	79.3	79.5	79.4	79.4	79.4	79.4	79.4	79.4	79.4	79.4
σ_4'	8.0	11.16	7.81	8.66	8.38	8.52	8.46	8.49	8.48	8.49	8.48	8.48	8.48

Negligibly small variations were obtained in the reliability for a 100% increase of the coefficients of variations of channel surface width and bed material size. Furthermore, triangular and uniform probability distributions were also tested. The results obtained using the triangular distribution agreed well with the initial computations. For smaller safety factors, uniform distribution yielded changes in reliability of up to 18%.

Nomenclature

D_{50}	median sediment size
D_{75}	characteristic sediment size
g(z)	performance function
SF	safety factor
SM	safety margin
Q	discharge
Q_c	channel capacity
S_0	bed slope
W	surface width of the channel
X	loading
Y	resistance
y	flow depth
Z	failure point
z	reduced variate
α	reliability
α_i	indexes in the AFOSM formulation
β	index in the AFOSM formulation
β_r	reliability index
μ	mean
σ	standard deviation
Ω	coefficient of variation
Ω_r	coefficient of variation of reliability
Φ	standard normal distribution function, and
ϕ	probability density ordinate.

References

Ang, A.H.S. and Tang, W.H. Probability Concepts in Engineering Planning and Design: Vol. 2, Wiley, New York, 1984.

Blench, T. Regime Behavior of Canals and Rivers, Butherworths Scientific Publications, London. 1957.

Chow, V.T. Open Channel Hydraulics, McGraw Hill, New York, 1959.

Chow, V.T., Maidment, D.R. and Mays, L.W. Applied Hydrology. McGraw Hill International Editions, Singapore, 1988.

Cornell C.A. "A Probability Based Structural Code", Journal of ACI, 66: 975-985, 1969.

Hasofer, A.M. and Lind, N.C. "An Exact and Invariant First Order Reliability Format", Journal of Engineering Mechanics, ASCE, 100(EM-1): 111-121, 1974.

Hong, H.P. "Simple Approximations for Improving Second Order Reliability Estimates", Journal of Engineering Mechanics, ASCE, 125: 592-595, 1999.

Johnson, P.A. "Uncertainty of Hydraulic Parameters", Journal of Hydraulic Engineering, ASCE, 122: 112-114, 1996.

Johnson, P.A. "Fault Tree Analysis of Bridge Failure due to Scour and Channel Instability", Journal of infrastructure systems, ASCE, 5:35-41, 1999.

Kennedy, R.G. "The Provision of Silting in Irrigation Canals," Proc. No:2826, Proc. ICE, Vol. 119, London, 1895.

Lacey, G. "Stable Channels in Alluvium," Paper No: 4736, Min. Proc. ICE, Vol. 229, London, 1930.

Lacey, G. "Sediment as a Factor in the Design of Unlined Irrigation Canals," Q20, R.12, 6^{th} Congress of ICID, Delhi, 1966.

Lindley, E.S. "Regime Channels," Proc. Punjab Engrg. Congress, 7, 1919.

Mays, L.W. and Tung, Y.K. Hydrosystems Engineering and Management. McGraw Hill Book Co., New York, 1992.

Maza Alvarez, J.A. and Echavarria Alfaro, F.J. "Contribution to the Study of General Scour", Proc., International Symposium on River Mechanics, IAHR, Bangkok, Thailand, 795-803, 1973.

Melchers, R.E. Structural Reliability Analysis and Prediction, John Wiley and Sons, Chichester, 2002.

Paloheimo, E. and Hannus, H. "Structural Design Based on Weighted Fractiles", Journal of Structural Division, ASCE, 100(ST-7), 1367-1378, 1974.

Rackwitz, N. and Fiessler, B. "Structural Reliability under Combined Random Load Sequences", Computers and Structures, 9, 489-494, 1978.

Ranganathan, R. Reliability Analysis and Design of Structures, Tata McGraw Hill, New Delhi, 1990.

Simons, D.B. and Albertson, M.L. "Uniform Water Conveyance Channels in Alluvial Material," Trans. ASCE, Vol. 128, Part 1, 1963.

Yanmaz, A.M. "Overtopping Risk Assessment in River Diversion Facility Design", Canadian Journal of Civil Eng. 27, 1-8, 2000.

Yanmaz, A.M. "Dynamic Reliability in Bridge Pier Scouring", Turk. J. Eng. and Env. Sci., 26, 367-375, 2002.

Yanmaz, A.M. "Reliability Simulation of Scouring Downstream of Outlet Facilities", Turk J. Engin and Environ Sci.,, 27, 65-71, 2003.

Yanmaz, A.M. and Çiçekdağ, Ö. "Composite Reliability Model for Local Scour around Cylindrical Bridge Piers", Canadian Journal of Civil Eng. 28, 520-535, 2001.

Yanmaz, A.M. and Üstün, İ. "Generalized Reliability Model for Local Scour Around Bridge Piers of Various Shapes", Turk. J. Engin. Environ. Sci., 25, 687-698, 2001.

Yen, B.C., Cheng, S.T. and Melching, C.S. Stochastic and Risk Analysis in Hydraulic Engineering. Water Resources Publications, Littleton, Colorado, 1986.