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Analysis of Viaduct Road Vibrations Using Cluster Filtering and Actuation

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Abstract

An infinite number of modes of distributed-parameter structures cause spillover destabilization creating problems in controlling vibrations. This paper introduces cluster filtering and actuation in the analysis of such systems to solve these problems. In this study, the cluster filtering method is proposed where all eigenfunctions are divided into finite number of groups having the same characteristics. The reciprocity in sensing and actuation is also verified. All these theoretical approaches are demonstrated on both flat and curved viaduct road models. This analysis also includes the FEM study of road models.

Key words: Distributed-parameter structures, Spillover effect, Cluster filtering and actuation, Viaduct road.

Introduction

The trend in highway road construction is toward steel structures instead of concrete because of their lightweight and resistance to earthquakes. One example of this was the Hanshin Great Earthquake that caused extension damage greater than expected on concrete highways. On the other hand, when the earthquake resistance of a steel road structure is increased, vibration and low frequency noise on such structures becomes a major problem due to flexibility. There is an increasing demand, especially in populated areas, to suppress vibration and noise on highway roads to reduce noise-related environmental pollution (Sivrioglu *et al.*, 1999).

Many research studies concerning distributed parameter structures have been conducted to find a reliable as well as simple and practical control approach that fits real systems. Unfortunately, most control approaches have reliability and robustness problems when applied to real structures. One of the reasons is that there is a spillover effect, which occurs in the control of distributed systems due to distortion of some portion of higher order modes inside infinite number of structural modes. The cluster control approach concerned with distributed parameter structures introduces a complete solution to the insufficient observation of spillover caused by lack of sensor measurements and control of spillover caused by incomplete actuation in distributed parameter structures (Kikushima *et al.*, 2000a).

Cluster control proposes a novel active vibration control that can suppress all structural modes belonging to the targeted groups with simple control methods. It is known that the eigenfunctions of a planar structure can be expressed as the (m,n) mode using modal indices m and n (Tanaka and Kikushima, 1998).

In order to represent a real viaduct road model, the plate is connected to the carrier columns by the end points of beams as pinned-pinned supports. In this research, it is demonstrated that modal separation and modal actuation of structural modes of a flat road using cluster filtering and cluster actuation depend on existing modal symmetry axes. Every eigenfunction of a two-dimensional structure can be expressed in a form of the "(m,n) modes", m and n being modal indices. Simulation results obtained using the FEM model of a flat road show only the certain mode groups such as odd/odd, even/odd, odd/even, and even/even ones. When studying a curved viaduct road, it is possible to classify mode groups as odd/odd or even and even/even or odd.

Mathematical Modeling of the Distributed Parameter System

The model of the viaduct road is presented in Figure 1. The model used is taken from those connecting the side roads to the main roads in Nagoya 1/10 scale. The road consists of a flat part and a quarter curved part. In this study, curved and flat parts are considered separately (Kikushima *et al.*, 2000a,b).

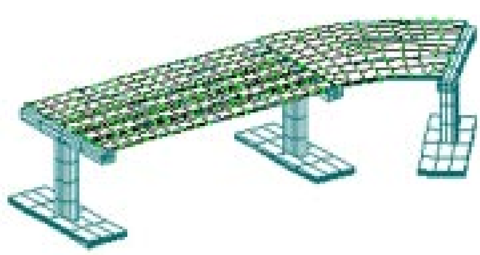

Figure 1. Viaduct road model.

Flat viaduct road part

Figure 2 shows the model of the flat road part. The dimensions of the plate and beams are $1.8 \times 0.9 \times 0.006$ m and $1.8 \times 0.02 \times 0.004$ m, respectively. These beams are connected to the carrier columns by the end points with pinned-pinned supports.

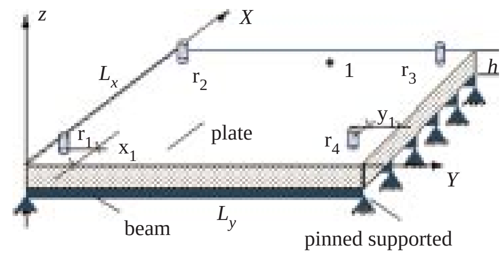

Figure 2. Flat road part.

The partial differential equation governing the motion of the flat road model can be written as:

$$m(\vec{x})\ddot{w}(\vec{x},t) + 2\xi\kappa^{1/2}\dot{w}(\vec{x},t) + \kappa w(\vec{x},t) = F(\vec{x},t) \tag{1}$$

Here, $w(\vec{x},t)$,$\dot{w}(\vec{x},t)$ and $\ddot{w}(\vec{x},t)$ are the displacement, velocity and acceleration of the structure. In addition, $m(\vec{x})$ is the mass density, $F(\vec{x},t)$ is the distributed force, and κ is a differential operator (Rudolph, 1994). The eigenvalue problem associated with differential equations is

$$\kappa\psi_n = \lambda_n m \psi_n \tag{2}$$

where λ_n is the nth eigenvalue, and ψ_n is the associated eigenfunction (mode shape function). In general, eigenvalues are related to the undamped natural frequency by

$$\lambda_n = \omega_n^2 \tag{3}$$

The displacement on the structure is expressed as

$$w(\vec{x},t) = \sum_{i=1}^{\infty} \psi_i(\vec{x}) w_i(t) \tag{4}$$

where $w_i(t)$ is the amplitude of the ith mode at time t, and $\psi_i(\vec{x})$ is value of associated mode shape function at location $\vec{x}$ on the structure (Meirovitch, 1990). Since the damping effect of the aluminum structural system in this study is too small, it will be neglected.

Curved viaduct road part

The curved viaduct road part is shown in Figure 3. It is supported by six beams Figure 3.

These beams are connected to the carrier columns by the end points with pinned-pinned supports (Kikushima *et al.*, 2000b). The equation of motion for a circular plate using cylindrical coordinates r, φ, z is given as follows:

$$\frac{\partial^2 w}{\partial t^2} + a^2 \nabla^2 \nabla^2 w = \frac{F(r,\varphi,t)}{\rho h} r, \varphi, z \tag{5}$$

$$\frac{\partial^2 w}{\partial t^2} + a^2 \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}\right)^2 w = \frac{F(r,\varphi,t)}{\rho h} \tag{6}$$

The same equations can be written in the form

$$\begin{aligned} &M(r,\varphi)\ddot{w}(r,\varphi,t) + 2\xi\kappa^{1/2}\dot{w}(r,\varphi,t) + \\ &\kappa w(r,\varphi,t) = F(r,\varphi,t) \end{aligned} \tag{7}$$

where $w(r,\varphi,t)$, $\dot{w}(r,\varphi,t)$ and $\ddot{w}(r,\varphi,t)$ are the displacement, velocity and acceleration of the structure respectively. In addition, $M(r,\varphi)$ is the mass density,$F(r,\varphi,t)$ is the distributed force, and κ is a differential operator (Rudolph,1994). The displacement on the structure is expressed as

$$w(r,\varphi,t) = \sum_{i=1}^{\infty} \psi_i(r,\varphi) w_i(r,\varphi,t) \tag{8}$$

where $\psi_i(r,\varphi)$ and $w_i(r,\varphi,t)$ denote the eigenfuctions and modal amplitudes, respectively. The analytical study of vibrations of a plate simply supported at two ends and free at the other two ends is very complex. Therefore, the solution is achieved numerically using a computer for finite element analysis so that the modes and mode shapes of the system can be calculated easily.

Classification of the Modes

The main idea in cluster filtering and actuation is to group infinite number of modes of the system according to their natural characteristics such as geometry and boundary conditions and to control one of the mode groups without affecting the others. There are three basics in cluster control. These are cluster sensing, cluster actuation and to control the desired mode group using a control method (Kikushima *et al.*, 2000a,b).

Classification of the modes of the flat road part

The cluster control approach begins with the derivation of cluster filtering equations based on symmetrical sensor points on the distributed parameter system (Figure 2) such as

$$\begin{array}{ll} r_1 = (x_1, y_1) & L_x = 0.9\ m \\ r_2 = (L_x - x_1, y_1) & L_y = 1.8\ m \\ r_3 = (L_x - x_1, L_y - y_1) & x_1 = 0.09\ m \\ r_4 = (x_1, L_y - y_1) & y_1 = 0.18\, m \end{array} \tag{9}$$

Utilizing these symmetrical points on the structure, four different kinds of modes such as odd/odd, even/odd, odd/even and even/even may be classified as follows:

$$\begin{array}{l} \psi^{o/o}(r_1) = \psi^{o/o}(r_2) = \psi^{o/o}(r_3) = \psi^{o/o}(r_4) \\ \psi^{e/o}(r_1) = -\psi^{e/o}(r_2) = -\psi^{e/o}(r_3) = \psi^{e/o}(r_4) \\ \psi^{o/e}(r_1) = \psi^{o/e}(r_2) = -\psi^{o/e}(r_3) = -\psi^{o/e}(r_4) \\ \psi^{e/e}(r_1) = -\psi^{e/e}(r_2) = \psi^{e/e}(r_3) = -\psi^{e/e}(r_4) \end{array} \tag{10}$$

Finally, the sensor output for each mode group may be found by summing the modal amplitudes at each sensor point. If point force actuators are placed instead of sensors at the symmetrical points on the distributed structure, a cluster actuation method, which excites only the certain mode groups such as odd/odd, even/odd, odd/even, and even/even, becomes possible. In this case, the force amplitudes and signs are taken as

$$\begin{array}{ll} F_1 = F_2 = F_3 = F_4 = 1 & \text{for odd/odd} \\ F_1 = -F_2 = -F_3 = F_4 = 1 & \text{for odd/even} \\ F_1 = F_2 = -F_3 = -F_4 = 1 & \text{for even/odd} \\ F_1 = -F_2 = F_3 = -F_4 = 1 & \text{for even/even} \end{array} \tag{11}$$

Using the force relations above, cluster actuation and sensing are verified using FEM analysis.

Classification of the modes of the curved road part

The cluster control approach begins with the derivation of cluster filtering equations based on symmetrical sensor points on the distributed parameter system such as

$$\begin{array}{lll} x_1 = (r_1, \varphi_1) & r_1 = 0.99\ m & \varphi_1 = 9^o \\ x_2 = (r_2, \varphi_2) & r_2 = 1.71\ m & \varphi_2 = 9^o \\ x_3 = (r_3, \varphi_3) & r_3 = 0.99\ m & \varphi_3 = 36^o \\ x_4 = (r_4, \varphi_4) & r_4 = 0.171\ m & \varphi_4 = 36^o \end{array} \tag{12}$$

Utilizing these symmetrical points on the structure, two different kinds of modes such as odd/odd-even and even/even-odd may be classified as follows:

$$\left.\begin{array}{l} \psi^{o/o-e}(r_1) = \psi^{o/o-e}(r_3) \\ \\ \psi^{o/o-e}(r_2) = \psi^{o/o-e}(r_4) \end{array} \quad \text{or} \right\} \text{ for odd/odd–even}$$

$$\left.\begin{array}{l} \psi^{e/e-o}(r_1) = -\psi^{e/e-o}(r_3) \\ \\ \psi^{e/e-o}(r_2) = -\psi^{e/e-o}(r_4) \end{array} \quad \text{or} \right\} \text{ for odd/odd–odd} \tag{13}$$

Finally, the sensor output for each mode group may be found by summing the modal amplitudes at each sensor point. If point force actuators are placed instead of sensors at the symmetrical points on the distributed structure, a cluster actuation method, which excites only certain mode groups such as odd/odd-even and even/even-odd, becomes possible. In this case, the force amplitudes and signs are taken as

$$\left.\begin{array}{l} F_1 = F_3 \\ \\ F_2 = F_4 \end{array} \quad \text{or} \right\} \text{ for odd/odd–even}$$

$$\left.\begin{array}{l} F_1 = -F_3 \\ \\ F_2 = -F_4 \end{array} \quad \text{or} \right\} \text{ for odd/odd–even} \tag{14}$$

Simulation

In this study, the road model used for FEM analysis and simulation consists of a flat aluminum plate supported by six beams underneath (Figure 1). While performing FEM analysis, the Nastran package program is used. Beams are fixed to the columns as pined-pined supports. It is observed that the system has 16 critical frequencies under 200 Hz. If sensor output is obtained from point r_1 when a force is applied to point 1 on the plate in Figure 2, the frequency response of the flat plate showing all modes under 200 Hz is produced as Figure 4.

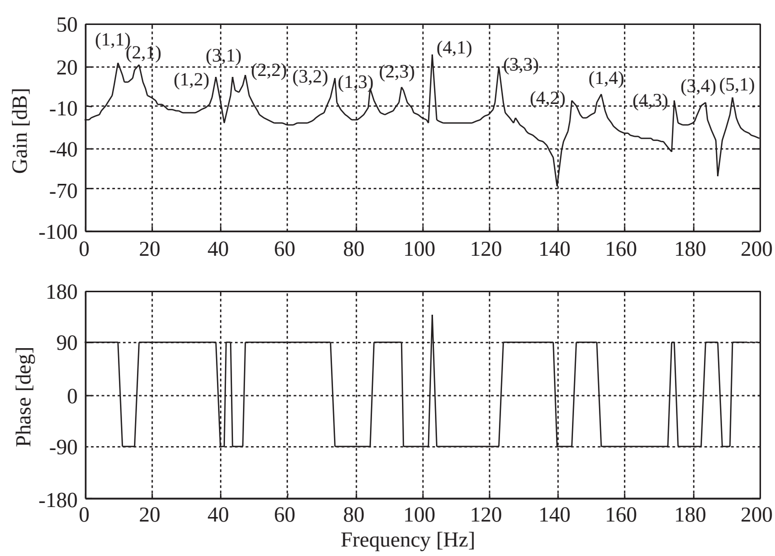

Figure 4. All modes under 200 Hz.

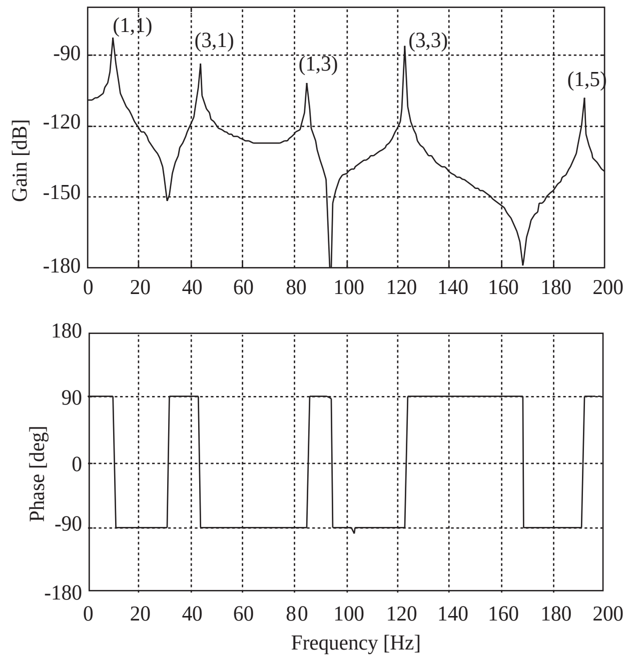

Figure 5. Odd/odd modes under 200 Hz.

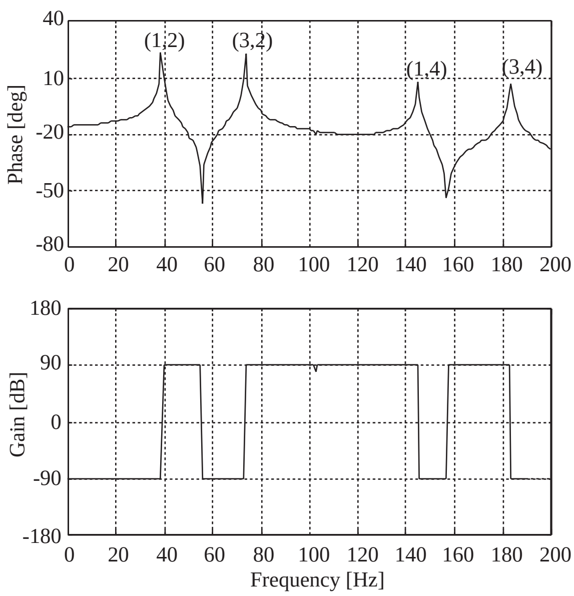

Figure 6. Odd/even modes under 200 Hz.

If the measurements are taken at four symmetrical points in Figure 2, only odd/odd modes are obtained. For other modes (odd/even, even/odd and even/even), the signs in Eq. (10) are used. The same application is true for the forces as shown in Eq. (11). If four forces are applied to the system all in the same direction and a frequency plot is obtained, only odd/odd modes are witnessed whereas others are absent, as shown in Figure 5. By changing the signs, i.e. the directions of forces in Eq. (11), other mode groups could be demonstrated. Thus, it can be shown for a flat plate infinite number of modes of the system can be classified into four groups. The plots demonstrating odd/even, even/odd and even/even modes are shown in Figures 6, 7 and 8, respectively.

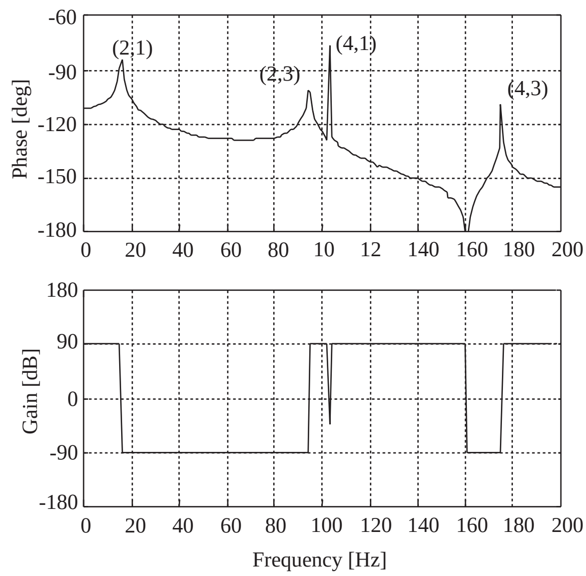

Figure 7. Even/odd modes under 200 Hz.

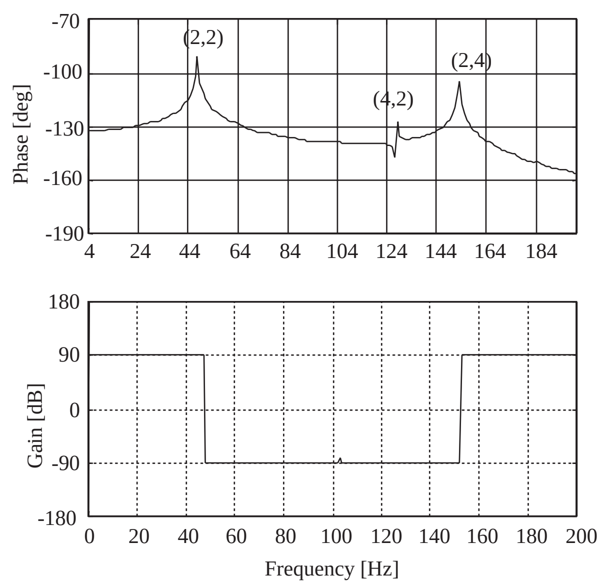

Figure 8. Even/even modes under 200 Hz.

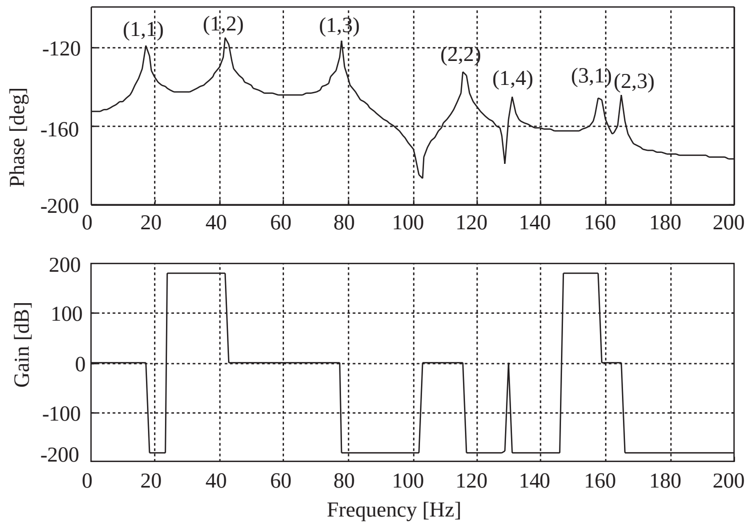

Figure 9. Curved viaduct road modes under 200 Hz.

Since there is symmetry only one direction of the curved plate, modes can be classified into two groups as odd/odd-even and even/even-odd. The frequency response of the curved plate showing all modes under 200 Hz is presented in Figure 9.

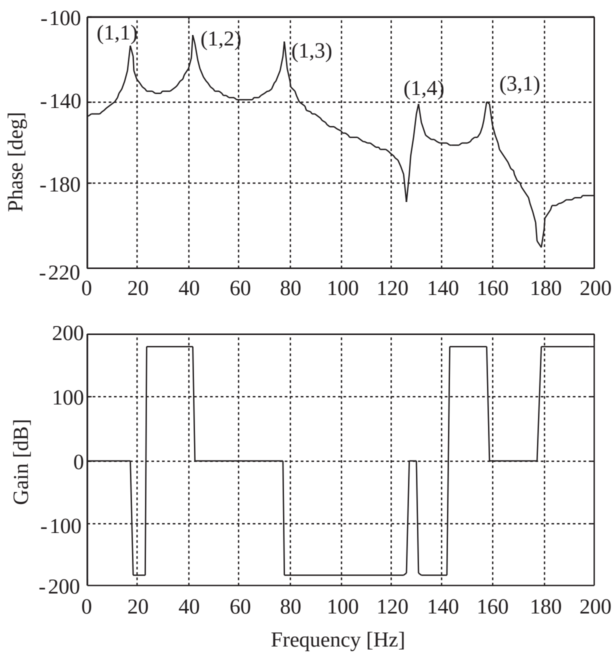

Figure 10. Odd/odd-even modes under 200 Hz.

If a changing force depending on frequency is applied at point r_1 in Figure 3 and magnitudes are estimated at r_3 and r_4 are summed, it is observed that only odd/odd-even modes are active, as shown in Figure 10. If r_3 is considered positive and r_4 negative, then only even/even-odd modes are active, as shown in Figure 11.

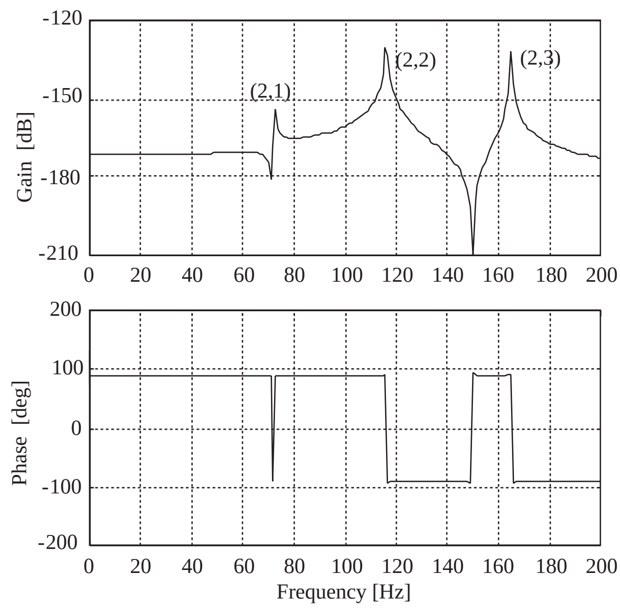

Figure 11. Even/even-odd modes under 200 Hz.

Conclusion

It is demonstrated that the modal separation and modal actuation of structural modes of a flat and curved road using cluster filtering and cluster actuation depend on the existing modal symmetry axes. Simulation results obtained using the FEM model of a flat road show odd/odd, odd/even, even/odd and even/even due to having two symmetrical axes. Simulation results obtained using the FEM model of a curved road show only the major odd/odd-even and even/even-odd modes due to having one symmetrical axis. Consequently, it is verified that the modes belonging to viaduct roads can be classified in certain groups depending on the geometrical symmetry axis of the model. One of the important problems in the vibration control of continuous systems is the spillover effect of other mode groups while controlling a certain mode group. To prevent this problem, it is possible to classify this mode group so that the other groups will not be affected. In this study, first, the grouping of the modes is introduced. It has been shown that the cluster filtering method allows four mode groups in a flat viaduct road and two mode groups in a curved road. Since this grouping approach is also possible in cluster actuation, the calculated control forces using the control algorithm results in controlling the related mode groups.

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